

Aga Khan University Examination Board

Notes from E-Marking Centre on SSC II Mathematics Examination May 2016

Introduction

This document has been produced for the teachers and candidates of SSC Part II (Class X) Mathematics. It contains comments on candidates' responses to the 2016 Secondary School Certificate (SSC-II) Examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E- Marking Notes

This includes overall comments on candidates' performance on every question and some specific examples of candidates' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that require candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

Candidates need to be aware that the marks allocated to the questions are related to the answer space provided on the examination paper as a guide to the length of the required response. A longer response will not in itself lead to higher marks. Candidates need to be familiar with the command words in the Student Learning Outcomes which contain terms commonly used in examination questions. However, candidates should also be aware that not all questions will start with or contain one of the command words. Words such as 'how', 'why' or 'what' may also be used.

Detailed Comments:

Question 1a

Find the highest common factor (H.C.F.) and the least common multiple (L.C.M.) of the expressions $y^3 + ay^2 + y^2 + ay$ and $a^3 + a^2y$.

Better responses indicated that candidates had command over the concepts of H.C.F. and L.C.M. they factorise the given expressions and correctly found the H.C.F. and L.C.M. First, they factorise the given expressions and correctly applied the concepts of H.C.F. and L.C.M.

Example:

$$\begin{aligned} & \cdot y^3 + ay^2 + y^2 + ay \\ & y^2(y+a) + y(y+a) \\ & (y+a)(y^2+y) \\ \\ & \cdot a^3 + a^2y \\ & a^2(a+y) \\ \\ & \text{H.C.F.} = (a+y) \\ & \text{L.C.M.} = a^2(a+y)(y^2+y) \\ & \quad \swarrow \quad \text{or} \\ & a^2y(a+y)(y+1) \end{aligned}$$

Weaker responses showed candidates had lack of understanding of the concepts of L.C.M. and H.C.F. and made mistakes in finding the L.C.M. and H.C.F. of the given algebraic expressions.

In few responses, factorisation was done as describe below.

$$y^3 + ay^2 + y^2 + ay = y \times y \times y + a \times y \times y + y \times y + a \times y$$

$$a^3 + a^2y = a \times a \times a + a \times a \times y$$

And then, H.C.F. and L.C.M. were written as $a \times y = ay$ and a^3y^2 respectively.

In few other responses, it was noted that after factorisation, the candidates added the factors instead of multiplying them to find the L.C.M.

Other weaker responses exhibited that candidates failed to factorise the given expressions and consequently, were unable to find the H.C.F. and L.C.M. In few other responses, candidates opted for the division method but failed to complete the process.

Example 1:

Handwritten work for Example 1:

$$y^3 + ay^2 + y^2 + ay = y(y \cdot y + a \cdot y + y + a)$$

$$a^3 + a^2y = a(a \cdot a + a \cdot y)$$

H.C.F. = $ay + a$ L.C.M. = ay

L.C.M. = $y^2 + a^2 + y + a + y$

Example 2:

Handwritten work for Example 2:

a) HCF

$$y^3 + ay^2 + y^2 + ay$$

$$= y \cdot y \cdot y + a \cdot y \cdot y + y \cdot y + a \cdot y$$

$$a^3 + a^2y$$

$$= a \cdot a \cdot a + a \cdot a \cdot y$$

HCF = a^2y

- LCM

common \times uncommon

$$= a^2y \times y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot a \cdot a \cdot a$$

$$= a^2y \times y^7 \times a^3$$

$$= a^5y^8$$

Question 1b

Find the value of b if the expression $4x^4 + 20x^3 + 33x^2 + 20x + b$ is a perfect square.

Better responses displayed that candidates confidently performed the division method to find square root of a given algebraic expression and finally compared the remainder with zero to find the value of b .

Example:

Finding square root:		for the expression to be a perfect square, $-4+b=0$ $-4+b=0$ $\Rightarrow b=+4$
$2x^2$	$4x^4 + 20x^3 + 33x^2 + 20x + b$	
$+ 2x^2$	$-4x^4$	
$4x^2 + 5x$	$20x^3 + 33x^2 + 20x + b$	
$+ 5x$	$(-) -20x^3 + 25x^2$	
$4x^2 + 10x + 2$	$8x^2 + 20x + b$	$b = +4$
$+ 2$	$(-) -8x^2 + 20x + 4$	
$4x^2 + 10x + 4$	$-4 + b$	

Weaker responses reported that candidates started correctly but failed in the middle of the process and made mistakes in subtraction of expressions or in writing the signs of the terms. Eventually, they failed to find the value of b correctly. In fewer responses, it was noted that candidates started with factorisation method, which was not a good choice with presence of an unknown in the given expression.

Example:

	$2x^2 + 5x^3 + 13x^2$
$2x^2$	$4x^4 + 20x^3 + 33x^2 + 20x + b$
$2x^2$	$4x^4$
$4x + 5$	$+ 20x^3 + 33x^2 + 20x + b$
5	$\pm 20x^3 \quad \pm 20x$
$4x + 10 + 3$	$+ 33x^2 + b$
	$\pm 33x^2$
	$+ b = 0$
	$= 2x^2 + 5x^3 + 13x^2$
	<i>Ans.</i>

Question 2

In resolving $\frac{3x-2}{(2x-1)(x+1)} = \frac{A}{(2x-1)} + \frac{B}{x+1}$ into its partial fraction, find the value of B .

Better responses exhibited that the candidates correctly multiplied both sides by L.C.M. $(2x-1)(x+1)$, substituted $x+1=0$ and simplified it correctly to find the value of B .

Example:

Multiplying both sides by $(2x-1)(x+1)$	$\Rightarrow -5 = B(-3)$
$3x-2 = A(x+1) + B(2x-1)$	$\Rightarrow -5 = -3B$
Taking $x+1=0$	$\Rightarrow 5 = 3B$
$\therefore x = -1$	$\Rightarrow 3B = 5$
$3(-1)-2 = A(0) + B\{2(-1)-1\}$	$\Rightarrow \boxed{B = \frac{5}{3}}$
$\Rightarrow -3-2 = 0 + B\{-2-1\}$	

Weaker responses reported that candidates made different types of mistakes, which included incorrect calculation of L.C.M., mistakes in multiplication on both sides by L.C.M. and instead of substituting $x+1=0$, they substituted $2x-1=0$ and eliminated B . In few other responses it was noted that candidates correctly wrote $3x-2 = A(x+1) + B(2x-1)$, but started using their own values by trial and error method to find the value of B .

Example 1:

$\frac{3x-2}{(2x-1)(x+1)} = \frac{A}{(2x-1)} + \frac{B}{x+1}$	
$3(1)-2 = B(1+1)$	$2x-1$
$3-2 = 2B$	$x=1$
$1 = B \cdot 2$	
$\boxed{B = \frac{1}{2}}$	

Example 2:

$$\frac{3n-2}{(2n-1)(n+1)} \times (2n-1)(n+1) = \frac{A \times (2n-1)(n+1)}{(2n-1)} + \frac{B \times (2n-1)(n+1)}{(n+1)}$$

$$3n-2 = A(n+1) + B(2n-1)$$

$$3n-2 = A\left(\frac{1}{2}+1\right) + B(2(-1)-1)$$

$$3n-2 = A\left(\frac{3}{2}\right) + B(-3)$$

$$\frac{3n-2}{(2n-1)(n+1)} = \frac{A}{(2n-1)} + \frac{B-3}{n+1} \quad \therefore B = -3$$

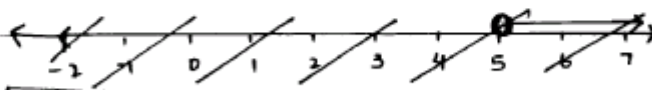
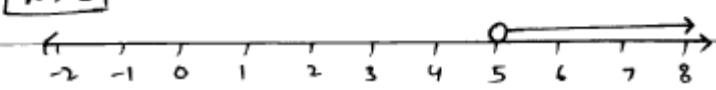
Question 3a

Find the solution of the following linear inequality and represent its solution on the number line.

$$\frac{3+x}{2} < \frac{5x-13}{3}, \text{ where } x \in R.$$

Better responses displayed that candidates multiplied both sides by L.C.M., performed simplification process correctly to get $x > 5$ and properly represented the solution on the number line.

Example:

$\Rightarrow \frac{3+x}{2} < \frac{5x-13}{3}$	Verification..
$\Rightarrow 3(3+x) < 2(5x-13)$	$\frac{3+5}{2} < \frac{25-13}{3}$
$\Rightarrow 9+3x < 10x-26$	$\frac{48}{12} < \frac{12}{12}$
$\Rightarrow 3x-10x < -26-9$	$4 < 6$
$\Rightarrow -7x < -35$	S.S. = $\{x/x \in R \wedge x > 5\}$
$\Rightarrow 7x > 35$	
$\Rightarrow x > \frac{35}{7}$	$x > 5$
$\Rightarrow x > 5$	

Weaker responses exhibited that candidates failed to solve given inequality. In some responses, the candidates converted the inequality to a linear equation and wrote $9 + 3x = 12x$ and $10x - 13 = -3x$. In few other responses, candidates wrote $-26 - 9 = +35$. One more mistake noted was that on simplification of $-7x < -5$ candidates wrote $7x < 5$. The weaker responses also exhibited that candidates failed to represent the solution on the number line. Few other mistakes can be noted in the following example.

Example:

$\Rightarrow \frac{3+x}{2} < \frac{5x-13}{3}$
By taking L.C.M
$\Rightarrow \frac{3 \times 3+x}{2} < \frac{5 \times 5x-13}{3}$
$\Rightarrow 9+3x < 10x-26$
$\Rightarrow 3x-10x < -26-9$
$-7x < -35$
$x < \frac{-35}{-7}$
$x < 5$

Question 3b

Find the solution set of the equation $\frac{|2x-4|}{5} = 2(3x-1)$, where $x \in R$.

Better responses showed that candidates correctly applied the concept of modulus to find the solution set of the given equation and solved the problem systematically.

Example:

$\frac{ 2x-4 }{5} = 2(3x-1) \Rightarrow \frac{ 2x-4 }{5} = 6x-2$	
$ 2x-4 = 5(6x-2)$	
$ 2x-4 = 30x-10$	
$2x-4 = \pm(30x-10)$	
$2x-4 = -30x+10$	$2x-4 = +30x-10$
$2x+30x = 10+4$	$2x-30x = -10+4$
$32x = 14$	$-28x = -6$
$x = \frac{14}{32} = \frac{7}{16}$	$x = \frac{-6}{-28} = \frac{3}{14}$
$x = \frac{7}{16}$	$x = \frac{3}{14}$
Solution Set = $\left\{ \frac{7}{16}, \frac{3}{14} \right\}$	

Weaker Responses displayed that candidates were having confusion in applying the concept of modulus. The weaker responses also showed mistakes in performing algebraic operation. For example, they wrote $(6x-2) \times 5 = 6x-10$ and similarly, $10(3x-1) = 30x-1$, etc. Few other mistakes can also be noted in the examples cited below.

Example 1:

$\frac{ 2n-4 }{5} = 2(3n-1)$	
$\frac{ 2n-4 }{5} = \pm 2(3n-1)$	
$\frac{2n-4}{5} = -2(3n-1)$	$\frac{2n+4}{5} = +2(3n+1)$
$\frac{-2n}{5} = \frac{-26n-2}{5}$	$\frac{2n}{5} = \frac{6n+2}{5}$
$2n = 34n$	$2n = 8n$
$5 = 4n$	$5 = 8n$
$\frac{2n}{5} = \frac{5}{2}$	$\frac{2n}{5} = \frac{5}{2}$

Example 2:

$\frac{ 2x-4 }{5} = 2(3x-1)$	$\frac{2x-4}{5} = -2(3x-1)$
	$2x-4 = -10(15x-5)$
$ 2x-4 = 10(15x-5)$	$2x-4 = -150x+50$
$2x-4 = 150x-50$	$2x+150x = 4-50$
$2x-150x = 4-50$	$152x = 46$
$-148x = -46$	$x = \frac{46}{152} = \frac{23}{76}$
$x = \frac{-46}{-148}$	$S.S = \left\{ \frac{12}{37}, \frac{23}{76} \right\}$
$x = \frac{46}{148} = \frac{12}{37}$	

Question 4

For the following system of simultaneous linear equations, find the value of y.

$$x + 2y = 159$$

$$2x + y = 147$$

Better responses displayed that mostly candidates applied the elimination method to solve the given system of linear equation to find the value of y. But some candidates also used substitution method and comparison method to solve the problem. In better responses, candidates got the value of x from equation $x + 2y = 159$ and substituted it in equation $2x + y = 147$ to find the value of y.

Example 1:

$$\begin{aligned}x + 2y &= 159 && \text{--- (i)} \\2x + y &= 147 && \text{--- (ii)}\end{aligned}$$

From eq (i), $y = \frac{171}{3}$

$$x = 159 - 2y \text{ --- (iii)}$$

From eq (ii) $y = 57$

$$x = \frac{147 - y}{2} \text{ --- (iv)}$$

Comparing eq (iii) and eq (iv),

$$159 - 2y = \frac{147 - y}{2}$$
$$2(159 - 2y) = 147 - y$$
$$318 - 4y = 147 - y$$
$$-4y + y = 147 - 318$$
$$-3y = -171$$

Example 2:

$$\begin{aligned}x + 2y &= 159 - \textcircled{i} \\2x + y &= 147 - \textcircled{ii} \\ \text{From eq } \textcircled{i} \\ x &= 159 - 2y - \textcircled{iii} \\ \text{Putting the value of } x \text{ in eq } \textcircled{ii} \\ 2(159 - 2y) + y &= 147 \\ 318 - 4y + y &= 147 \\ 318 - 3y &= 147 \\ -3y &= 147 - 318 \\ -3y &= -171 \\ y &= \frac{-171}{-3} \\ y &= 57.\end{aligned}$$

Weaker Responses displayed various types of confusions, some are listed below.

- Candidates created table and randomly filled the values of x and y
- They wrote $x = 159 - 2y$ and then substituted different values of y
- In simplification the sign mistake is very common error was in adding the alike terms, for example, $x + y = 12$ is transformed as $y = 12x$.
- In multiplying both sides by a number, only left hand side of the equation was multiplied.

Example 1:

$x + 2y - 2x - y = 147$	
$-x - y = 12$	$x + 2y - (2x - y)$
$y = x - 12$	$x + 2y - 2x + y$
$x + 2y = 2x + 12$	$-x - y = 12$
	$-y = 12 + x$
	$y = -12 - x$
	$x + 2(-12 - x)$
	$x + 2(-24 - 2x) = 159$
	$3x = 159 + 24$
	$x = \frac{183}{3} = 61$
	$61 + 2y = 159$
	$2y = 159 - 61$
	$y = 49$

Example 2:

(i) $x + 2y = 159$	(ii) $2x + y = 147$
$y = \frac{159 - x}{2}$	$y = 147 - 2x$
If $x = 0$ then	$\therefore x = 34$ then
$y = \frac{159 - 0}{2}$	$y = 147 - 2(34)$
$y = 79.5$	$y = 79$
$\therefore x = 3$ then	$\therefore x = 34.5$ then
$y = \frac{159 - 3}{2}$	$y = 147 - 2(34.5)$
$y = 78$	$y = 78$
$\therefore x = 5$ then	$\therefore x = 35$ then
$y = \frac{159 - 5}{2}$	$y = 147 - 2(35)$
$y = 77$	$y = 77$

Question 5

Find the solution set of the equation $6x^2 + x - 2 = 0$.

It was generally a well attempted question. Candidates used quadratic formula, factorisation method and, in fewer cases, use of completing square method is also observed.

Better responses reported that candidates were well-versed with the use of different methods to solve quadratic equations for example breaking of middle term method, completing square method or quadratic formula method. Hence, successfully solved and wrote the solution set of the given equation.

Example:

$6x^2 + x - 2 = 0$	
$6x^2 - 3x + 4x - 2 = 0$	
$3x(2x-1) + 2(2x-1) = 0$	
$(2x-1)(3x+2) = 0$	
By splitting the quadratic equation, we get:-	
(i) $2x - 1 = 0$	
(ii) $3x + 2 = 0$	
$2x - 1 = 0$	$3x + 2 = 0$
$2x - 1 + 1 = 0 + 1$	$3x + 2 - 2 = 0 - 2$
$2x = 1$	$3x = -2$
$x = \frac{1}{2}$	$x = -\frac{2}{3}$
Solution Set = $\{\frac{1}{2}, -\frac{2}{3}\}$	

Weaker responses indicated that candidates wrote wrong quadratic formula, did wrong identification of a , b and c or made mistake in simplification after substituting values in the formula. The candidates which used factorisation method, made mistake in breaking of middle term or failed to take the common properly to complete the factorisation process and, consequently, failed to find the solution set.

Example 1:

$$6n^2 + n - 2 = 0$$

$$6n^2 + 4n - 3n - 2$$

$$2n(3n + 2) - 1(3n + 2)$$

$$(2n - 1)(3n + 2)$$

Example 2:

$$a = 6, b = 1, c = -2$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-1 \pm \sqrt{(1)^2 - 4(6)(-2)}}{2(6)}$$

$$\frac{-1 \pm \sqrt{1 + 48}}{12}$$

$$\frac{-1 \pm \sqrt{49}}{12}$$

$$-1 + \frac{7}{12}, \quad -1 - \frac{7}{12}$$

Question 6a

A map is drawn on a Cartesian plane. Town A is located at (4, 3) and town B is located at (10, 11). If a bus travels in a straight line from town A to town B, then find the distance between the two towns in kilometres?
(Note: One map unit equal to one kilometre.)

Most of the candidates opted for part a. Generally it was a well attempted question.

Better responses exhibited that the candidates wrote the correct distance formula, substituted values of the coordinates correctly and were able to find the distance between the two towns.

Example:

to find the distance between town A and town B we will use the distance formula i.e. $d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$$d = \sqrt{|10 - 4|^2 + |11 - 3|^2}$$
$$d = \sqrt{|6|^2 + |8|^2}$$
$$d = \sqrt{36 + 64}$$
$$d = \sqrt{100}$$
$$d = 10 \text{ km.}$$

The distance between the two towns is 10 kilometers if the bus travels in a straight line.

Weaker responses showed that candidates were unable to comprehend the questions and made different types of mistakes. Few are as follows:

- Wrote wrong formula. For example, $|AB| = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$ or $|AB| = \sqrt{(y_2 - x_1)^2 + (x_2 - y_1)^2}$ or $AB = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Made mistake in substitution of values. For example, $|AB| = \sqrt{(11 - 10)^2 + (3 - 4)^2}$
- Made mistakes in simplification. For example, the following working highlights such mistakes $|AB| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2 + (-2)} = \sqrt{-4} = -2$

Example 1:

Using distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Putting values of x & y in formula

$$d = \sqrt{(4 - 10)^2 + (3 - 11)^2}$$

$$d = \sqrt{(-6)^2 + (-8)^2}$$

$$d = \sqrt{36 + 64}$$

$$d = \sqrt{100}$$

The distance covered by the bus is $\sqrt{100}$ km.

Example 2:

$$= A(4, 3) \quad B(10, 11)$$

$$= \left(\frac{x_1 + x_2}{2} \right) \left(\frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{4 + 10}{2} \right) \left(\frac{3 + 11}{2} \right)$$

$$= \left(\frac{14}{2} \right) \left(\frac{14}{2} \right)$$

$$= (7, 7) \text{ kilometers.}$$

Question 6b

If midpoint of a line segment lies at the origin and one of the end points is $(2a, -3b)$, then find the other endpoint of the line segment.

This question was based on the application of mid point formula but it was less attempted. The candidate who attempted this question generally performed well.

Better responses indicated that candidates comprehended the question well and used correct formula and substituted values appropriately to find the coordinates of the other end of the given line segment.

Example:

Midpoint of a line segment = $O(0,0)$
One of the endpoints = $A(2a, -3b)$
The other endpoint = $B(x,y) = ?$

i/ $\frac{2a+x}{2} = 0$ ii/ $\frac{-3b+y}{2} = 0$

$\Rightarrow 2a+x=0$ $\Rightarrow -3b+y=0$
 $\Rightarrow x = -2a$ $\Rightarrow y = 3b$

The other endpoint of the line segment is $B(-2a, 3b)$

Weaker responses displayed that candidates did not read the question carefully, hence, they treated $(0, 0)$ and $(2a, -3b)$ as endpoints and failed to fulfil the requirement of the question. In some responses it is noted that candidates used the distance formula instead of midpoint formula.

Example 1:

b) Mid point of $A(0,0), B(x_2, y_2) = (2a, -3b)$
According to Mid-point formula: $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$
 $(\frac{0+x_2}{2}, \frac{0+y_2}{2}) = 2a, -3b$

$\frac{0+x_2}{2} = 2a$	$\frac{0+y_2}{2} = -3b$	According to this Mid point of $(0,0), (x_2, -6b) = 2a, -3b$
$x_2 = 2a \times 2$	$y_2 = 2(-3b)$	
$x_2 = 4a$	$y_2 = -6b$	
For the other point of line segment is $B(4a, -6b)$		

Example 2:

$$\begin{aligned} \textcircled{b} \quad 0 &= \frac{2a+x}{2} & 0 &= \frac{-3b+y}{2} \\ (-2a)x \quad 2 &= x & (3b)x \quad 2 &= y \\ \boxed{x = -4a} & & \boxed{y = 6b} & \\ \underline{\underline{(x, y)}} &= (-4a, 6b) & & \\ \downarrow & & & \\ &\rightarrow \text{unknown end point of the line segment} & & \end{aligned}$$

Question 7a

Verify that $(1 + \tan^2 \theta)(1 - \cos^2 \theta) = \tan^2 \theta$.

This question offered choice between **part a** and **part b** of the question. Candidates mostly opted **part a**.

The question was based on the application of trigonometric identities to verify $(1 + \tan^2 \theta)(1 - \cos^2 \theta) = \tan^2 \theta$.

Better responses showed that candidates were knowledgeable about trigonometric formulae and appropriately applied these formulae to verify the required identity.

Example:

(a) $(1 + \tan^2 \theta)(1 - \cos^2 \theta) = \tan^2 \theta$
 $\therefore 1 + \tan^2 \theta = \sec^2 \theta$
 $\therefore 1 = \cos^2 \theta + \sin^2 \theta \Rightarrow 1 - \cos^2 \theta = \sin^2 \theta$
 $\rightarrow (\sec^2 \theta)(\sin^2 \theta) = \tan^2 \theta$
 $\therefore \frac{1}{\cos^2 \theta} = \sec^2 \theta$
 $\rightarrow \left(\frac{1}{\cos^2 \theta}\right)(\sin^2 \theta) = \tan^2 \theta$
 $\rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$
 $\rightarrow \left(\frac{\sin \theta}{\cos \theta}\right)^2 = \tan^2 \theta \Rightarrow \therefore \tan \theta = \frac{\sin \theta}{\cos \theta}$
 $\rightarrow (\tan \theta)^2 = \tan^2 \theta$
 $\Rightarrow \tan^2 \theta = \tan^2 \theta, \text{ proved!}$

Weaker responses displayed that candidates wrote wrong formulae and made different types of mistakes, few are listed below:

- $1 - \cos^2 \theta = \sec^2 \theta$
- $\tan^2 \theta = \frac{\sin \theta}{\cos \theta}$
- $1 - \tan^2 \theta = 1 + \sec^2 \theta$
- $\sin^2 \theta = \frac{1}{\cos^2 \theta}$
- $\frac{\sin^2 \theta - \cos^2 \theta}{\cos^4 \theta} = \frac{\sin^2 \theta - \cos \theta}{\cos^2 \theta}$

Example 1:

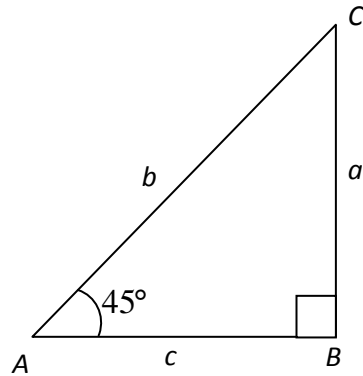
$$\begin{aligned} &\Rightarrow (1 + \tan^2 \theta)(1 - (\tan^2 \theta + 1)) = \tan^2 \theta \\ &\Rightarrow (1 + \tan^2 \theta)(1 - \tan^2 \theta - 1) = \tan^2 \theta \\ &\Rightarrow (1 + \tan^2 \theta)(-\tan^2 \theta) \\ &\Rightarrow \tan^2 \theta - (\tan^2 \theta)(\tan^2 \theta) = \tan^2 \theta \\ &\Rightarrow \tan^2 \theta = \tan^2 \theta = \tan^2 \theta \\ &\Rightarrow \boxed{\tan^2 \theta = \tan^2 \theta} \quad \text{Proved.} \end{aligned}$$

Example 2:

$$\begin{aligned} &\text{Taking L.H.S.} \\ &(1 + \tan^2 \theta)(1 - \cos^2 \theta) = \tan^2 \theta \\ &(\cos^2 \theta) + (\sin^2 \theta) = \tan^2 \theta \\ &\tan^2 \theta = \tan^2 \theta \\ &\text{Proved L.H.S.} = \text{R.H.S.} \quad \text{Q.E.D.} \end{aligned}$$

Question 7b

Using the given figure, calculate the values of $\sin 45^\circ$ and $\tan 45^\circ$ independent of a , b and c .



NOT TO SCALE

Better responses reported that the candidates applied characteristics of isosceles triangle and applied Pythagorean Theorem to develop the relation between a , b and c . Then, they used correct formula of sine and tangent to find the required trigonometric ratio.

Example:

b)		
$\angle A + \angle B + \angle C = 180^\circ$		
$45^\circ + 90^\circ + \angle C = 180^\circ$	by using pythagoras theorem	
$135 + \angle C = 180^\circ$	$P^2 + B^2 = H^2$	$\sin 45^\circ = \frac{P}{H} = \frac{x}{\sqrt{2}x}$
$\angle C = 180^\circ - 135^\circ$	$(a)^2 + (c)^2 = (b)^2$	
$\angle C = 45^\circ$	$(x)^2 + (x)^2 = (b)^2$	$= \frac{1}{\sqrt{2}}$
When two angles in a triangle are equal their opposite sides are also equal that means	$x^2 + x^2 = b^2$	$\sin 45^\circ = \frac{1}{\sqrt{2}}$
	Root on both sides	
$c = x \therefore$ let x be the same sides	$\sqrt{2x^2} = \sqrt{b^2}$	$\tan 45^\circ = \frac{P}{B} = \frac{x}{x}$
$a = x$	$\sqrt{2}x = b$	
$b = ?$	Now, $c = x$	$= 1$
for the value of b	$a = x$	$\tan 45^\circ = 1$
	$b = \sqrt{2}x$	

Weaker responses displayed that candidates directly wrote the value of $\sin 45^\circ$ and $\tan 45^\circ$. In few other responses, it is noted that candidates wrote the values of $\sin 45^\circ$ and $\tan 45^\circ$ in terms of a , b and c without developing relation between these quantities. In certain weaker responses, candidates supposed the values of a and b on their own without any mathematical justification.

Example 1:

Suppose value of $a = 2$ and $b = 2$
By applying pythagoras theorem
$(b)^2 = (a)^2 + (c)^2$
$(b)^2 = (2)^2 + (2)^2$
$(b)^2 = \sqrt{4 + 4}$
$b = 3$
$\sin 45 \Rightarrow \frac{\text{perp}}{\text{Hyp}} \Rightarrow \frac{2}{3}$
$\tan 45 \Rightarrow \frac{\text{Perp}}{\text{Base}} \Rightarrow \frac{2}{2} \Rightarrow 1$

Example 2:

$\sin 45^\circ = \frac{1}{\sqrt{2}}$
$\sin 45^\circ = 0.7071135 \text{ ans}$
$\tan 45^\circ = \sqrt{3}$
$\tan 45^\circ = 1.7320508 \text{ ans}$

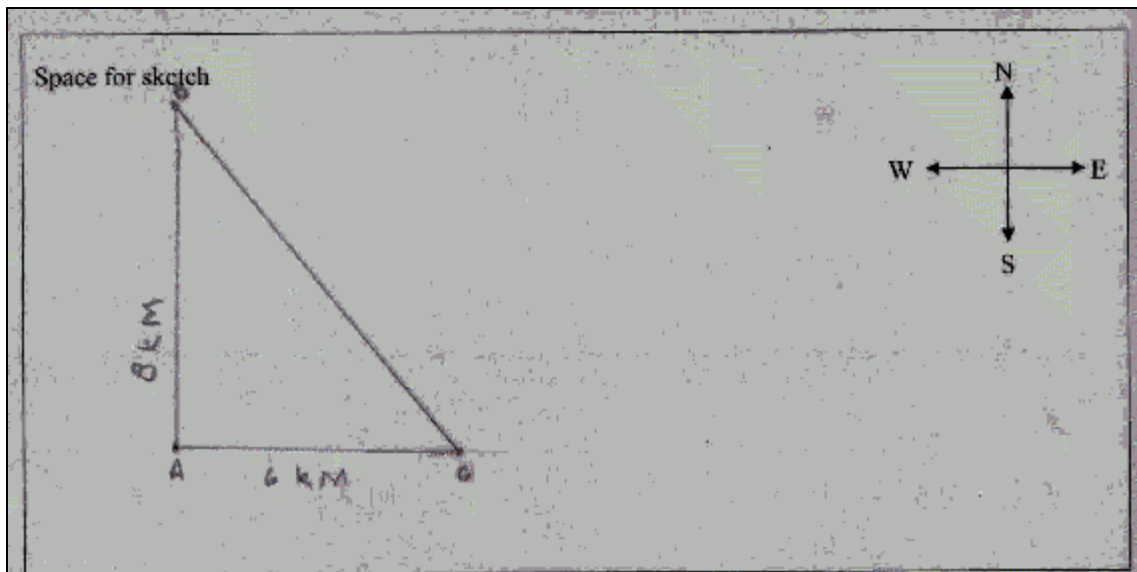
Question 8a

Sara started riding a bicycle from her home and travelled 6 km west to reach a point A. Then she travelled 8 km north and reached a point B. Illustrate the situation by drawing a sketch and calculate the shortest distance between point B and her home.

This is generally a well attempted question which offered a choice between **part a** and **part b**. Candidates mostly attempted **part a** of the question which was based on Pythagorean Theorem and its application.

Better responses exhibited that candidates comprehended the question well. They drew the sketch followed by application of Pythagorean Theorem to find the required shortest distance.

Example 1:



Space for sketch

8 km

6 km

N

W ← → E

S

$$H^2 = B^2 + P^2$$
$$H^2 = (6)^2 + (8)^2$$
$$H^2 = 36 + 64$$
$$H^2 = 100$$
$$\sqrt{H^2} = \sqrt{100}$$
$$H = 10 \text{ km}$$

Distance between point B and home is 10 km.

Weaker responses indicated that candidates started with wrong sketch of the given situation which led to wrong application of Pythagorean Theorem. Consequently, such responses failed to find the required distance. It was also noted that weaker responses failed to supply the unit of the distance.

Example 1:

Space for sketch

Pythagoras theorem.

$$h^2 = b^2 + p^2$$

$$8^2 = b^2 + 6^2$$

$$64 = b^2 + 36$$

$$64 - 36 = b^2$$

$$\sqrt{28} = \sqrt{b^2}$$

$$5.2 = b$$

The shortest distance between point B and her home is 5.2 km.

Example 2

Space for sketch

$$|D| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(6 - 0)^2 + (8 - 0)^2}$$

$$= \sqrt{6^2 + 8^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100}$$

$$|D| = 10$$

Question 8b

The lengths of the sides of a rectangle are 8 cm and 6 cm.

- i. Find the length of its diagonal.
- ii. If the length of the diagonal of this rectangle is the same as that of a square, then find the side of the square.

Better responses indicated that candidates understood the question well and were able to find the length of the diagonal in the first part. While attempting the second part, they correctly wrote the relation $x^2 + x^2 = (10)^2$, where x is the side of the square, and successfully fulfilled the requirement of the question.

Example:

Consider a rectangle ABCD. To find out the length of its diagonal we will apply pythagoras theorem	If a square has the same length of its diagonal as the rectangle then to find out the ^{length} of the other side we will apply pythagoras theorem. Consider the side as x .
$H^2 = P^2 + B^2$	$H^2 = x^2 + x^2$
$H^2 = (6)^2 + (8)^2$	$H^2 = 2x^2$ (the length of sides is equal or same in a ^{square})
$H^2 = 36 + 64$	$(10)^2 = 2x^2$
$H^2 = 100$	$\Rightarrow 100 = 2x^2$
$\sqrt{H^2} = \sqrt{100}$	$\Rightarrow \frac{100}{2} = x^2$
$H = 10$	$\Rightarrow \sqrt{50} = \sqrt{x^2} \Rightarrow \cancel{2\sqrt{5}} = x = 5\sqrt{2}$
The length of the diagonal of the rectangle is 10cm.	The side of the square is 2$\sqrt{5}$ $5\sqrt{2}$

Weaker responses showed that candidates were able to find the length of the diagonal of the rectangle but failed to attempt the second part in which sides of square were required.

Example 1:

<small>side of the square:</small>	
(i) Data	
x (diagonal length) = ?	$= \sqrt{x^2} = \sqrt{100}$
$P = 6\text{cm}$	$\Rightarrow \boxed{x = 10}$
$B = 8\text{cm}$	The diagonal is 10cm
Solution :-	
$x^2 = P^2 + B^2$	(ii) since all sides of square are same
$x^2 = (6)^2 + (8)^2$	therefore, if its diagonal is 10cm, then the
$x^2 = 100$	side of square will be $\boxed{S = 10\text{cm}}$
Square root both side	$S = S$, Area.
	$10 = 10\text{cm}$ $S \times S = 100\text{cm}^2$

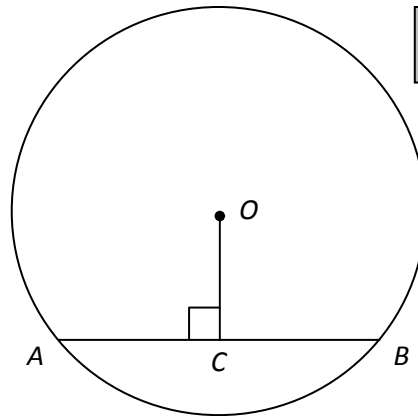
Example 2:

i. $h^2 = b^2 + p^2$	ii $S^2 = h \times h$
$h^2 = (8)^2 + (6)^2$	$S^2 = 10 \times 10$
$h^2 = 64 + 36$	$S^2 = 100$
$h^2 = 100$	$\sqrt{S^2} = \sqrt{100}$
$\sqrt{h^2} = \sqrt{100}$	$S = 10\text{cm}$
$h = 10\text{cm}$	
The length of its diagonal	The side of the square
is 10cm	is 10cm.

Question 9

A circle with centre O is shown in the diagram.

- If $m\overline{AC} = 5$ cm, then find $m\overline{AB}$ and write the reason to justify your answer.
- Is $m\overline{AC} > m\overline{OA}$? Write the reason to justify your answer.



NOT TO SCALE

This question was based on the theorem related to the circle and its properties.

Better responses showed that candidates had command over the theorems and their application. Candidates were able to find the required measurement and were able to justify their answer with proper reason.

Example:

(i) As we know $m\overline{AC} = 5 \text{ cm} = \overline{CB}$
So $m\overline{AB} = \overline{AC} + \overline{CB}$
$m\overline{AB} = 5 + 5 = 10 \text{ cm}$
$m\overline{AB} = 10 \text{ cm}$.
Reason: \rightarrow A line drawn from centre perpendicular to the chord bisect it into two equal parts
(ii) NO; $m\overline{AC} > m\overline{OA}$
Reason; If we make a triangle OCA , then \overline{OA} will be its Hypotenuse, which is greater side.
\Rightarrow OR; Greater angle have greater side opposite to it

Weaker responses indicated that the candidates failed to comprehend the properties of circles and, therefore, were unable to find the required measurement or justify their answers.

Example:

$m \overline{AB} = 5\text{cm}$
Because a line perpendicular p to the chord from the center bisects it equally in two parts
 $m \overline{AC}$ is greater $m \overline{OA}$ Because any point perpendicular to the center is at highest distance from it.

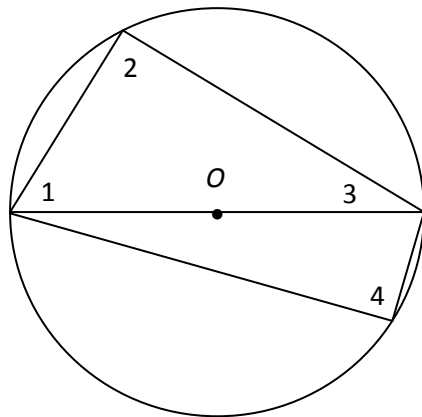
Example:

① the $\overline{AC} = 5\text{cm}$ when we add 5 to find CB Because the half of $AB = AC + CB$
 $AB = 5 + 5$
 $AB = 10\text{cm}$
② ~~yes $\overline{AC} > \overline{OA}$~~
"Yes, $AC > OA$ Because the hypotenuse (AO) is greater than the AC perpendicular. in between of these there is 90° angle is front of hypotenuse.

Question 10

A circle with centre O is shown in the diagram.

- Find $m\angle 2 + m\angle 4$ and write the reason to justify your answer.
- Is $m\angle 1 + m\angle 3 > m\angle 4$? Write the reason to justify your answer.



NOT TO SCALE

Better responses displayed that candidates had command over the theorems, applied relevant theorems to find $m\angle 2 + m\angle 4$ and justified their answer. Similarly, they correctly responded to the question asked in the **part ii** with proper justification by referring correct theorems.

Example:

- i) The sum of $m\angle 2$ and $m\angle 4$ is 180° OR $m\angle 2 + m\angle 4 = 180^\circ$
Reason: According to the theorem, the opposite angles of an inscribed quadrilateral to a given circle are always supplementary.
- ii) No, $m\angle 1 + m\angle 3 = m\angle 4$ because
 $\angle 2 = 90^\circ$ and $\angle 4 = 90^\circ$
In $\triangle ABC$ $\angle 1 + \angle 2 + \angle 3 = 180^\circ$
 $\angle 1 + \angle 3 = 180^\circ - 90^\circ = 90^\circ$ so $\angle 1 + \angle 3 = \angle 4$

Weaker responses indicated that the candidates were unable to comprehend the theorem to be applied in the given question. Hence, they failed to fulfill the requirement of the question and found it difficult to justify their answers.

Example 1:

i- $m\angle 2$ & $m\angle 4$ will be same because in Parallelogram opposite angles are same therefore they both are same
ii- As $m\angle 1$ & $m\angle 3$ are opposite angles so if we add them so their sum will be greater than $m\angle 4$.

Example 2:

(i) $m\angle 2 = 90^\circ$ and $m\angle 4$ also = 90°
Reason:- Theorem: In semicircle the angle on the circumference is always 90° (Perpendicular to the chord)
(ii)
Yes $m\angle 1 + m\angle 3 > m\angle 4$
Reason: The sum of angle of two sides of triangle is greater than the third angle

Question 11

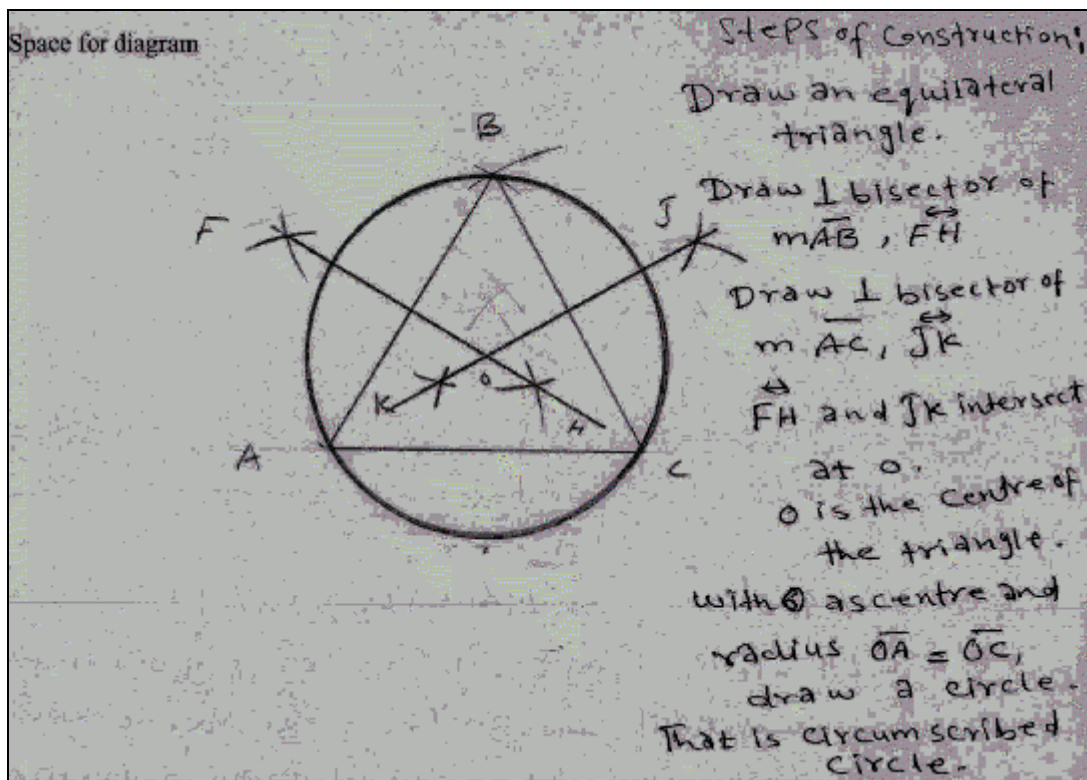
Construct an equilateral triangle with measure of each side equal to 5 cm and draw the circumscribed circle of the triangle.

This question was based on the construction of geometrical figures and it was a well attempted question.

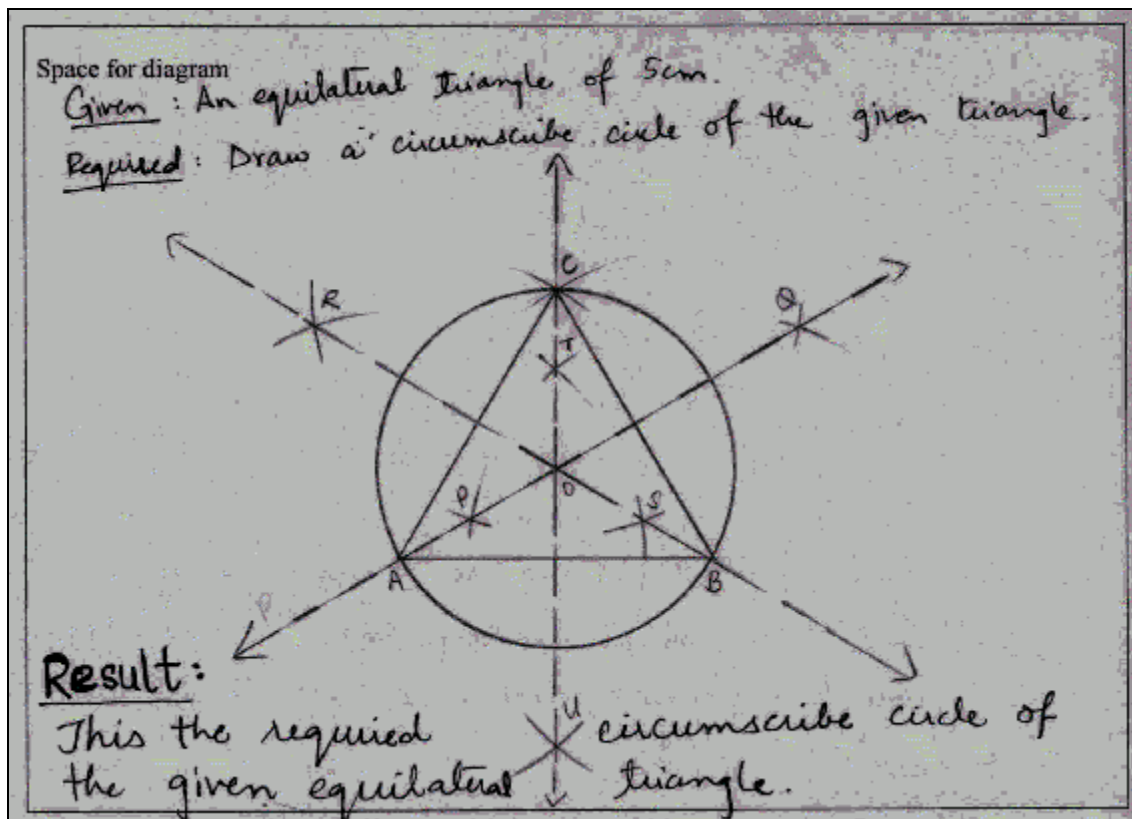
Better responses displayed that candidates had good command over geometrical construction and had clear understanding of the process of drawing requiring circumscribed circle. Although for circumscribed circle, point of intersection of right bisectors of a triangle is needed, but for equilateral triangle right bisector, angle bisector, median and altitude, all are along the same line.

Few candidates used the property of equilateral triangles and with the help of angle bisectors drew the required the circumscribed circle as cited in example 2.

Example 1:



Example 2:



Weaker responses indicated that candidates had lack of understanding of basic rules of practical geometry and failed to draw the required triangle and consequently circumscribed circle associated to the given triangle.

Example:

