Aga Khan University Examination Board

Notes from E-Marking Centre on SSC II Mathematics Examination May 2015

Introduction

This document has been produced for the teachers and candidates of SSC Part II (Class X) Mathematics. It contains comments on candidates' responses to the 2015 Secondary School Certificate (SSC-II) Examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E-Marking Notes

This includes overall comments on students' performance on every question and some specific examples of students' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that require candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

Weaker responses revealed that candidates had problems with conversion of verbal phrases into the mathematical operations to solve word problems, presentation of a solution set on number line, understanding theorems related to cords and arcs of a circle, tangents to a circle and angle in a segment of a circle. In general, questions based on trigonometric identities and applications of theorems were not well attempted.

Detailed Comments:

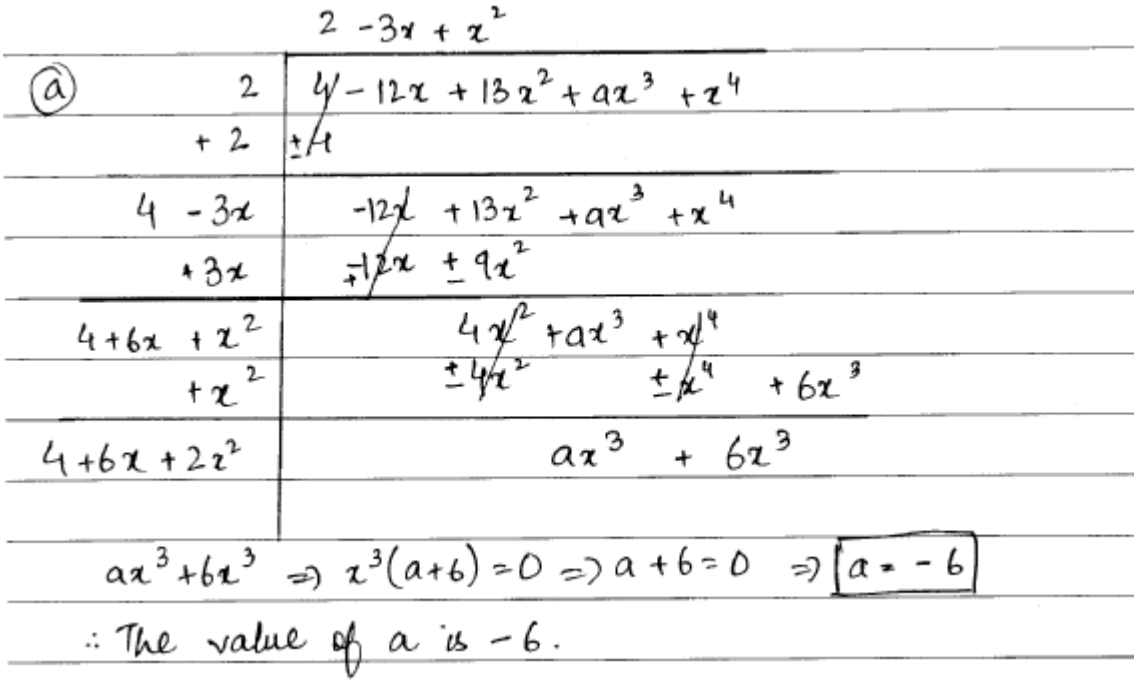
Question 1

This question offered a choice between part a and b. A vast majority of students attempted part a, which was also attempted better by students as compared to part b.

Question 1a
If $4-12x+13x^2+ax^3+x^4$ is a perfect square, find the value of *a*.

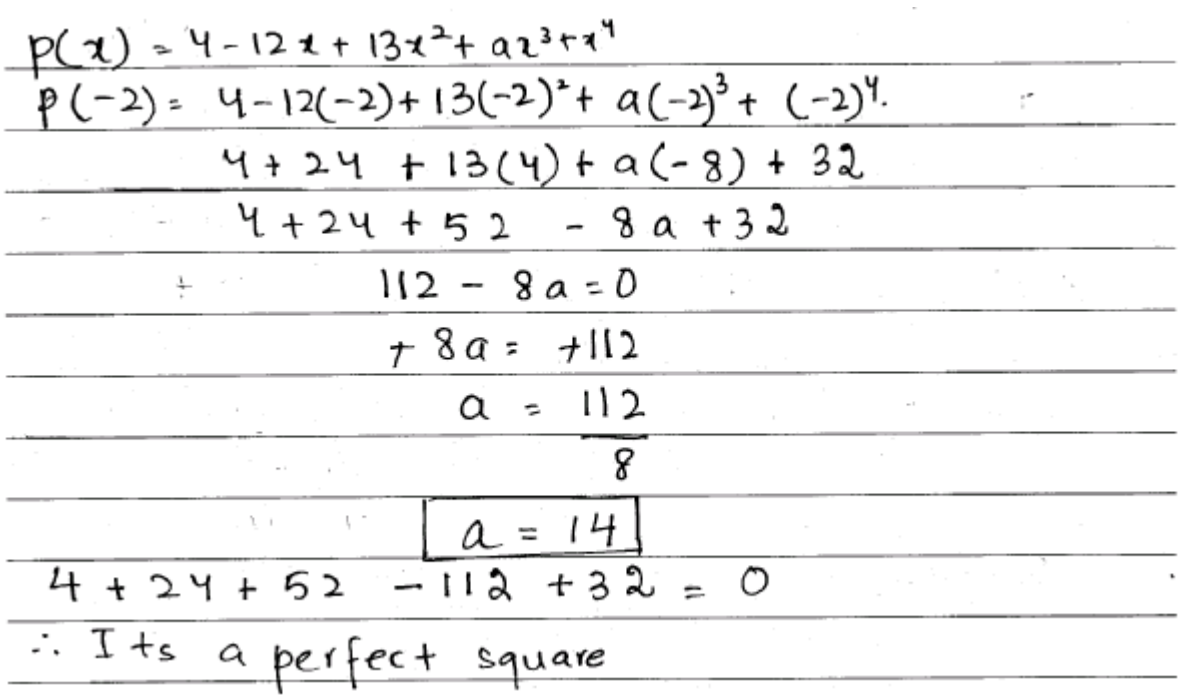
Better responses exhibited the candidates were able to understand the question and had a strong grasp on the method of long division. The remainder theorem was also applied in the last step to find the value of *a*.

Example:



Weaker responses rearranged the given polynomial in descending order and then tried to solve it with long division at which the candidates could not succeed. Majority left it in the middle. Errors in algebraic solutions were frequent in responses where the polynomial was not arranged in descending order.

Example:

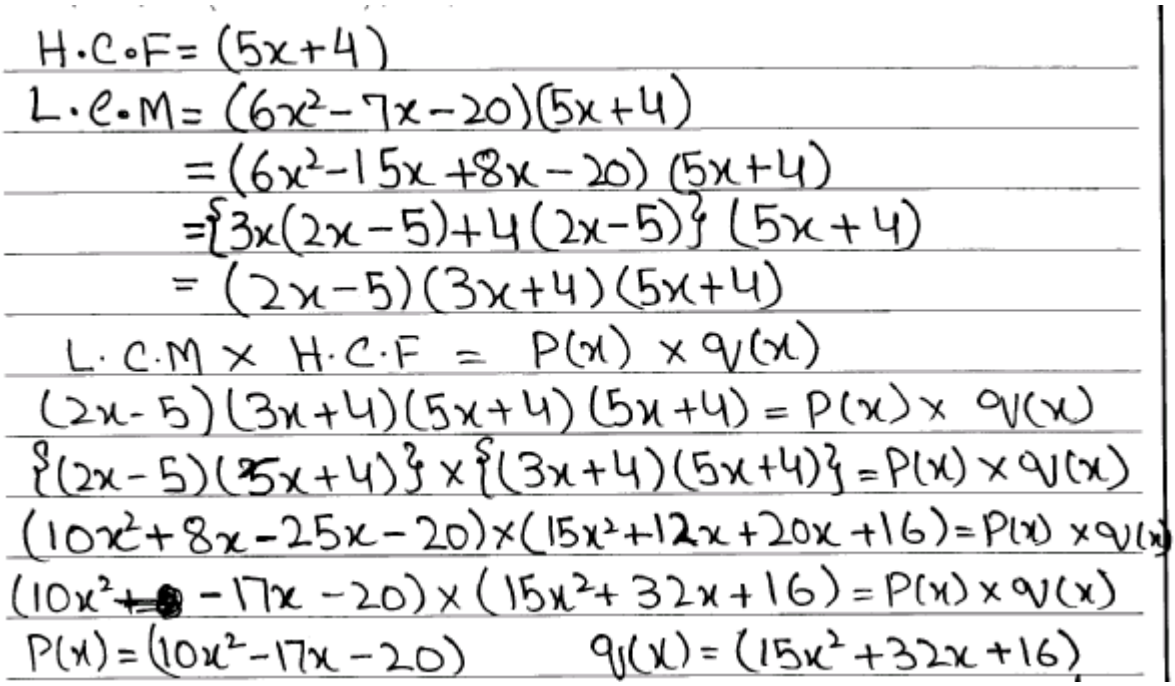


Question 1b

The highest common factor and the least common multiple of TWO quadratic expressions are $(5x+4)$ and $(6x^2-7x-20)(5x+4)$ respectively. Find these two quadratic expressions.

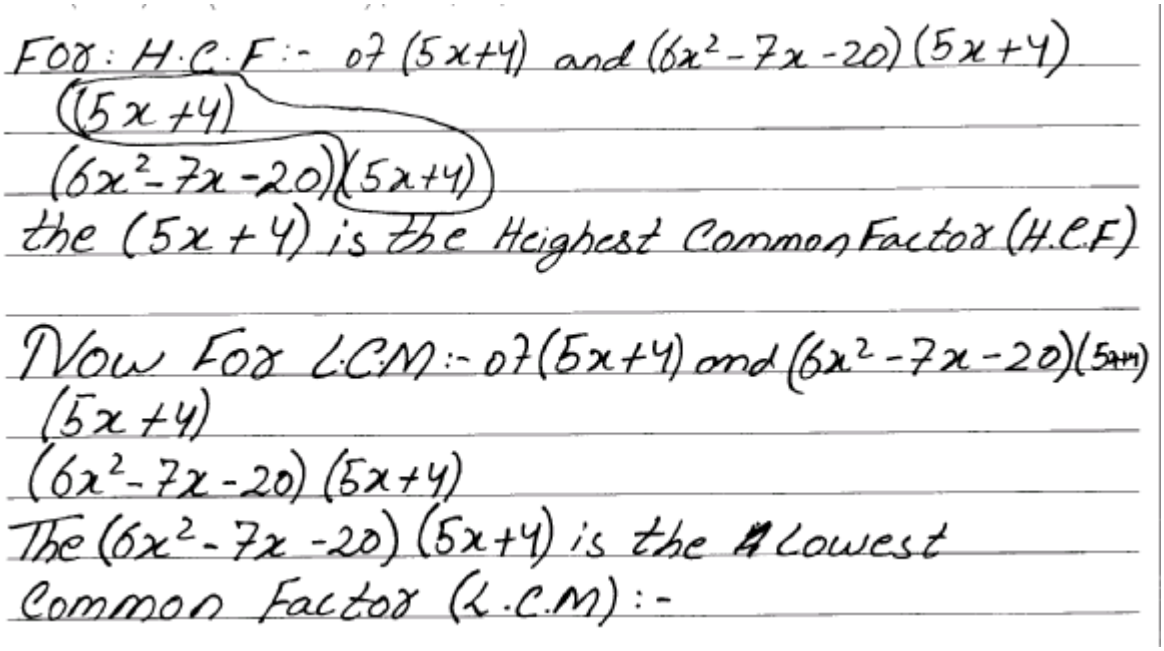
Better responses understood the question and expressed the least common multiple as $(3x+4)(2x-5)(5x+4)$. Candidates used the definition of L.C.M. and H.C.F. to find their answers

Example:



Weaker responses failed to understand the question. Instead of finding the two quadratic expressions, candidates assumed that the given H.C.F. and L.C.M. in the question are the two expressions, and using these expressions they either found the Greatest Common Divisor by division method or they used the formula *product of two polynomials = Their G.C.D × Their L.C.M*

Example:



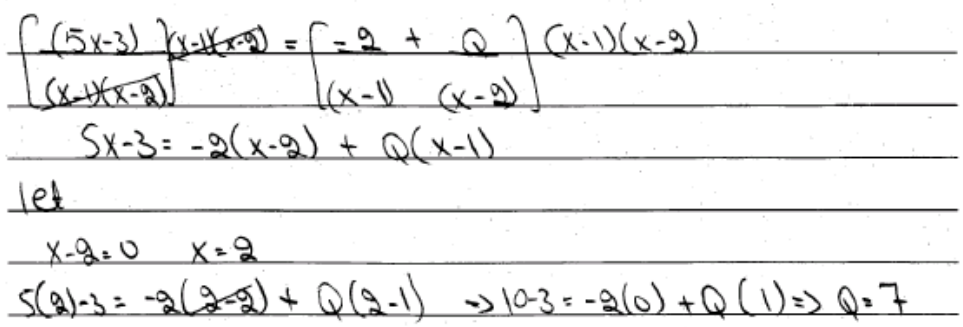
Question 2

If $\frac{5x-3}{(x-1)(x-2)} = -\frac{2}{x-1} + \frac{Q}{x-2}$, find the value of Q.

This question was attempted well by most of the candidates.

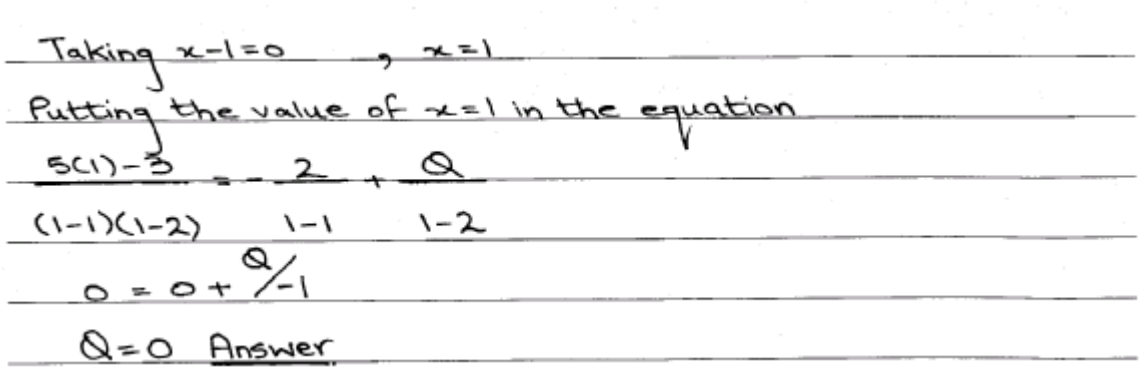
Better responses reflected that candidates had no difficulty clearing the given equation of fractions. Some candidates equated the coefficients of the like terms while others supposed $x-2=0$. Both ways they got the results easily.

Example:



Weaker responses reflected that the candidates were confused between putting $x-2=0$ and $x-1=0$ to get the answer. Some candidates skipped the step of clearing the equation of fractions while others made minor errors that led them to the wrong answer.

Example:



Question 3

This question offered a choice between part a and b. Majority of the candidates chose part b**,** which was also attempted slightly better than part a but both parts were not attempted well by the candidates.

Question 3a

i. Find the value of *x* for $-6 < \frac{3-3x}{2} \leq 12,\ x \in R$

ii. Represent the solution of $-6 < \frac{3-3x}{2} \leq 12$ on a number line.

Better responses solved the liner equalities, breaking the given equation into $\frac{3-3x}{2} > -6$ and $\frac{3-3x}{2} \leq 12$, and represented the solution on the number line.

Example:

$$-6 < \frac{3-3x}{2} \quad , \quad \frac{3-3x}{2} \leq 12$$

$$-12 < 3-3x \quad , \quad 3-3x \leq 12(2)$$

$$-12-3 < -3x \quad , \quad 3-3x \leq 24$$

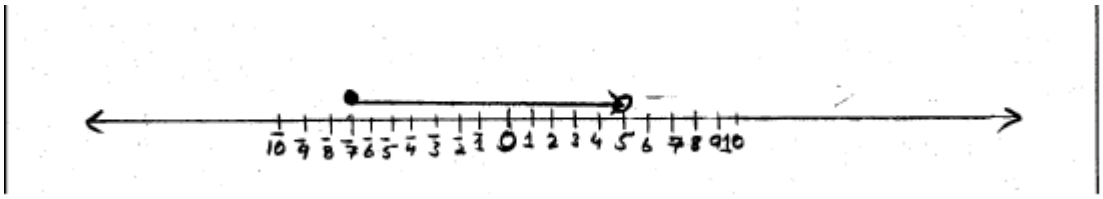
$$-15 < -3x \quad , \quad -3x \leq 24-3$$

$$\frac{-15}{-3} < x \quad , \quad -3x \leq 21$$

$$, \quad x \leq \frac{21}{3}$$

$$5 > x \quad , \quad x \geq -7$$

$$-7 \leq x < 5 \text{ or } 5 > x \geq -7$$



Weaker responses failed to break the given equation into correct terms. The most frequent error was $\frac{3-3x}{2} < -6$. Candidates represented the solution of the two equations separately on the number line but did not represent the final solution. They were also confused about when to use opened and closed dots.

Example:

$-6 < \frac{3-3x}{2}$

$-12 < 3-3x$

$-3-12 < -3x$

$-15 < -3x$

$\frac{+15}{+3} < \frac{-3x}{-3}$

$x > 5$

$\frac{3-3x}{2} \leq 12$

$3-3x \leq 12$

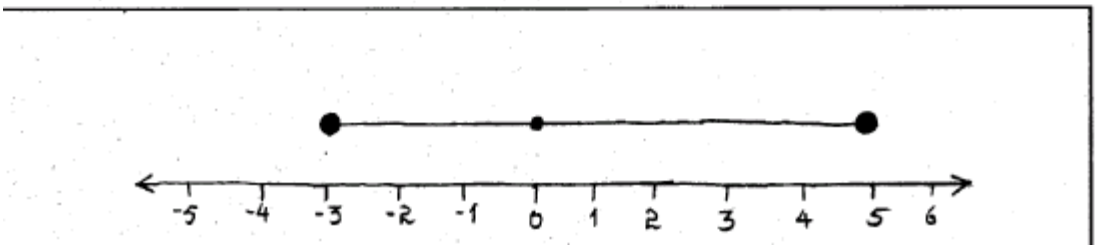
$-3x \leq 12-3$

$-3x \leq 9$

$\frac{+3x}{+3} \leq \frac{9}{-3}$

$x \geq -3$

ii. Represent the solution of $-6 < \frac{3-3x}{2} \leq 12$ on a number line. (2 Marks)

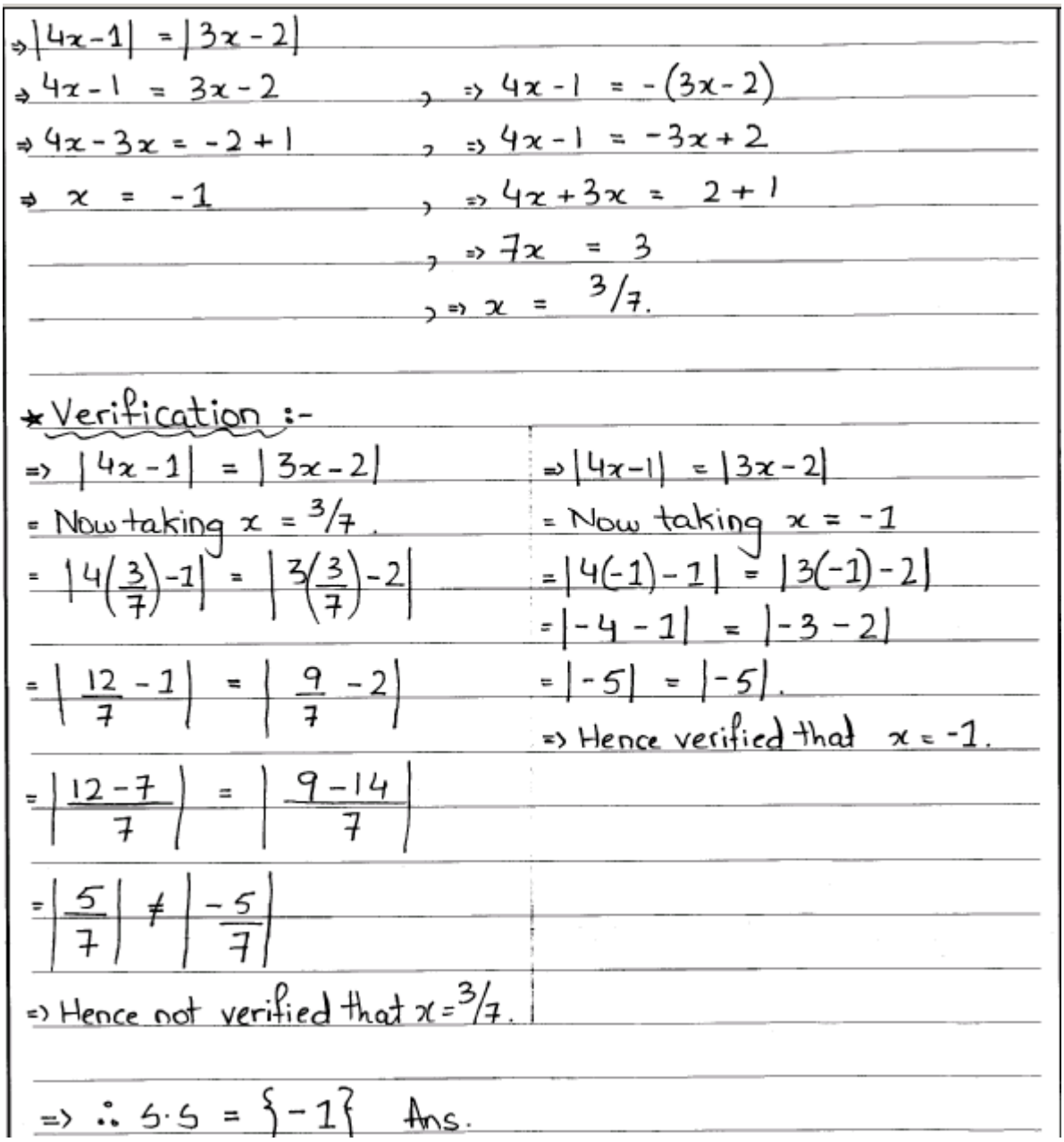


Question 3b

Solve $|4x-1| = |3x-2|$. Also verify your answer.

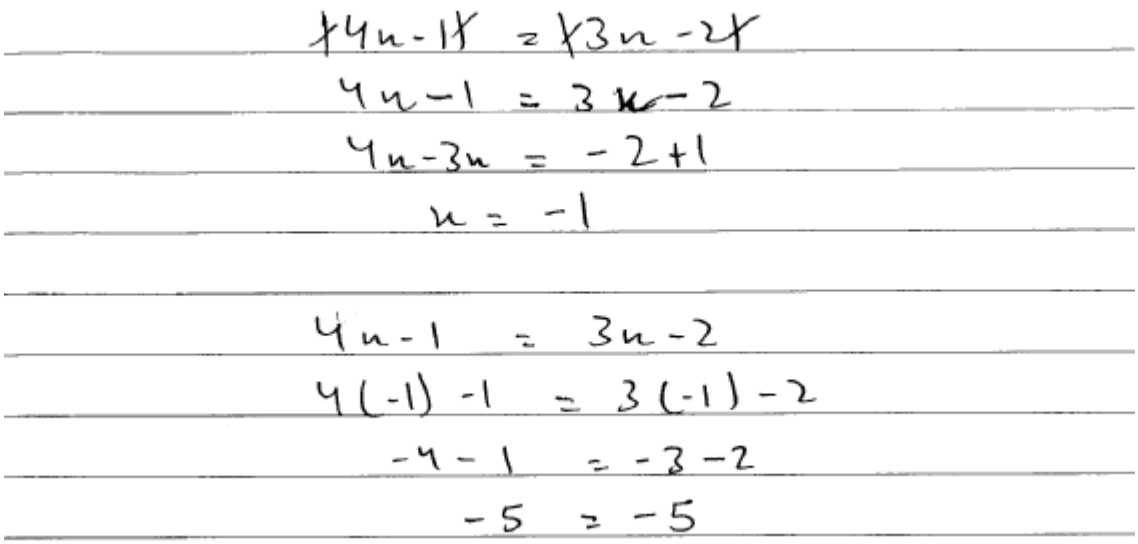
Better responses exhibited correct solutions of linear inequalities involving absolute value and found the two values of *x*. The candidates verified both the answers by putting them in the equation given equation.

Example:



Weaker responses reflected that candidates could not understand that since both sides of the equation contain absolute value bars so the only way the two sides are equal will be if the two quantities inside the absolute value bars are equal or equal but with opposite signs. A few candidates extracted only $4x-1=3x-2$ from the given equation and ignored the other equation. Another frequent error made was that candidates did not use the original equation to verify the answers; instead, they used the extracted equations due to which they could not verify the answers. Some candidates did use the original equation but they failed to verify the solution because they did not find absolute values in the end.

Example:



Question 4

A family of 3 adults and 2 children enjoyed a movie at the cinema for Rs 3,100. This can be represented by the equation $3x+2y=3100$, where x and y represent the cost of each adult and child ticket respectively. Next week, their uncle and his 2 children join them for another movie and the total cost this time is Rs 4,800. Assuming the cost of tickets remain the same, what is the cost of a child's ticket?

Candidates exhibited average performance in this question.

Better responses indicated that candidates understood the given situation in the question and they successfully converted it into mathematical equation $4x+4y=4800$. Methods of substitution and elimination were applied correctly by the candidates to solve the equations simultaneously and find the value of y i.e. the cost of child's ticket. Many candidates found the values of x and y both; they were given full credit.

Example:

$3x+2y=3100 \Rightarrow$ equ (i)

$4x+4y=4800$ (after joining uncle and two kids) $\Rightarrow$ equ (ii)

Taking equation (i) value of y will be:-

$3x+2y=3100$

$y=\frac{3100-3x}{2} \Rightarrow$ equ (iii)

Putting equation (iii) in equ (ii)

$4x+4y=4800$

$4x+4\left(\frac{3100-3x}{2}\right)=4800$

$4x+\frac{12400-12x}{2}=4800$

$\frac{8x+12400-12x}{2}=4800$

$-4x+12400=9600$

$12400-9600=4x$

$2800=4x$

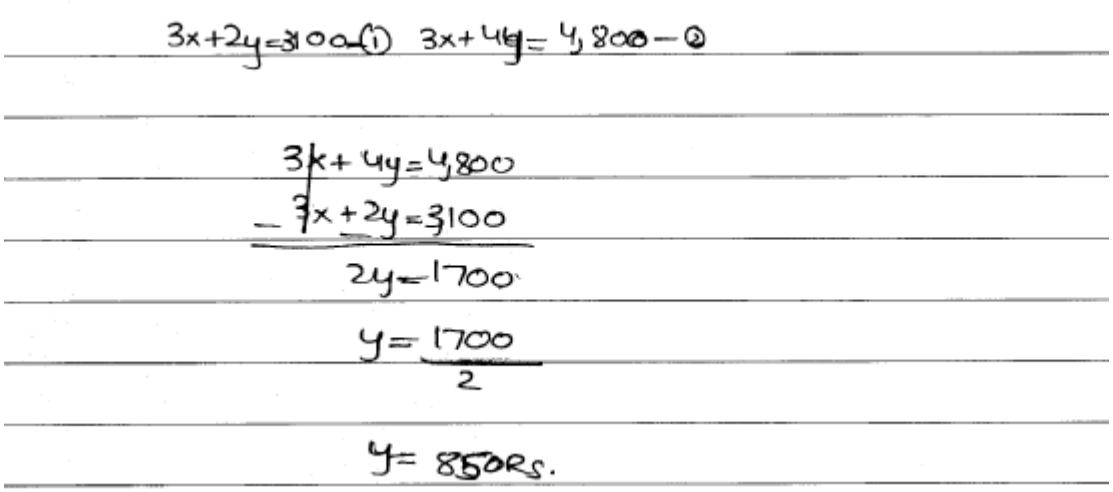
$\boxed{700=x}$ Cost of adult's ticket

Putting x value in equ (iii)

$y=\frac{3100-3(700)}{2}=\frac{3100-2100}{2}=\frac{1000}{2}=\boxed{500}$ Cost of child's ticket

Weaker responses represented that the candidates could not understand the question and they failed to convert the given data into a mathematical equation. Candidates made a lot of errors in solution of simultaneous equations due to application of incorrect algebraic operations. Many candidates found the cost of adult's ticket which was *x* instead of child's ticket which was *y*.

Example:



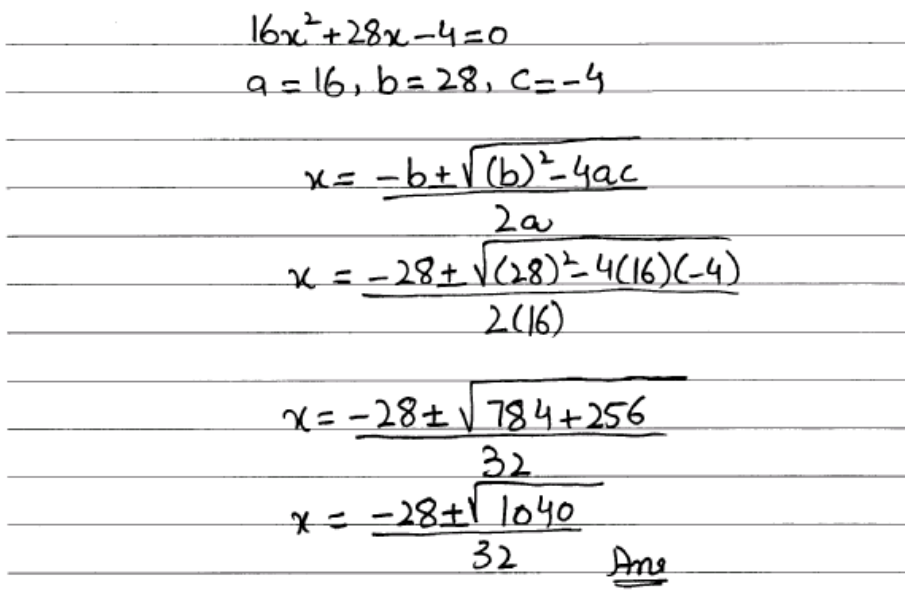
Question 5

Use **quadratic formula** to find the value of *x* for $16x^2 + 28x = 4$.

Candidates performed very well in the question.

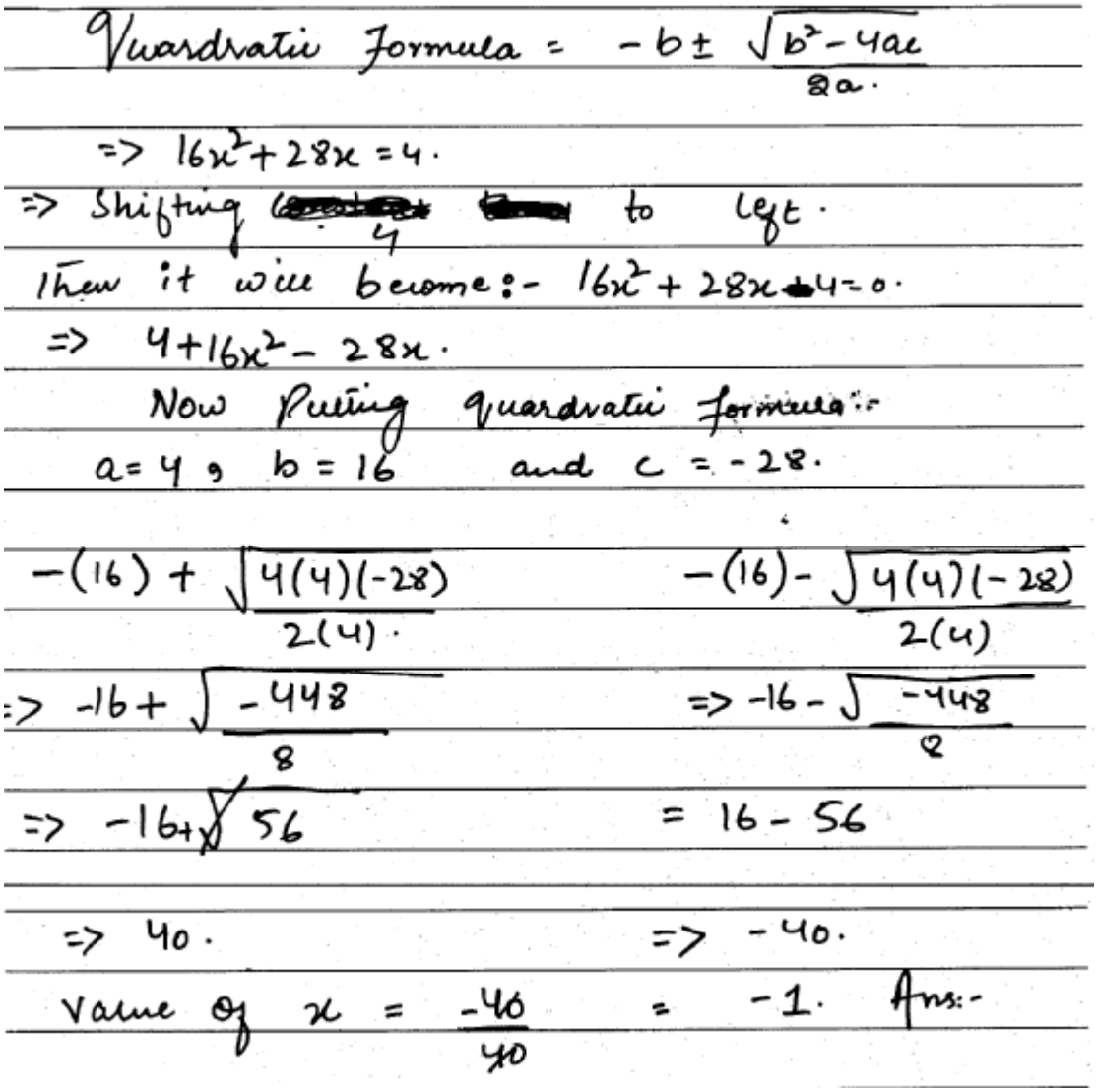
Better responses expressed the given equation in general form, stated the correct quadratic formula, identified the correct values of *a*, *b* and *c*, and solved to get the correct values of *x*.

Example:



Weaker responses reflected that the candidates could not recall the quadratic formula. Most of the candidates were able to identify the correct values of *a*, *b* and *c*, but they made minor errors in solution.

Example:

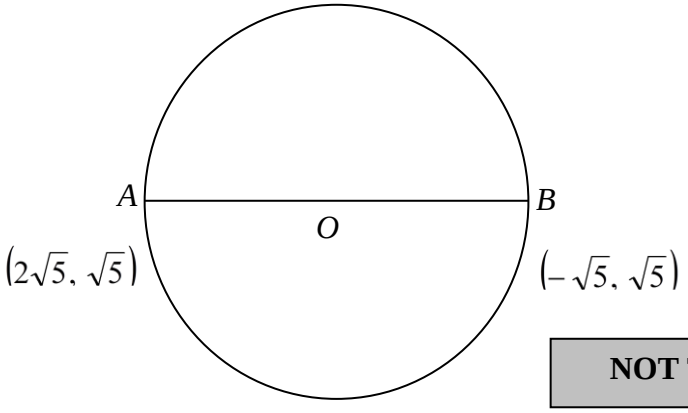


Question 6

This question offered a choice between part a and b. Majority of the candidates chose part a. Candidates performed well in this questions.

Question 6a

A circle having centre *O* is shown below. Find the coordinates of point *O*.



Better responses used the midpoint formula to find the coordinates of *O*. They applied the formula correctly to get the required results.

Example:

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}.$$

$$\frac{2\sqrt{5} + (-\sqrt{5})}{2}, \frac{\sqrt{5} + \sqrt{5}}{2}$$

$$\frac{2\sqrt{5} - \sqrt{5}}{2}, \frac{1 + 1\sqrt{5}}{2}$$

$$\frac{\sqrt{5}}{2}, \frac{2\sqrt{5}}{2}$$

Ans = $\sqrt{5}/2, \sqrt{5}$ $\{(\sqrt{5}/2, \sqrt{5})\}$

Weaker responses either used the distance formula or made wrong substitutions in the midpoint formula. The most frequently used incorrect formula was $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$.

Example:

Mid point gamula: $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

$\frac{2\sqrt{5} + \sqrt{5}}{2}, \frac{-\sqrt{5} + \sqrt{5}}{2}$

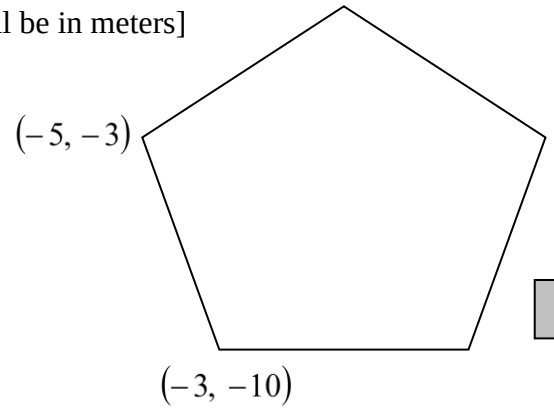
$\frac{3\sqrt{5}}{2}, 0/2$

$\frac{3\sqrt{5}}{2}, 0$

Co-ordinates of point $O(3\sqrt{5}/2, 0)$

Question 6b

The diagram shows a small garden which is in the shape of a **regular pentagon** (a polygon having five equal sides). If a child walks on all the edges of this park once, what is the distance that he covers?
[Note: The distance found will be in meters]



NOT TO SCALE

Better responses used the distance formula which reflects that the candidates understood the question. The distance $\sqrt{53}$ was multiplied by 5 to find the total distance covered.

Example:

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d = \sqrt{(-5-(-3))^2+(-10-(-3))^2}$

$d = \sqrt{(-5+3)^2+(-10+3)^2}$

$d = \sqrt{(-2)^2+(-7)^2}$

$d = \sqrt{4+49}$

$d = 7.28$ m

Total $d = 7.28 + 7.28 + 7.28 + 7.28 + 7.28$

$= 36.4$ m

Weaker responses either used the midpoint formula or used the incorrect distance formula; the most frequently used incorrect formula was $\sqrt{(x_1+x_2)^2+(y_1+y_2)^2}$.

Example:

Distance Formula $= \left(\frac{x_1+x_2}{2}\right), \left(\frac{y_1+y_2}{2}\right)$

$\left(\frac{-5+(-3)}{2}\right), \left(\frac{-3+(-10)}{2}\right)$

$= \frac{-5-3}{2}, \frac{-3-10}{2}$

Answer: $\frac{-8}{2} = -4 \quad \frac{-13}{2} = -6.5$

$x = \{-4, -6.5\}$

Question 7

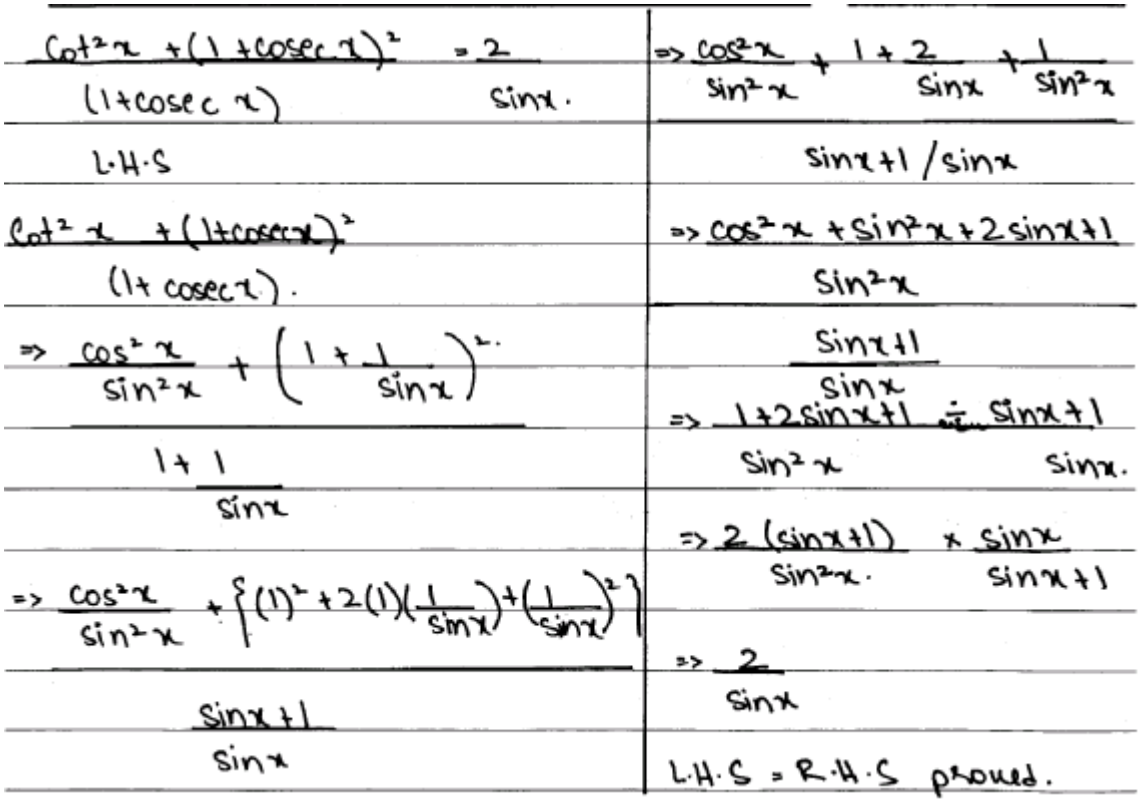
This question offered a choice between part a and b. Most of the candidates chose part a, however, part b was attempted slightly well by candidates than part a. This was not a well attempted question.

Question 7a

Show that $\dfrac{\cot^2 x + (1+\operatorname{cosec} x)^2}{(1+\operatorname{cosec} x)} = \dfrac{2}{\sin x}$.

Better responses exhibited thorough knowledge and application of trigonometric identities. The candidates mostly chose the left hand side of the equation, applied various trigonometric identities, and proved it equal to the right hand side. Some candidates converted the left hand side in terms of sin *x* and cos *x* and successfully got the answer.

Example:



Weaker responses represented confusion in trigonometric identities. A large number of candidates did not apply the correct identity to expand $(1+\operatorname{cosec} x)^2$ and wrote $1+\operatorname{cosec}^2 x$ instead of $1+2\operatorname{cosec} x+\operatorname{cosec}^2 x$. Also, $\operatorname{cosec}^2 x+1=\cot^2 x$ instead of $\cot^2 x+1=\operatorname{cosec}^2 x$ was frequently used by candidates that led them to incorrect answers.

Example:

$$\frac{\cot^2 x+(1+\operatorname{cosec} x)^2}{(1+\operatorname{cosec} x)}$$

$$\frac{\cos^2 x}{\sin^2 x}+(1+\operatorname{cosec} x)$$

$$\frac{\cos^2 x+(1+\operatorname{cosec} x)(\sin^2 x)}{\sin^2 x}$$

$$\frac{\cos^2 x+\sin^2 x+\operatorname{cosec} x\sin^2 x}{\sin^2 x}$$

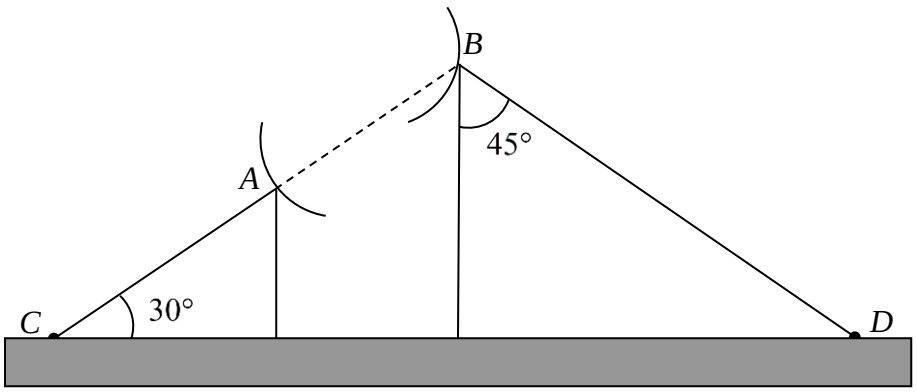
$$\frac{1+\operatorname{cosec} x\sin^2 x}{\sin^2 x}\ ;\ \sin^2 x+\cos^2 x=1$$

$$\frac{1+\sin^2 x}{\sin^3 x}=\frac{1+1}{\sin x}$$

$$=2/\sin x : \text{Hence, proved.}$$

Question 7b

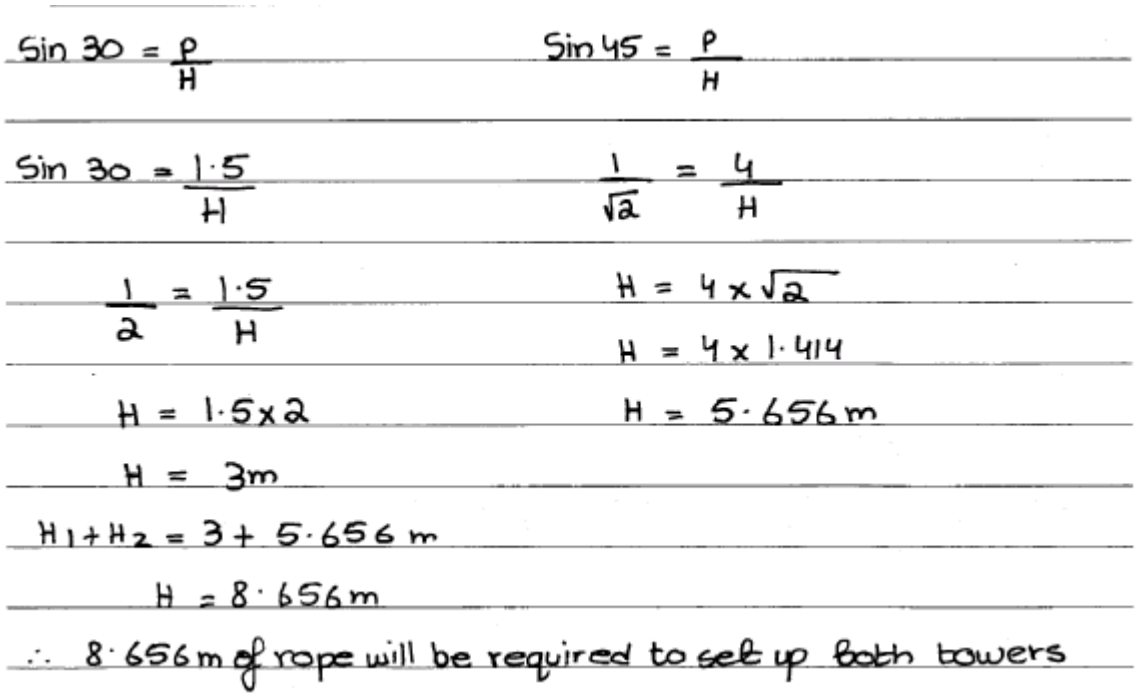
A signal link has to be set up between two microwave dishes A and B as shown below. The towers holding the dishes are to be set vertical with the help of ropes on either side from points C and D on ground. If the height of A and B is 1.5 m and 4 m respectively, find the total length of the rope required to set up both towers.



NOT TO SCALE

Better responses reflected that the candidates understood the question and could relate it to the given diagram. They used the correct formula of sine and cosine to find the values of $\overline{AC}$ and $\overline{BD}$.

Example:



Weaker responses exhibited that the candidates could not understand that length of the rope is $\overline{AC} + \overline{BD}$. A large number of candidates either used incorrect tangent formula or the wrong sine/ cosine formula to find the values of $\overline{AC}$ and $\overline{BD}$.

Example:

Consider Δ OBD

$45 + 90 + x = 180$ Sum of a Ls of Δ

$135 + x = 180$

$x = 180 - 135 = 45°$

$\sin 90° = P/H$

$1/2 = 4/H$

$H = 4 \times 2$

$H = 8\ m$

Consider Δ ACM

$\sin 30 = P/H$

$1/2 = 1.5/H$

$H = 1.5 \times 2$

$H = 3$

The length of rope from A to B:-

$BD = BC$

$8 = 3 + x$

$8 - 3 = x$

$5 = 8$

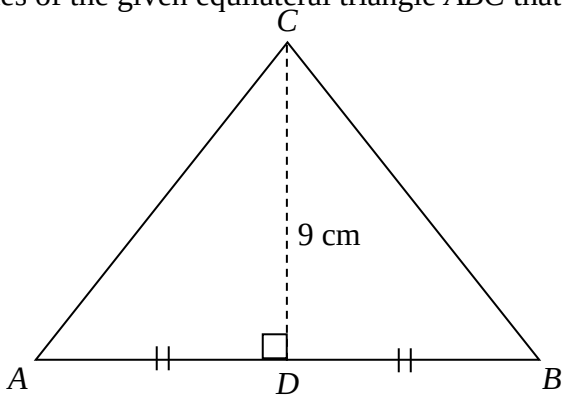
∴ The total length of a rope is 16m

Question 8

This question offered a choice between part a and b. A vast majority of candidates chose part a, but part b was attempted better than part a. Candidates exhibited average performance in this question.

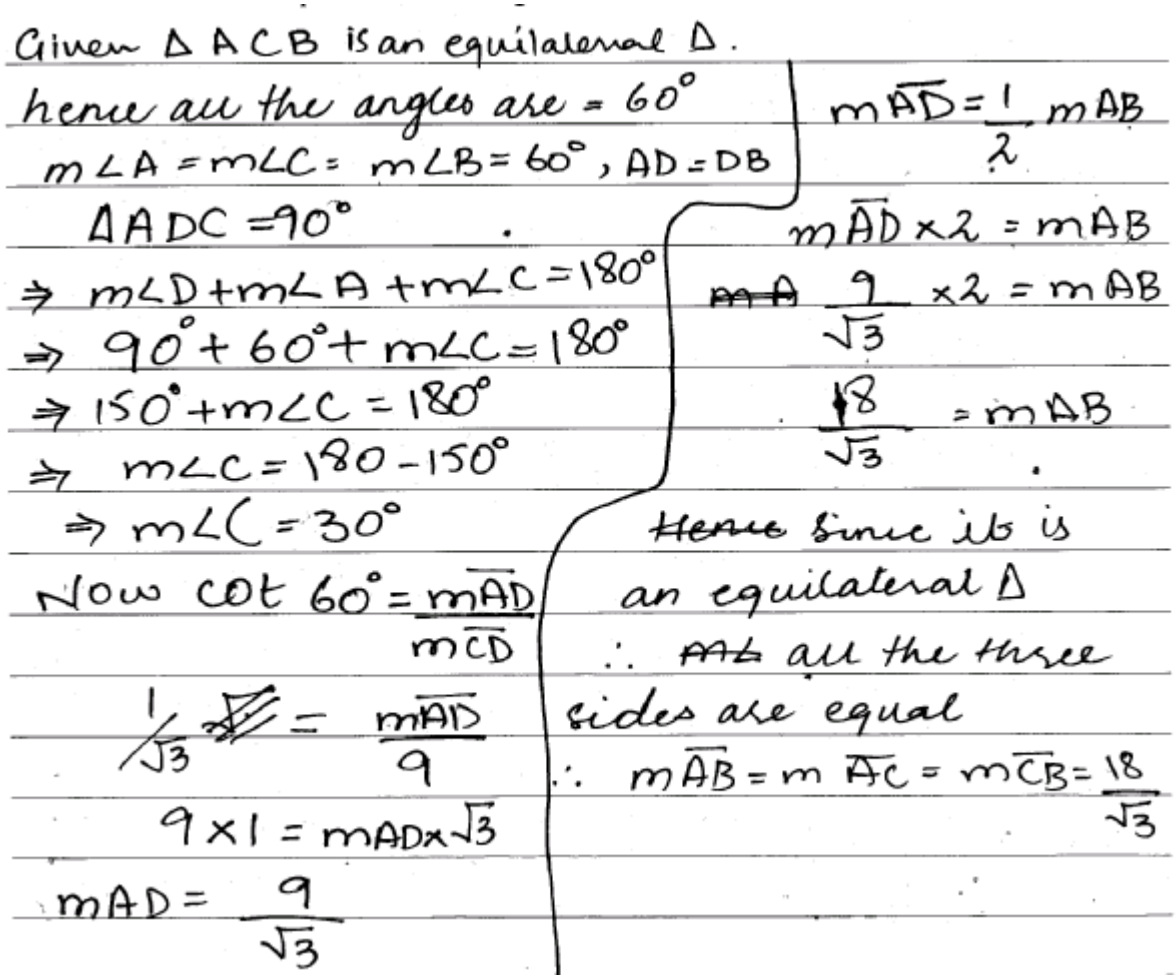
Question 8a

Find the length of the sides of the given equilateral triangle *ABC* that has a height of 9 cm.



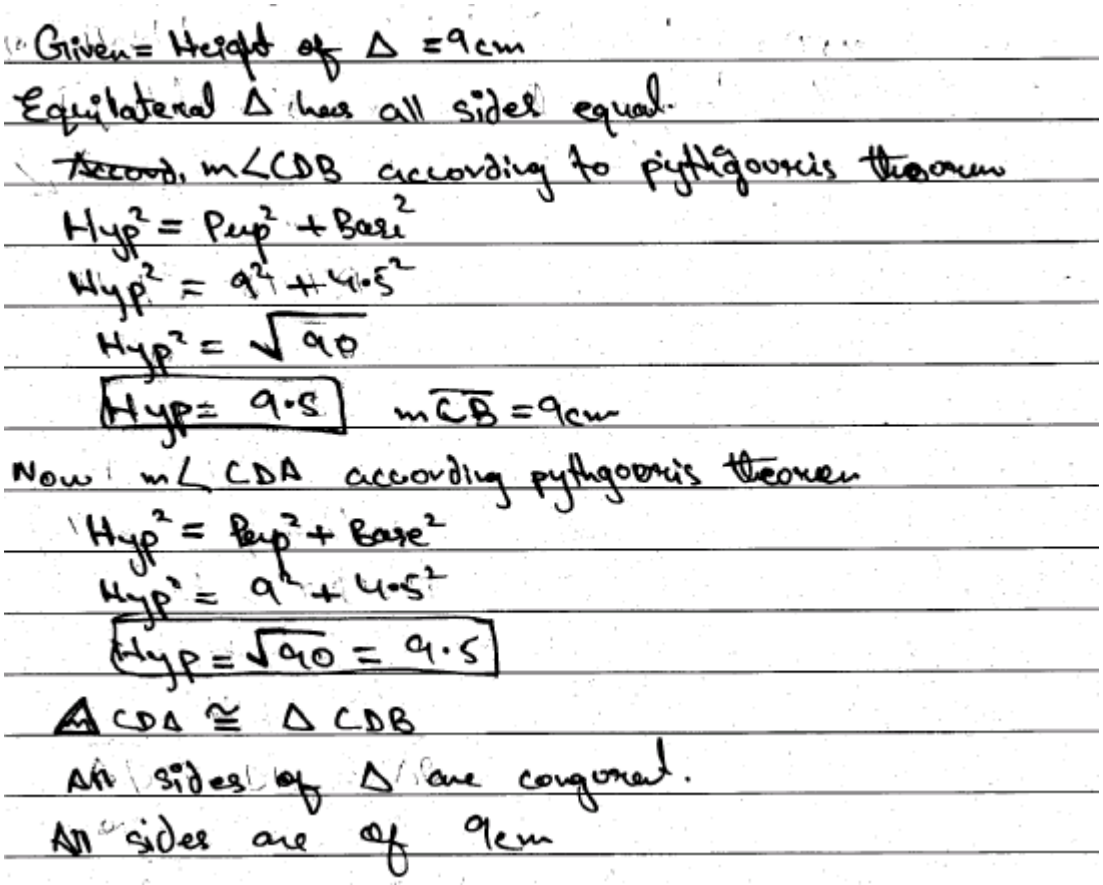
Better responses exhibited that the candidates understood what an equilateral is and could relate it to the given figure. Pythogras' theorem was used correctly to find the length of each side of the given triangle. Many candidates used trigonometric ratios to solve this question and got full credit for it.

Example:



Weaker responses indicated that candidates assumed $\overline{CD} = \overline{AD} = \overline{DB} = 9$ cm or $\overline{AD} = \overline{DB} = 4.5$ cm. Hence, application of Pythogras' theorem on these values led to incorrect answers.

Example:

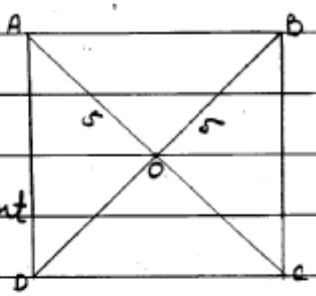


Question 8b
Find the area of a square that has a diagonal of 10 cm.

Better responses exhibited a variety of solutions. Most of the candidates found the length of each side $5\sqrt{2}$ cm by using pythogras' theorem first and then squared it to find the area while others realised this would be just a repeated step and left the answer as $x^2 = 50$, since x^2 is the area of square with length *x.* In both cases, the candidates were given full credit. Surprisingly, a few candidates used compass and ruler to construct a square with 10 cm diagonal; length of each side was then measured with a ruler and was squared to find the area! Such responses were also given full credit.

Example:

(b)



In a square ABCD, $\overline{AC}$ & $\overline{BD}$ are the diagonals where as $\overline{AC}$ & $\overline{BD}$ are also prependicular bisectors on point O. Concider ΔAOB in a square

$AO = 5cm$, $OB = 5cm$

$(AB)^2 = (AO)^2 + (OB)^2$

$(AB)^2 = (5)^2 + (5)^2$

$(AB)^2 = 25 + 25$

$(AB)^2 = 50\ cm$

$\overline{AB} = 7.07cm$

as all the sides of square are equal hence

$\overline{AB} = \overline{BC} = \overline{CD} = \overline{DA}$

Area of square is = $L \times W$, as all sides are equal so we can write $L \times L$ so

Area $= 7.07 \times 7.07 = 49.98cm^2$ approx. $50cm^2$

Weaker responses displayed that the candidates could not understand the question. Often rough sketches drawn or the solution itself reflected the lack of understanding of basic concepts such as the length of each side of a square is the same but not equal to the length of the diagonal.

Example:

$(10)^2 = (x)^2 + (x+1)^2$

$100 = x^2 + x^2 + 2x + 1$

$100 = 2x^2 + 2x + 1$

$100 - 1 = 2x^2 + 2x$

$99 = 2(x^2 + x)$

$\frac{99}{2} = x(x+1)$

$49.5\text{cm} = x(x+1)$

Length $(x) = 49.5\text{cm}$

Breadth $= x + 1$

$= 49.5 + 1$

$= 50.5\text{ cm}$

Area $= 2(l + b)$

$= 2(49.5 + 50.5)$

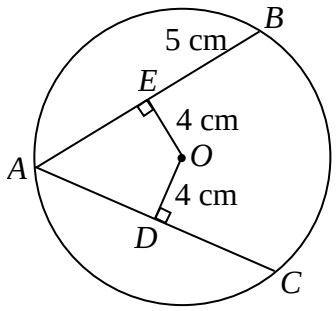
$= 2(100)$

$= 200\text{ cm}^2$ Answer

Question 9

In the given circle with centre O, $\overline{BE} = 5$ cm and $\overline{OE} = \overline{OD} = 4$ cm.

i. Find the length of $\overline{DC}$ and write a statement to justify your answer.
ii. Find the length of $\overline{AB}$ and write a statement to justify your answer.



NOT TO SCALE

This was a well attempted question.

Better responses stated the correct reasons with correct values. For part (i), $\overline{DC}$ = $\overline{EB}$ = 5 cm since both are halves of equidistant chords from the centre of the circle and for

part (ii) $\overline{AB}$ = 2 $\overline{EB}$ = 10 cm since $\overline{OE}$ is perpendicular to $\overline{AB}$, it bisects the chord $\overline{AB}$.

Example:

Ans (i) (Since $m\overline{BE} \cong m\overline{DC}$), $m\overline{DC} = 5$cm Ans.

∴ The triangles within the circle with centre O i.e ΔOBE and ΔODC are congruent, hence the measurements of corresponding sides are also congruent.

Ans (ii) (∴ $\overline{OE}$ is a perpendicular bisector of the chord $\overline{AB}$ and $m\overline{BE} = 5$cm)

$\Rightarrow m\overline{AB} = 2(m\overline{BE})$

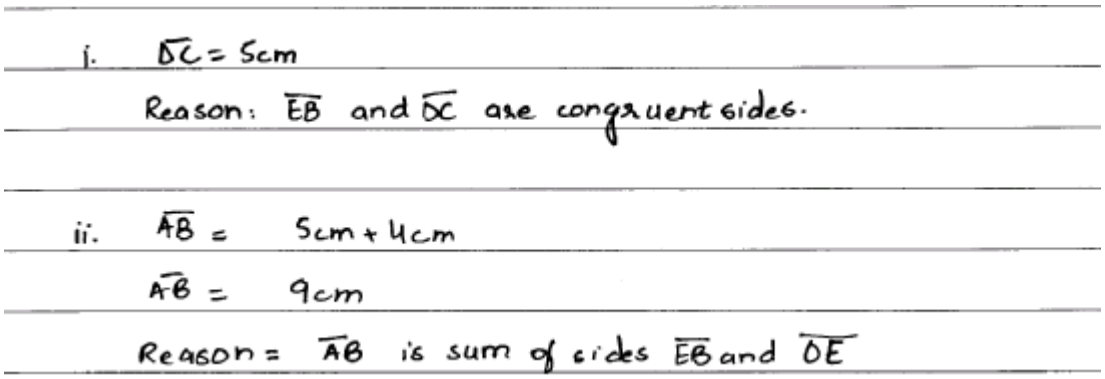
$\Rightarrow m\overline{AB} = 2(5)$

$\Rightarrow m\overline{AB} = 10$ cm.

∴ It is a property of circles that any perpendicular bisector to a chord always touches the centre of the circle, hence $\overline{OE}$ is a perpendicular bisector of chord $\overline{AB}$.

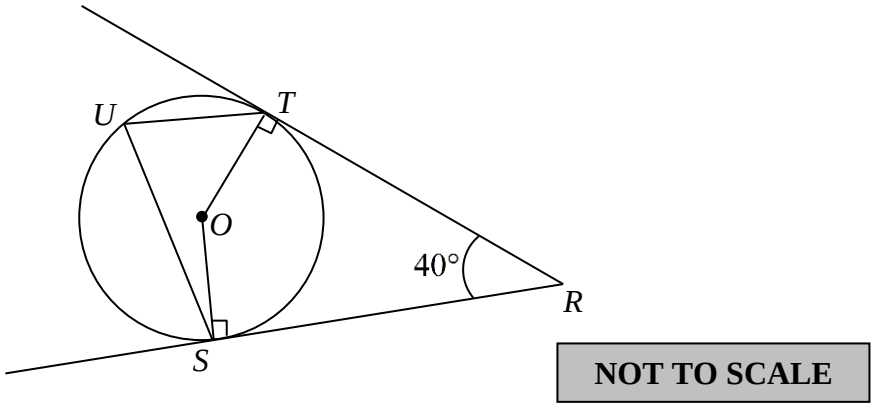
Weaker responses reflected lack of knowledge of theorems of circles. Many candidates could deduce the answers visually from the figure given but they could not justify their answers with the correct reasoning.

Example:



Question 10

$\overline{RS}$ and $\overline{RT}$ are tangents to a circle with centre *O*. If $m\angle SRT = 40°$, find $m\angle SUT$.



This was not a well attempted question.

Better responses either equated the sum of angles in a quadrilateral *OTRS* equal to $360°$ or found $\angle SOR$ by putting $\angle SRO = 20°$ to calculate $\angle SOT$. Both ways, they were able to find $\angle SUT$ by using the correct theorem.

Example:

TOSR is a Quadrilaterals having total sum of angle = 360° ∴

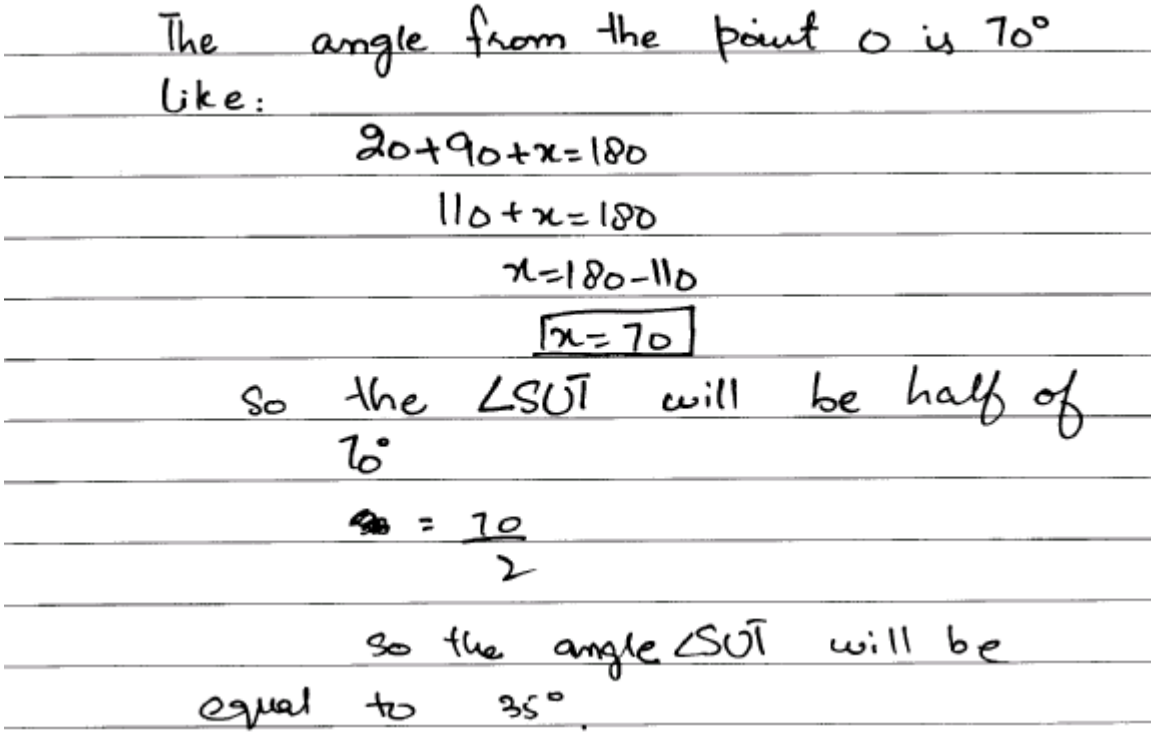
$$40 + 90 + 90 + O = 360$$
$$220 + O = 360$$
$$O = 360 - 220$$
$$O = 140°$$

According to Theorem U is half of the angle made on O ∴

$$U = \frac{O}{2}$$
$$= \frac{140}{2}$$
$$U = 70°$$

Weaker responses reflected that very few candidates equated the sum of angles in a quadrilateral *OTRS* equal to $360°$. Most of the candidates found $\angle SOR$ by putting $\angle SRO = 20°$ but could not decide what to do next. They also assumed that $\angle SUT = \frac{1}{2}\angle SOR$ or $\angle SUT = 2\angle SOR$. Overall the candidates demonstrated feeble concepts of theorems of circle.

Example:



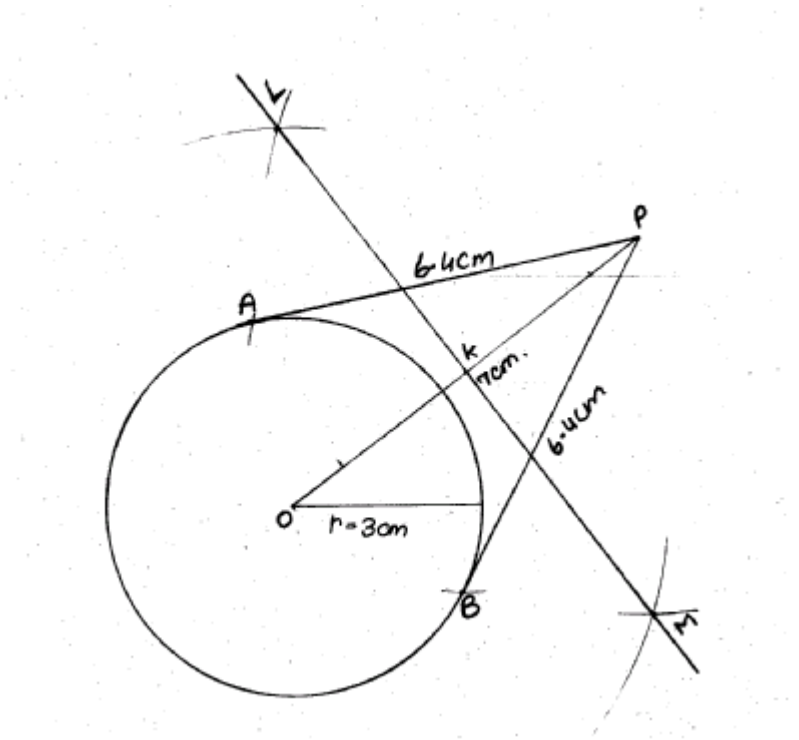
Question 11

Draw a circle of radius 3 cm, having centre *O*. Draw tangents to this circle from a point *P*, which lies 7 cm away from the centre of this circle, and write down the length of the tangents from point *P* to the point of tangency to the circle.

This was a well attempted question.

Better responses displayed clear concepts and accurate skills of construction circles and tangents. Some candidates used perpendicular bisectors to find the midpoint of the line *OP* while others used a ruler to mark it. Using the midpoint as centre, some candidates drew a complete circle with diameter *OP* while others just made arcs on the circle having radius 3 cm to make the tangents. All such responses were given full credit. Surprisingly, some candidates used SLO 24.1.1 (i) to draw tangents with the help of protractor. They were also given full credit.

Example:



Weaker responses reflected that the candidates could not understand the question. Some candidates assumed that point *P* lies on the circle while others drew direct or transverse common tangents.

Example: