Aga Khan University Examination Board

Notes from E-Marking Centre on HSSC-II Mathematics Examination April/ May 2019

Introduction

This document has been produced for the teachers and candidates of Higher Secondary School Certificate (HSSC) part-II Mathematics. It contains comments on candidates' responses to the 2019 HSSC-II Examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E-Marking Notes

This includes overall comments on students' performance on every question and *some* specific examples of students' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that require candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

Candidates need to be aware that the marks allocated to the questions are related to the answer space provided on the examination paper as a guide to the length of the required response. A longer response will not in itself lead to higher marks. Candidates need to be familiar with the command words in the SLOs which contain terms commonly used in examination questions. However, candidates should also be aware that not all questions will start with or contain one of the command words. Words such as 'how', 'why' or 'what' may also be used.

Key observations:

This year candidates did not perform well in questions based on applications of differentiation, integration and transformation. Candidates had problems with the derivation of functions involving *ln* function and finding equation of tangents from any point to a circle. They performed well in questions related to limits, partial fractions, parabola, ellipse and hyperbola.

Detailed Comments:

Constructed Response Questions (CRQs)

Question 1a:

Find the limit of the following functions.

$$\lim_{x\to 3}\frac{\left(x^2-9\right)^2}{2x^2-12x+18}$$

Better responses exhibited that candidates correctly factorised the expression in the numerator and denominator and cancelled the common factors to simplify the given rational expression to find the required limit.

Example 1:

Simplifying: $\frac{(x^2-9)^2}{2x^2-12x+18}$ => can be written as $\frac{(x^2-3^2)^2}{2x^2-6x-6x+18}$

=> $\frac{\{(x+3)\cancel{(x-3)}\}^2}{(2x-6)\cancel{(x-3)}}$ => $\frac{(x+3)^2(x-3)}{2x-6}$

Taking '2' common on denominator:

=> $\frac{(x+3)^2\cancel{(x-3)}}{2\cancel{(x-3)}}$ => $\frac{(x+3)^2}{2}$ => $\frac{x^2+6x+9}{2}$

Applying limit:

$\lim_{x\to 3}\frac{(3)^2+6(3)+9}{2}$ => $\boxed{18}$ → Answer!

Example 2:

$$\lim_{x \to 3} \frac{(x^2-9)(x^2-9)}{2(x^2-6x+9)}$$

$$\lim_{x \to 3} = \frac{(x+3)\cancel{(x-3)}\ (x^2-9)}{2[\cancel{(x-3)}(x-3)]}$$

$$\lim_{x \to 3} \frac{(x+3)\ (x+3)\cancel{(x-3)}}{2[\cancel{(x-3)}]}$$

Apply limit $x \to 3$. $\quad \frac{(3+3)(3+3)}{2}$.

18 ans.

Weaker responses showed that the candidates have misconceptions in the concept of limit and made errors in calculating the given limit. The errors made were during the process of factorisation, cancellation and substitution of limits. Few candidates applied limit directly and wrote answer of $\frac{0}{0}$ as ∞ , which was incorrect. In few other responses, it is noted that candidates took x^4 or x^2 of common from numerator and denominator, which was again a wrong technique to find the given limit. The mistakes in the arithmetical operations are also evident in some responses.

Example 1:

$$\lim_{x \to 3} \frac{(x^2)^2 - 2(x^2)(9) + (9)^2}{2x^2 - 12x + 18}$$

$$\lim_{x \to 3} \frac{x^4 - 18x^2 + 81}{2x^2 - 12x + 18}$$

applying limit $x \Rightarrow$

$$\frac{(3)^4 - 18(3)^2 + 81}{2(3)^2 - 12(3) + 18}$$

$$\frac{81 - 162 + 81}{18 - 36 + 18}$$

$$\frac{0}{0} = \propto \text{(infinity)}$$

Example 2:

$$\lim_{x \to 3} \frac{(x-3)^{\cancel{2}}(x+3)^2}{\cancel{(x-3)}(x-2)}$$

$$\lim_{x \to 3} \frac{(x-3)(x+3)^2}{(x-2)}$$

[illegible] $x = 3$.

$$\frac{(3-3)(3+3)^2}{(3-2)}$$

$$\frac{(0)(36)}{1}$$

$$\Rightarrow 0$$

Example 3:

$$\lim_{x\to 3} \frac{x^4 - 18x^2 + 81}{2x^2 - 12x + 18}$$

$$\lim_{x\to 3} \frac{x^2 - 18 + 81/x^2}{2 - 12/x + 18/x^2}$$

$$\frac{(3)^2 - 18 + 81/9}{2 - 12/3 + 18/9}$$

$$\frac{\cancel{9} - 18 + \cancel{9}}{\cancel{2} - 4 + \cancel{2}}$$

$$\frac{+16}{+4}$$

Hence,

$$\lim_{x\to 3} \frac{(x^2-9)^2}{2x^2-12x+18} = 18/4$$

Question 1b:

Find the limit of the following functions.

$$\lim_{x\to 0}\left(\frac{e^{e^x} - e^x}{e^{e^x} + e^x}\right)$$

It was based on the two simple steps to find the limit. The first step was to put the limit directly and second step was to simplify it.

Better responses exhibited that candidates applied the limit correctly and they were knowledgeable of the fact that $e^{e^o} = e$ not $e^{e^o} = 1$ and therefore, they found the required limit appropriately.

Example 1:

$$= \frac{\lim_{x\to 0}\left(e^{e^x} - e^x\right)}{\lim_{x\to 0}\left(e^{e^x} + e^x\right)} = \frac{\lim_{x\to 0} e^{e^x} - \lim_{x\to 0} e^x}{\lim_{x\to 0} e^{e^x} + \lim_{x\to 0} e^x}$$

$$= \frac{e^{e^0} - e^0}{e^{e^0} + e^0} = \frac{e^1 - 1}{e^1 + 1} = \frac{e-1}{e+1}$$

Example 2

$x\to0 \left(e^{e^x} + e^x\right)$

$$\frac{\lim\limits_{x\to0} e^{e^x}-e^x}{\lim\limits_{x\to0} e^{e^x}+e^x} = \frac{\lim\limits_{x\to0} e^{e^x} - \lim\limits_{x\to0} e^x}{\lim\limits_{x\to0} e^{e^x} + \lim\limits_{x\to0} e^x}$$

$$\frac{e^{e^0}-e^0}{e^{e^0}+e^0} = \frac{e^1-1}{e^1+1}$$

Weaker responses showed that candidates applied different techniques that separated the numerator and denominator or tried to simplify $\left(\dfrac{e^{e^x}-e^x}{e^{e^x}+e^x}\right)$, which was not required. Few other weaker responses reported that candidates failed to write $e^{e^o}=1$ and hence, were unable to find the required limit.

Example 1:

$$\lim_{x\to0}\left(\frac{e^{ex}}{e^{ex}+e^x} - \frac{e^x}{e^{ex}+e^x}\right)$$

apply limit $x\to0$ $\left(\dfrac{e^{e(0)}}{e^{e(0)}+e^0} - \dfrac{e^0}{e^{e(0)}+e^0}\right)$ $\therefore e^0 = 1$

$+\dfrac{1}{2} - \dfrac{1}{2} = 0$ Answer $= 0$.

Example 2:

$$\lim_{x \to 0}\left(\frac{\cancel{e^{ex}}}{\cancel{e^{ex}}} + \frac{-e^{x}}{e^{x}}\right) \qquad 1 + \left(\frac{-e^{0}}{e^{0}}\right)$$

$$\lim_{x \to 0}\left(1 + \left(\frac{-e^{x}}{e^{x}}\right)\right) \qquad \cancel{1+\frac{1}{1}} \qquad 1 + \left(\frac{-1}{1}\right)$$

Apply limit $\qquad 1 - 1/1 = 0$

Example 3:

$$\lim_{x \to 0} \frac{\left(e^{ex} - e^{x}\right)}{e^{ex} + e^{x}} \qquad \therefore e^{0} = 1$$

$$\lim_{x \to 0} \frac{e^{e(0)} - e^{0}}{e^{e(0)} + e^{(0)}} = \frac{e^{0} - e^{0}}{e^{0} + e^{0}} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

Question 2ai:

If $y = e^{x+\tan x}$, then show that $\frac{dy}{dx} = 2y + y\tan^2 x$.

Better responses exhibited that the candidates were systematic in their approach they correctly found derivative of $y = e^{x+\tan x}$ as $\frac{dy}{dx} = e^{x+\tan x}\left(1+\sec^2 x\right)$ and substituted $\sec^2 x = 1+\tan^2 x$ and $y = e^{x+\tan x}$ to prove the required result skillfully.

Example 1:

$y = e^{x+\tan x}$

$\frac{dy}{dx} = e^{x+\tan x} \frac{d}{dx}(x + \tan x)$

$\frac{dy}{dx}$ $\cancel{\frac{d}{dx}}$ $= e^{x+\tan x}(1 + \sec^2 x)$

$\cancel{= e^{x+\tan x}(\ldots \tan^2 x)}$

$\frac{dy}{dx} = e^{x+\tan x}(+1+1-1+\sec^2 x)$

$\frac{dy}{dx} = e^{x+\tan x}(2 + \cancel{\sec^2 x}\ \tan^2 x)$

$\frac{dy}{dx} = 2e^{x+\tan x} + e^{x+\tan x}(\tan^2 x$

$y = e^{x+\tan x}$; which is equal to

$\frac{dy}{dx} = 2y + y\tan^2 x$ proved

Example 2:

$\frac{dy}{dx} = (1+\sec^2 x)(e^{x+\tan x})$

$dy/dx = e^{x+\tan x} + (e^{x+\tan x})(\sec^2 x)$ $\because \sec^2 x = 1+\tan^2 x$

$dy/dx = e^{x+\tan x} + (e^{x+\tan x})(1+\tan^2 x)$

$dy/dx = e^{x+\tan x} + e^{x+\tan x} + (\tan^2 x)(e^{x+\tan x}) \longrightarrow (1)$

as given in question $y = e^{x+\tan x}$ ∴ replace it in above eq

$y + y + \tan^2 x(y)$

$\frac{dy}{dx} = 2y + \tan^2 x(y)$ proved

Weaker responses reflected lack of understanding of the concepts of derivatives of exponential function coupled with trigonometric function. They failed to find the derivative of the given function. In case the derivative was found, they were unable to make the proper substitutions to prove the required result, i.e. $\frac{dy}{dx} = 2y + y\tan^2 x$.

Example 1:

$$\frac{dy}{dx} = \frac{d}{dx} e^{x+\tan x}.$$

$$\frac{dy}{dx} = e^{x+\tan x}.\ 1+\sec^2 x.$$

Example 2:

$$\frac{dy}{dx} = e^{x+\tan x} \frac{d}{dx}(x+\tan x)$$

$$\frac{dy}{dx} = e^{x+\tan x}(1+\sec^2 x)$$

$$\because \tan^2 x = 1+\sec^2 x$$

$$= e^{x+\tan x} \tan^2 x$$

Example 3:

$$\frac{dy}{dx} = e^{x+\tan x} \frac{d}{dx}(x+\tan x)$$

$$\frac{dy}{dx} = e^{x+\tan x} \cdot -\cot x$$

$$\frac{dy}{dx} =$$

Question 2aii:

For $y = \left(\sqrt{3x-2}\right)\left(\sqrt{3x+2}\right)$**, show that** $\dfrac{dy}{dx} = \dfrac{9x}{\sqrt{9x^2-4}}$.

This was generally a well-attempted question.

Better responses exhibited that candidates applied the correct formula of $a^2 - b^2$ and took the derivative of y with respect to x to prove the required result.

Example 1:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{3x-2}\right)\left(\sqrt{3x+2}\right)$$

$$= \frac{3}{2\sqrt{3x-2}}\left(\sqrt{3x+2}\right) + \frac{3}{2\sqrt{3x+2}}\left(\sqrt{3x-2}\right)$$

$$= \frac{3}{2}\left\{\frac{\sqrt{3x+2}}{\sqrt{3x-2}} + \frac{\sqrt{3x-2}}{\sqrt{3x+2}}\right\}$$

$$= 3/2\left\{\frac{\left(\sqrt{3x+2}\right)^2 + \left(\sqrt{3x-2}\right)^2}{\left(\sqrt{3x-2}\right)\left(\sqrt{3x+2}\right)}\right\}$$

$$= 3/2\left\{\frac{6x}{\sqrt{9x^2+6x-6x-4}}\right\}$$

$$\frac{dy}{dx} = \frac{9x}{\sqrt{9x^2-4}}$$

Example 2:

$$y = \left[(3x-2)(3x+2)\right]^{1/2}$$

$$y = (9x^2-4)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}(9x^2-4)^{1/2}} \times \overset{9}{\cancel{18}}x$$

$$\frac{dy}{dx} = \frac{9x}{\sqrt{9x^2-4}} \qquad \text{proved}$$

Weaker responses reflected that the candidates either failed to apply the correct formula of $a^2 - b^2$ or made mistakes in finding the derivative of $y = \left(\sqrt{9x^2 - 4}\right)$ and therefore, failed to show the required result.

Example 1:

$y = (\sqrt{3x-2})(\sqrt{3x+2})$

$u \cdot v$

$u \frac{d}{dx} v' + v \frac{d}{dx} u$

$(\sqrt{3x-2}) \frac{d}{dx} \frac{1}{2\sqrt{3x+2}} + (\sqrt{3x+2}) \frac{d}{dx} \frac{1}{\sqrt{3x-2}}$

$\frac{dy}{dx} = \frac{9x}{\sqrt{9x^2-4}}$ proved

Example 2:

$\frac{dy}{dx} = \frac{9x}{\sqrt{9x^2-4}} \times \frac{\sqrt{9x^2-4}}{\sqrt{9x^2-4}}$

$\frac{dy}{dx} = \frac{9x\sqrt{9x^2-4}}{9x^2-4}$

$\frac{dy}{dx} = \frac{9x\sqrt{(3x)^2-(2)^2}}{(3x)^2-(2)^2}$

$\frac{dy}{dx} = \frac{9x\sqrt{(3x-2)(3x+2)}}{(3x-2)(3x+2)}$

$\therefore y = (\sqrt{3x-2})(\sqrt{3x+2})$

$\frac{d}{dx}(9x) = 1$

$\frac{d}{dy} = (\sqrt{(3x-2})(\sqrt{3x+2})$

Hences proved

Example 3:

another way $= ((3x-2)(3x+2))^{1/2}$ $\quad (\cancel{3x})\ (3x)$

$(9x^2 + \cancel{6x} - \cancel{6x} - 4)^{1/2}$ $\quad \sqrt{(3x)^2 - (-2)^2}$

$(9x^2 - 4)^{1/2}$ $\quad 9x$

$\frac{d}{dx}\sqrt{9x^2-4}$ $\quad \therefore \frac{d}{dx}\sqrt{x} = \frac{1}{\sqrt{x}}$ $\quad \{(\sqrt{3x-\cancel{2}})(\sqrt{3x+\cancel{2}})$

$3x \times 3x$

$\frac{dy}{dx} = \frac{9x}{\sqrt{9x^2-4}}$ Ans $\quad = 9x$

Multiplication

$\sqrt{3x-2}\,\frac{d}{dx}\frac{1}{2}(3x+2)^{-1/2} + \sqrt{3x-2}\,\frac{d}{dx}(3x+2)^{-1/2}$

Question 2b:

For $y = \ln\left(\frac{\sqrt{x}\cos x}{\sin x}\right)$,

i. find $\frac{d^2y}{dx^2}$

ii. Hence, find the value of $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{4}$

Generally this was a poorly attempted question and the basic reason was that the majority of the candidates failed to apply the laws of logarithm, i.e. $y = \frac{1}{2}\ln x + \frac{1}{2}\ln\cos x - \ln\sin x$.

Better responses indicated that the candidates understood the requirement of the question and they first applied the laws of logarithm and solved the questions. But failure to use the laws of logarithm made it lengthy for the candidates. In such cases, candidates did extra efforts and spent more time to find the required derivative and its value at $x = \frac{\pi}{4}$.

Example 1:

$$y = \ln \frac{\sqrt{x\cos x}}{\sin x}$$

$$y = \ln\sqrt{x\cos x} - \ln \sin x$$

$$y = \frac{1}{2}\ln x\cos x - \ln \sin x$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\ln x\cos x) - \frac{d}{dx}\ln \sin x$$

$$= \frac{1}{2}\left[\frac{1}{x\cos x}\frac{d}{dx}(x\cos x)\right] - \frac{\cos x}{\sin x}$$

$$= \frac{1}{2}\left[\frac{1}{x\cos x}\cdot \cos x - x\sin x\right] - \cot x$$

$$= \frac{\cos x - x\sin x}{2x\cos x} - \cot x$$

$$= \frac{\cancel{\cos x}}{2x\cancel{\cos x}} - \frac{\cancel{x}\sin x}{2\cancel{x}\cos x} - \cot x$$

$$= \frac{1}{2x} - \frac{1}{2}\tan x - \cot x$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{2x} - \frac{1}{2}\tan x - \cot x\right)$$

$$= \frac{1}{2}\frac{d(x^{-1})}{dx} - \frac{1}{2}\frac{d}{dx}\tan x - \frac{d}{dx}\cot x$$

$$= \frac{-1}{2x^2} - \frac{1}{2}\sec^2 x + \operatorname{cosec}^2 x$$

Weaker responses reflected various types of errors including failure or wrong application of the laws of logarithm, incorrect application of product and quotient rule of differentiation. They also reflected the wrong derivatives of sin$x$ and cos$x$. Particularly, they had difficulty in finding the second derivatives.

Example 1:

~~we~~ Apply Quotient law $\frac{d}{dx} = U/V = \frac{U'V - V'U}{V^2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\ln \frac{\sqrt{x\cos x}}{\sin x}\right]$$

$\frac{d}{dx}\sqrt{x\cos x}$ $\Rightarrow \frac{d}{dx}\sin x$

$\frac{dV}{dx} = \frac{1}{2\left(\sqrt{x\cos x}\right)}$ $\frac{dV}{dx} = \cos x$

$$\frac{dy}{dx} = \frac{1}{\frac{\sqrt{x\cos x}}{\sin x}} \quad \frac{d}{dx}\frac{\sqrt{x\cos x}}{\sin x} \begin{matrix} \rightarrow U \\ \rightarrow V \end{matrix}$$

$$\frac{dy}{dx} = \frac{\sin x}{\sqrt{x\cos x}} \times \left[\frac{\left(\frac{1}{2\sqrt{x\cos x}} \sin x\right) - \cos x \sqrt{x\cos x}}{\sin^2 x}\right]$$

$$\frac{dy}{dx} = \frac{\sin x}{\sqrt{x\cos x}}\left[\frac{\frac{\sin x - 2x\cos^2 x}{2\sqrt{x\cos x}}}{\sin^2 x}\right]$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x\cos x}} \times \left[\frac{\sin x - 2x\cos^2 x}{2\sqrt{x\cos x}\,\sin x}\right]$$

$$\frac{dy}{dx} = \frac{\sin x - 2x\cos^2 x}{2\cos^2 x \sin x}$$

Example 2:

$\cancel{\frac{dy}{dx} = \frac{\sqrt{x\cos x}}{\sin x} \frac{d}{dx}}$

$\frac{dy}{dx} = \frac{\sin x}{\sqrt{x\cos x}} \frac{d}{dx}\left(\frac{\sqrt{x\cos x}}{\sin x}\right) = \frac{\sin x}{\sqrt{x\cos x}}\left\{\sin x \frac{1}{2\sqrt{x\cos x}}(\cos x - x\sin x) - \sqrt{x\cos x}\ \cos x\right\}$

$= \frac{\sin x}{\sqrt{x\cos x}}\left(\frac{\sin x(\cos x - x\sin x) - \sqrt{x\cos x}\cos x}{2\sqrt{x\cos x}}\right)$

$= \frac{\sin x}{\sqrt{x\cos x}} \frac{\sin x}{2x\cos x}\left[\sin x\cos x - x\sin^2 x - 2x\cos^2 x\right] = \frac{\sin x}{2x\cos x}\left(\sin x\cos x - x - x\cos^2 x\right)$

$= \frac{\sin^2 x}{2x} - \frac{\sin x}{2\cos x} - \frac{\sin x\cos x}{2} = \frac{1}{2}$

$= \frac{1}{2}\left(\frac{\sin^2 x}{x} - \tan x - \sin x\cos x\right) =$

$\frac{d^2y}{dx^2} = \frac{1}{2}\left(x \cdot 2\sin x\cos x + \sin^2 x - \sec^2 x - \cos^2 x + \sin^2 x\right)$

Question 2c:

A water tank is cylindrical in shape. If the volume of the tank is 64 m^3, then what would be the radius and height of the water tank to have the least surface area? Write you answer to two decimal places.

(**Note**: The volume of a cylinder is $V = \pi r^2 h$ and surface area of a cylinder is $S = 2\pi r^2 + 2\pi rh.$ Where r is the radius of the cylinder and h is the height of a cylinder.)

This question was based on the application of the differentiation process. It was generally a least attempted question.

Better responses indicated that the candidates understood the question well and correctly applied all the necessary steps in sequence to find the height and radius of the water tank to calculate least surface area. They first found the value of $h = \frac{64}{\pi r^2}$ and substituted the value in $S = 2\pi r^2 + 2\pi rh$ to get $S = 2\pi r^2 + 128r^{-1}$. Then, they found the stationery point or the value of r with the help of first derivative. They applied the second derivative test to ensure the minimum value of r and found the height and radius to ensure the minimum value of surface area correctly.

Example 1:

$V = 64 = \pi r^2 h$

$64 = \pi r^2 h$

$h = \dfrac{64}{\pi r^2}$ —i,

$S = 2\pi r^2 + 2\pi r h$

$S = 2\pi r^2 + 2\cancel{\pi} r\left(\dfrac{64}{\cancel{\pi} r^{\cancel{2}}}\right)$

$S = 2\pi r^2 + \dfrac{128}{r}$

$\dfrac{dS}{dr} = 4\pi r + \dfrac{128(-1)}{r^2}$

$\dfrac{dS}{dr} = 4\pi r - \dfrac{128}{r^2}$

For Stationary Point $\frac{dS}{dr} = 0$

$0 = 4\pi r - 128/r^2$

$\dfrac{128}{r^2} = 4\pi r$

$128 = 4\pi r^3$

$r^3 = 128/4\pi$

$\boxed{r = 2.17\,m}$

$\dfrac{d^2S}{dr^2} = 4\pi - \dfrac{128(-2)}{r^3}$

$\dfrac{d^2S}{dr^2} = 4\pi + \dfrac{128}{r^3}$

$r = 2.17$

$d^2S/dr^2 = 4\pi + 128/(2.17)^3$

$d^2S/dr^2 = 25.09 \ge 0$

$d^2S/dr^2 > 0 \rightarrow$ relative minimum value

i, $\Rightarrow h = \dfrac{64}{\pi(2.17)^2}$

$\boxed{h = 4.33\,m}$

Radius should be 2.17 m and height should be 4.33 m.

Weaker responses reflected a lack of understanding of the process of minimization. They tried to solve it arithmetically which was surely not the right way to solve such problems. Few responses showed that candidates solved the problem by applying the integration method which is also not a solution to such problems.

Example 1:

$\pi r^2 h = 64$ $\quad\quad V = 64m^3$

$3.1416r^2h = 64$ $\quad\quad r = ?$

$64/3.1416 = r^2 h$ $\quad\quad h = ?$

$\frac{64}{3.1416xh} = r^2$ $\quad\quad S = 2\pi r^2 + 2\pi rh$

$r = \sqrt{64/3.1416xh}$

$\boxed{r = 8/\sqrt{3.14xh}}$

$\boxed{h = 64/\pi r^2}$

$\cancel{h = 64/3.14(64/3.14xh)}$

Example 2:

$V = \pi r^2 h$

$S = 2\pi r^2 + 2\pi rh \quad$ —ii,

$64 = \pi r^2 h$

From ii, $\quad\quad \frac{64}{h} = \pi r^2$

$S = 2\pi r^2 + 2\pi rh \; h$

$\int \frac{64}{h} = \int \pi r^2$

Question 3a:

i. Convert $\dfrac{3x}{(x^2-4)(x^2+4)}$ into its partial fractions.

Better responses correctly converted the given rational expression into its partial fractions. The candidates used the suitable form, i.e. $\dfrac{3x}{(x^2-4)(x^2+4)} = \dfrac{A}{x-2} + \dfrac{B}{x+2} + \dfrac{Cx+D}{x^2+4}$ to correctly find the values of *A, B, C* and *D* to fulfil the requirement of the question.

Example:

$$\frac{3x}{(x+2)(x-2)(x^2+4)} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{Cx+D}{x^2+4}$$

$$\frac{3x}{(x+2)(x-2)(x^2+4)} = \frac{A(x-2)(x^2+4) + B(x+2)(x^2+4) + (Cx+D)(x^2-4)}{\cancel{(x+2)(x-2)(x^2+4)}}$$

Put $x=2$, $3(2) = A(0) + B(4)(8) + (2C+D)(0)$

$6 = B32$ Put $x=-2$, $3(-2) = A(-4)(8) + 0 + 0$

$\boxed{B = 3/16}$ $-6 = -32A$ $\boxed{A = 3/16}$

$3x = A(x^3+4x-2x^2-8) + B(x^3+4x+2x^2+8) + Cx^3 - 4Cx + Dx^2 - 4D$

$3x = 4Ax + 4Bx - 4Cx$, $3x = x(4A+4B-4C)$, $3 = 3/4 + 3/4 - 4C$ $\boxed{C = -3/8}$

$0 = -8A + 8B - 4D$, $0 = -8(3/16) + 8(3/16) - 4D$ $\boxed{D=0}$

$$\frac{3x}{(x^2-4)(x^2+4)} = \frac{3}{16(x+2)} + \frac{3}{16(x-2)} + \frac{-3}{8(x^2+4)}$$

Weaker responses exhibited that candidates were failed to select the appropriate form of the partial fractions. As a consequence, they failed to find the required values of *A, B, C* and *D*. In few responses, it was noted that candidates used the correct form but due to calculation mistake, were unable to find the values of constants involved in the partial fractions.

Example 1:

$$\frac{3x}{(x+2)(x-2)(x-2)^2} = \frac{A}{(x+2)} + \frac{B}{(x-2)} + \frac{C}{(x+2)^2}$$

$$3x = A(x-2)(x+2) + B(x+2)^2 + C(x-2) \quad -1$$

$$3x = Ax^2 - 4A + Bx^2 + 4Bx + 4B + Cx - 2C \quad -2$$

put $x=-2$ in eq 1

$$-6 = A(0) + B(0) + C(-4)$$

$$-\cancel{6}^{3} = -\cancel{4}^{2}C$$

$$\boxed{C = 3/2}$$

put $x = 2$ in eq 1

$$6 = A(0) + B(4)^2 + C(0)$$

$$\cancel{6}^{3} = \cancel{16}^{8}B$$

$$\boxed{B = 3/8}$$

$$0 = A + B \Rightarrow 0 = A + 3/8 \Rightarrow \boxed{A = -3/8}$$

$$\boxed{\frac{-3}{8(x+2)} + \frac{3}{8(x-2)} + \frac{3}{2(x+2)^2}}$$

Example 2:

$$\frac{3x}{(x^2-4)(x^2+4)} = \frac{A}{x^2-4} + \frac{B}{x^2+4}$$

$$\underline{Ax^2 + 4A + Bx^2 - 4B}$$

$$x - 16.$$

$\boxed{\therefore x=2}$ $A(2)^2 + 4A + B(2)^2 - 4B / 2 - 16$

$= 4A + 4A + \cancel{4B} - \cancel{4B} / -14$

$= 8A / -14 \Rightarrow 8A = 14 \Rightarrow 14/8$

$A = 7/4$

$\boxed{\therefore x=-2}$ $A(-2)^2 + 4A + B(-2)^2 - 4B / -2 - 16$

$= 4A + 4A + \cancel{4B} - 4B / -18$

$8A / -18 \Rightarrow 8A = 18 \Rightarrow A = 18/8 = 9/4$

Comparing co-efficient of x^2.

$2 = A + B.$

$2 = 7/4 + B.$

$2 \times 4 = 7 + B \Rightarrow 8 - 7 = B \Rightarrow 1 = B.$

Example 3:

$$\frac{3x}{(x^2-4)(x^2+4)} = \frac{A}{x-4} + \frac{B}{x+4} + \frac{Cx+D}{x^2+4}$$

xply and divide by $(x^2-4)(x^2+4)$

$$3x = A(x+4)(x^2+4) + B(x-4)(x^2+4) + (Cx+D)(x-4)(x+4)$$

Put $x = -4$

$$3(-4) = B(-4-4)((-4)^2+4) \Rightarrow -12 = B(-8)(20)$$

$-12 = B(-160)$

$\boxed{B = \frac{3}{40}}$

Put $x = 4$

$3(4) = A(4+4)(16+4)$

$12 = A(8)(20)$

$\boxed{A = \frac{3}{40}}$

Comparing coefficients of x^3

$0 = A + B + C$

$0 = \frac{3}{40} + \frac{3}{40} + C$

$0 = \frac{6}{40} + C$

$-\frac{6}{40} = C \Rightarrow \boxed{C = -5}$

comparing coefficients of x^2

$0 = 4A - 4B + D$

$0 = 4\left(\frac{3}{40}\right) - 4\left(\frac{3}{40}\right) + D$

$0 = \frac{3}{10} - \frac{3}{10} + D$

$0 = \frac{0}{10} + D$

$\boxed{D = 0}$

$$\boxed{\frac{3x}{(x^2-4)(x^2+4)} = \frac{3}{40(x-4)} + \frac{3}{40(x+4)} - \frac{5x}{x^2+4}}$$

Question 3a:

ii. Hence, find $\int \frac{3x}{(x^2-4)(x^2+4)}dx$

Better responses correctly converted the given rational expression into its partial fractions. The candidates therefore, were succeeded in finding the required integral by using the formula of $\int \frac{dx}{x}dx = \ln x + C.$ They also converted $\int \frac{x}{x^2+4}dx$ into $\frac{1}{2}\int \frac{2x}{x^2+4}dx$ by multiplying and dividing by 2 to get a proper form to integrate $\int \frac{x}{x^2+4}dx.$

$$\frac{6}{32}\int \frac{1}{x+2}dx + \frac{6}{32}\int \frac{1}{x-2}dx - \frac{3}{4}\int \frac{x}{(x^2+4)}dx$$

$$\frac{6}{32}\ln|x+2| + \frac{6}{32}\ln|x-2| - \frac{3}{4}\times\frac{1}{2}\int \frac{2x}{x^2+4}dx$$

$$\frac{6}{32}\ln|x+2| + \frac{6}{32}\ln|x-2| - \frac{3}{8}\ln|x^2+4| + C$$

Weaker responses suggested that candidates were failed to convert the given fraction into its partial fractions in part **i** hence, failed to find $\int \frac{3x}{(x^2-4)(x^2+4)}dx$. Few other responses reported that candidates were able to find the partial fractions but applied wrong technique or wrong formula and they were totally clueless about the correct way of integration of $\int \frac{3x}{(x^2-4)(x^2+4)}dx$.

Example 1:

$$\int \frac{A}{(x^2-4)} + \frac{B}{(x^2+4)}\,dx$$

$$\ln|x^2-4| + \ln|x^2+4$$

Example 2:

$$\Rightarrow \int \frac{3}{2(x-2)} + \frac{6}{32(x+2)} + \frac{\frac{1}{16}x + {}^{21}/_{8}}{x^2+4}\,dx$$

$$\Rightarrow \frac{3}{2}\ln(x-2) + \frac{6}{32}(\ln(x+2)) + \int \frac{\frac{x+336}{12}}{x^2+4}\,dx$$

$$\Rightarrow \frac{3}{2}\ln(x-2) + \frac{6}{32}(\ln(x+2)) +$$

Question 3b:

Evaluate the following indefinite integrals.

i. $\int \left(4x^4 - 12x^2 + 9\right)^{\frac{3}{2}} x\,dx$

Better responses exhibited that the candidates correctly applied the formula of $(a+b)^2$ to convert $\left(4x^4 - 12x^2 + 9\right)^{\frac{3}{2}}$ into $\left(2x^2 - 3\right)^2$ to get $\int\left(2x^2-3\right)^3(x)dx$, then they applied correct technique of integration to find $\int \left(4x^4 - 12x^2 + 9\right)^{\frac{3}{2}} x\,dx$. Few other responses indicated that the candidates also applied the substitution method.

Example 1:

$$\int\left(4x^4 - 6x^2 - 6x^2 + 9\right)^{3/2} x$$

$$\int\left(2x^2-3\right)^{2\times 3/2} x\,dx \qquad u = 2x^2 - 3$$

$$\frac{1}{4}\int\left(2x^2-3\right)^3 4x\,dx \qquad du = 4x\,dx$$

$$\frac{1}{4}\int u^3\,du = \frac{1}{4}\,\frac{u^4}{4} + c = \frac{(2x-3)^4}{16} + c$$

Weaker responses reflected lack of concept of integration techniques and they made different type of errors in the process of integration. These errors were in the simplification and expansion of the given expression and therefore, they failed to complete the process of integration.

Example 1:

$$\int\left(4x^{4}\right)^{\frac{3}{2}} - \int\left(12x^{2}\right)^{\frac{3}{2}} + \int(9)^{\frac{3}{2}}$$

$$\cancel{\int 4x^6 - \int 12x^3 +}$$

$$\frac{4x^{4+\frac{3}{2}+\frac{1}{2}}}{4+1} - \frac{12x^{2+\frac{3}{2}+\frac{1}{2}}}{2+1} + \cancel{9^{\frac{3}{2}}}$$

$$\boxed{\frac{4x^6}{5} - \frac{12x^4}{3} + c}$$

Example 2:

$$\Rightarrow \int \left(\left(2x^2-3\right)^2\right)^{3/2} dx \Rightarrow \int \left(2x^2-3\right)^3 dx$$

$$\Rightarrow \int \left(8x^6 - 12x^4 - 24x^4 - 36x^2 + 18x^2 + 27\right) dx$$

$$\Rightarrow \int \left(8x^6 - 36x^4 - 18x^2 - 27\right) dx$$

$$\Rightarrow \frac{8}{7}x^7 - \frac{36}{5}x^5 - \frac{18}{3}x^3 - 27x + C$$

Question 3b:

Evaluate the following indefinite integrals.

ii. $\int \frac{x}{(x+b)^2}\,dx$

Better responses exhibited that candidates were conceptually well versed with the concept of integration. They applied correct techniques, i.e. proper substitution or added or subtracted b in the denominator to get $\int \frac{x+b-b}{(x+b)^2}\,dx = \int \frac{dx}{(x+b)} - \int \frac{b}{(x+b)^2}dx$ and hence, they were able to effectively demonstrate their skills in integration.

Example 1:

Let $x+b=y$

$x = b-y$

$\frac{dy}{dx} = 1$

$dy = dx$

$\int \frac{x\,dx}{(x+b)^2} = \int \frac{b-y}{y^2}\,dy$

$= \int \frac{b\,dy}{y^2} \int \frac{\cancel{y}}{y^{\cancel{2}}}\,dy$

$= b\int \frac{1}{y^2}\,dy - \int \frac{1}{y}\,dy$

$= b\int y^{-2} - \ln y + c_1$

$= b\frac{y^{-2+1}}{-2+1} - \ln y + c_1 + c_2$

$= -by^{-1} - \ln y + c \quad \because c_1 + c_2 = c$

$= -\frac{b}{y} - \ln y + c$

Replacing y by $x+b$

$= \frac{-b}{x+b} - \ln|x+b| + c$

Example 2:

$\int \frac{x}{(x+b)^2}$

$= \int \frac{x+b-b}{(x+b)^2}\,dx$

$\int \left(\frac{\cancel{x+b}}{(x+b)^{\cancel{2}}} - \frac{b}{(x+b)^2}\right) dx$

$\int \frac{1}{x+b}\,dx - b\int \frac{1}{(x+b)^2}\,dx$

$\ln(x+b) - b\int (x+b)^{-2}\,dx$

$\ln(x+b) - \frac{b(x+b)^{-2+1}}{-2+1} + c$

$\ln(x+b) + \frac{b}{(x+b)} + c$

Ans

Weaker responses reflected a lack of concept of integration. The candidates were failed to bring the given expression in the proper form to integrate it. They used wrong formulae or techniques and hence, were unable to find the required integral.

Example 1:

$$\int \frac{x}{x^2+2bx+b^2}\,dx$$

$$\left(\int (x)\,dx\right)\left(\int (x^2+2bx+b^2)\,dx\right)$$

$$\left(\frac{x^2}{2}\right)\left(\frac{x^3}{3}+2b\frac{x^2}{2}\right)+C$$

Example 2:

$$\int (x)(x+b)^{-2}\,dx$$

let $u = x+b$, $x = u-b$

$$\frac{du}{dx} = 1, \quad du = dx.$$

$$\int (u-b)(u)^{-2}\,dx.$$

Question 3c:

Evaluate the following definite integrals by showing all the necessary steps. Give your answer in terms of *a*.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{a + a\cos x}$$

It was generally not a well attempted question.

Better responses exhibited a good understanding of concept of integration and its different techniques. In this question, they multiplied and divided $\frac{dx}{a+a\cos x}$ by $1-cox$ to get $=\frac{1}{a}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos ec^2 x dx - \frac{1}{a}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos ecx \cot xdx$ and were able to integrate the given function. Finally, they skillfully applied the limits to find the definite value of the given integral.

Example 1:

$$= \int_{-\pi/4}^{\pi/4} \frac{dx}{a(1+\cos x)}$$

$$= \frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{1}{1+\cos x} \times \frac{1-\cos x}{1-\cos x}\, dx$$

$$= \frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{1-\cos x}{1-\cos^2 x}\, dx$$

$$= \frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{1-\cos x}{\sin^2 x}\, dx$$

$$= \frac{1}{a}\int_{-\pi/4}^{\pi/4} \left(\frac{1}{\sin^2 x} - \frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}\right) dx$$

$$= \frac{1}{a}\left[\int_{-\pi/4}^{\pi/4} \operatorname{Cosec}^2 x\, dx - \int_{-\pi/4}^{\pi/4} \operatorname{Cot} x \operatorname{Cosec} x\, dx\right]$$

$$= \frac{1}{a}\left(-\left|\operatorname{Cot} x\right|_{-\pi/4}^{\pi/4} - \left|\operatorname{Cosec} x\right|_{-\pi/4}^{\pi/4}\right)$$

$$= \frac{1}{a}\left[-\left\{\operatorname{Cot}\left(\frac{\pi}{4}\right) - \operatorname{Cot}\left(\frac{-\pi}{4}\right)\right\} - \left\{\operatorname{Cosec}\left(\frac{\pi}{4}\right) - \operatorname{Cosec}\left(\frac{-\pi}{4}\right)\right\}\right]$$

$$= \frac{1}{a}\left[-\left(1-(-1)\right) - \left(\sqrt{2} - (-\sqrt{2})\right)\right]$$

$$= \frac{1}{a}\left(\left[-(1+1)\right] - \left[\sqrt{2}+\sqrt{2}\right]\right)$$

$$= \frac{1}{a}\left(-2 - 2\sqrt{2}\right)$$

$$= \frac{-2-2\sqrt{2}}{a} = \boxed{\frac{-2(1+\sqrt{2})}{a}} \text{ Ans}$$

Example 2:

$$\int_{-\pi/4}^{\pi/4} \frac{dx}{a(1+\cos x)}$$

$$\frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos x} \times \frac{(1-\cos x)}{(1-\cos x)}$$

$$\frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{dx\,(1-\cos x)}{1-\cos^2 x}$$

$$\frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{dx\,(1-\cos x)}{\sin^2 x}$$

$$\frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{(1-\cos x)\,dx}{\sin^2 x}$$

$$\frac{1}{a}\int_{-\pi/4}^{\pi/4} \operatorname{cosec}^2 x\,dx - \frac{1}{a}\int_{-\pi/4}^{\pi/4} \cot x \operatorname{cosec} x\,dx$$

$$\frac{1}{a}\left(-\cot x\right)_{-\pi/4}^{\pi/4} - \frac{1}{a}\left(-\operatorname{cosec} x\right)_{-\pi/4}^{\pi/4}$$

$$\frac{1}{a}\left[\left(-\cot\frac{\pi}{4}\right) - \left(-\cot\left(-\frac{\pi}{4}\right)\right)\right] + \frac{1}{a}\left[\operatorname{cosec}\frac{\pi}{4} - \operatorname{cosec}\left(-\frac{\pi}{4}\right)\right]$$

$$\frac{1}{a}\left[(-1)-(1)\right] + \frac{1}{a}\left[\sqrt{2}+\sqrt{2}\right]$$

$$\frac{-2}{a} + \frac{2\sqrt{2}}{a}$$

$$\frac{2\sqrt{2}-2}{a}$$

Weaker responses reflected that the candidates were unable to rationalise the denominator of $\frac{dx}{a + a\cos x}$. They made different kind of mistakes and failed to find the value of the given definite integral. It was noted that the candidates used wrong trigonometric identities and were unable to bring it in a proper form to complete the integration process.

Example 1:

$$\int_{-\pi/4}^{\pi/4} \frac{1}{a(1+\cos x)}\,dx.$$

$$= \frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{1}{1+\cos x} \times \frac{1-\cos x}{1-\cos x}.$$

$$\int_{-\pi/4}^{\pi/4} \frac{1-\cos x}{1-\cos^2 x}$$

$$\int_{-\pi/4}^{\pi/4} \frac{1-\cos x}{\sin^2 x}.$$

$$\int_{-\pi/4}^{\pi/4} \frac{1}{\sin^2 x} - \int_{-\pi/4}^{\pi/4} \frac{\cos x}{\sin^2 x}.$$

$$\left|\sin^3 x\right|_{-\pi/4}^{\pi/4} - \left|\sin^3\right|_{-\pi/4}^{\pi/4}$$

$$\left(\sin \pi/4 - \sin \pi/4\right) - \left(\cos \pi/4 - \cos \pi/4\right)$$

Example 2:

$$\frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos x} \times \frac{1-\cos x}{1+\cos x}$$

$$\frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{1-\cos x \; dx}{\sin^2 x}.$$

$$\frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{1-\cos x \; dx}{\frac{1-\cos 2x}{2}}$$

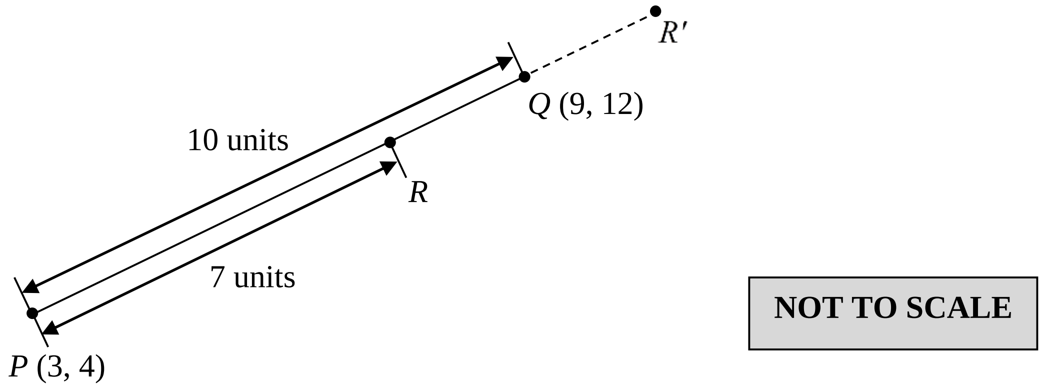
$$\frac{1}{a}\int_{-\pi/4}^{\pi/4} \frac{2(1-\cos x)\,dx}{1-\cos 2x}$$

$$\Rightarrow \frac{2}{a}\int_{-\pi/4}^{\pi/4} \frac{1}{1-\cos 2x} - \int_{-\pi/4}^{\pi/4} \frac{\cos x}{1-\cos 2x}$$

Question 4a:

a. In the given diagram, the length of PQ and PR are 10 units and 7 units respectively.

i. Find the coordinates of R.



Better responses exhibited a good understanding of application of ratio formula for the point of internal division. The candidates were able to clearly differentiate between the formula of point of internal division and external division. Candidates were able to find the correct ratio and find the coordinates of the point *R*.

Example 1:

$x_1=3 \quad y_1=4$

$x_2=9 \quad y_2=12$

$k_1\ 7:3\ k_2$

$$\left(\frac{k_1 x_2 + k_2 x_1}{k_1+k_2}, \frac{k_1 y_2 + k_2 y_1}{k_1+k_2}\right)$$

$$\left(\frac{7(9)+3(3)}{7+3}, \frac{7(4)+3(12)}{7+3}\right)$$

$$\left(36/5, 48/5\right)$$

Example 2:

$PQ = PR+RQ$

$10 = 7+RQ$

$RQ=3.$

Ratio of PR to RQ is 7 : 3.

$$(x,y) = \left(\frac{7(9)+(3)(3)}{10}, \frac{7(12)+(3)(4)}{10}\right)$$

$$(x,y) = \left(\frac{63+9}{10}, \frac{84+12}{10}\right)$$

$$(x,y) = \left(\frac{72}{10}, \frac{96}{10}\right)$$

$$(x,y) = \left(\frac{36}{5}, \frac{48}{5}\right)$$

Weaker responses reflected that candidates were unable to decide between the formulae of points internal and external divisions. Some weaker responses reported the use of distance formula to find the coordinates of point *R*, which was surely an incorrect choice. They failed to find the correct ratio and hence, failed to solve the question correctly.

Example 1:

$$R(x, y) = \left(\frac{m_1 \cdot x_2 + m_2 \cdot x_1}{m_1 + m_2}, \frac{m_1 \cdot y_2 + m_2 \cdot y_1}{m_1 + m_2} \right)$$

$$= \left(\frac{7 \cdot 9 + 10 \cdot 3}{10 + 7}, \frac{7 \cdot 12 + 10 \cdot 4}{10 + 7} \right)$$

$$= \left(\frac{93}{17}, \frac{124}{17} \right)$$

$$= (5.4705, 7.2941) \text{ Ans}$$

Example 2:

$P(3, 4)$

$|PR| = 7 \text{unit}, \quad |RQ| = 3 \text{unit}$

$\because |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|PR| = \sqrt{(x-3)^2 + (y-4)^2}$

$(7)^2 = \left(\sqrt{(x-3)^2 + (y-4)^2} \right)^2$

$49 = (x-3)^2 + (y-4)^2 \rightarrow eq(1)$

$|RQ| = \sqrt{(9-x)^2 + (12-y)^2}$

$(3)^2 = \left(\sqrt{(9-x)^2 + (12-y)^2} \right)^2$

$9 = (9-x)^2 + (12-y)^2 \rightarrow eq(2)$

$eq(2) \rightarrow \cancel{x^2} + \cancel{y^2} - 18x - 24y + 216 = 0$

$eq(1) \rightarrow \cancel{x^2} + \cancel{y^2} - 6x - 8y + 25 = 0$

$ \; (+) \quad (+) \quad (-)$

$-12x - 16y + 191 = 0$

$12x + 16y = -191$

$x = 17/4$

$y = 13/2$

$R\left(\frac{17}{4}, \frac{13}{2} \right)$

Example 3:

$$\frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}, \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2}$$

$m_1 = 7, \quad m_2 = 3$

$$\frac{(7)(9) - (3)(3)}{7 - 3}, \frac{(7)(12) - (3)(4)}{7 - 3}$$

$$\left(\frac{27}{2}, 18 \right)$$

Example 4:

Ratio theorem:- $k_1 : k_2 \rightarrow k_1 = \overline{PR_1} = 2,\ k_2 = \overline{QR} = 1$

For R:- $\left(\frac{k_1x_2 + k_2x_1}{k_1+k_2}, \frac{k_1y_2 + k_2y_1}{k_1+k_2}\right) = (x, y)R$

Let:-

$P(3,4) = P(x_1,y_1)$ and $Q(9,12) = Q(x_2,y_2)$

$R(x,y) = \frac{2(9)+1(3)}{2+1}, \frac{2(12)+1(4)}{2+1}$

$\Rightarrow \frac{18+4}{3} = 7, \frac{24+4}{3} = 9.3$. Hence $\boxed{R(x,y) = (7, 9.3)}$

Question 4a:

ii. If PQ is produced to a point R' such that $RQ = QR'$, then find the coordinates of R'.

Better responses exhibited a good understanding of the concept of midpoint of a line segment. The candidates aptly applied either ratio formula for the point of external division or the midpoint formula. Hence, they were able to find the correct coordinates of the point R'.

Example:

(2 Marks)

Q is mid Point of RR'

$\frac{\frac{36}{5} + x_1}{2} = 9$	$\frac{\frac{48}{5} + y_1}{2} = 12$	coordinates of R'
$x_1 = 18 - \frac{36}{5}$	$y_1 = 24 - \frac{48}{5}$	are $\left(\frac{54}{5}, \frac{72}{5}\right)$
$x_1 = \frac{54}{5}$	$y_1 = \frac{72}{5}$	

Weaker responses reflected that candidates were unable to find the coordinates of the point R'. They used incorrect formula or were unable to identify the correct ratio. It was also noted that candidates were failed to identify x_1, x_2, y_1 and y_2 correctly.

Example 1:

$$m_1 : m_2 = 7:3$$

$$R'(x, y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

$$= \left(\frac{63-6}{4}, \frac{84-12}{4}\right)$$

$$= \left(\frac{57}{4}, \frac{72}{4}\right) \Rightarrow R'(x, y) = \left(\frac{57}{4}, 18\right)$$

Example 2:

external distance $= \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$

$$R' = \left(\frac{63-9}{4}, \frac{119-12}{4}\right)$$

$$R' = \left(\frac{54}{4}, \frac{107}{4}\right)$$

$$R' = \left(\frac{27}{2}, \frac{107}{4}\right)$$

Example 3:

$$R' = \left(\frac{k_1x_2 - k_2x_1}{k_1 - k_2}, \frac{k_1y_1 - k_2y_1}{k_1 - k_2}\right)$$

$$R' = \left(\frac{7(9)-3(3)}{7-3}, \frac{7(12)-3(4)}{7-3}\right)$$

$$R' = \left(\frac{27}{2}, 18\right)$$

Example 4:

PQ = QR'

$\sqrt{(9-3)^2+(12-4)^2} = \sqrt{(x-9)^2+(y-12)^2}$

Squ on both sides

$100 = (x-9)^2 + (y-12)^2$

$100 = x^2 - 18x + 81 + y^2 - 24y + 144$

Question 4b:

Find the equation of the line passing through the point of intersection of the line $x+3y=6$ and $3x+2y=11$, which

i. is perpendicular to the line $5x-y=3$.

Better responses exhibited a good understanding of the equation of straight lines subjected to different conditions. Candidates were able to find the required equation. Mostly the candidates found the point of intersection of the given lines $x+3y=6$ and $3x+2y=11$. Some other responses used the relation $l_1+kl_2=0$ and the condition of perpendicularity to find the slope and hence, were able to find the required equation of the straight line. They also found the slope of the required line and applied the point-slope form correctly.

Example 1:

i. is perpendicular to the line $5x - y = 3$. (3 Marks)

$x+3y=6$

$x=6-3y$ — ①

$3x+2y=11$ put ① in this eq

$\Rightarrow 3(6-3y)+2y=11$

$18-9y+2y=11$

$-7y=11-18$

$y=1$ put value of y in ①

$\Rightarrow x=6-3(1)$

$x=3$

point of intersection is $(3,1)$

slope of line $5x-y=3$ is m_1 (l_1)

compare with $y=mx+c$

$y=5x-3$

$m_1=5$

since required line $\perp$ to l_1

$m_1 \times m_2 = -1$

$m_2 = -1/5$ (slope of required line)

$P(3,1)$ (x_1, y_1) $m_2=-1/5$

$y-y_1=m(x-x_1)$

$y-1=\frac{-1}{5}(x-3)$

$5y-5=-x+3$

$x+5y-5-3=0$

$\boxed{x+5y-8=0}$

Example 2:

solving $x+3y=6$ & $3x+2y=11$ simultaneously to find the point of intersection

$x=3$ $y=1$

$(3,1)$

since the unknown line is perpendicular to $5x-y=3$

& slope of $5x-y=3$ = $\frac{-5}{-1}=5$

so the slope of this unknown line must be $-1/5$.

using slope point form

$(y-y_1)=m(x-x_1)$

$(y-1)=\frac{-1}{5}(x-3)$

$5y-5=-x+3$

$x+5y-8=0$.

Weaker responses reflected that candidates were confused in finding the equation of a straight line under the given conditions. Few responses exhibited that the candidates were able to find the point of intersection of given two lines but failed to proceed further. It was also noted that the candidates were unclear about the correct form of the equation, i.e. (y - y_1) = m(x - x_1) to and, hence failed to find the equation.

Example 1:

$x+3y=6 \quad , \quad 3x+2y=1$

$m_1=1 \qquad m_2=3$

$\frac{m_1+m_2}{1-m_1m_2} \Rightarrow \frac{1+3}{1-3} = \frac{4}{2} = 2$

$y-y_1 = m(x-x_1)$

$y+1 = 2(x-5)$

$y+1 = 2x-10$

$2x-11-y=0$

$2x-y-11=0$

Example 2:

$x+3y=6$

$3x+2y=11$

by solve simultaneously

we get

$(a,b) = (3,1)$

$\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{3}+\frac{y}{1}=1$

$\boxed{x+3y-3=0}$

$5x-y=3$

$m=-\frac{b}{a}$

not $\boxed{m=\frac{1}{5}}$

$\boxed{m=-5}$

Example 3:

$x+3y=6$ — 1 , $3x+2y=11$ — ii

$5x-y=3$ — iii

$x+3y=6$

$x=6-3y$

Put in iii

$5(6-3y)-y=3$

$30-15y-y=3$

$30-16y=3$

$30-3=16y$

$27=16y$

$\boxed{\frac{27}{16}=y}$

$3y=6-x$

$y=\frac{6-x}{3}$

$5x-\frac{6+x}{3}=3$

$\frac{15x-6+x}{3}=9$

$16x-6=9+6$

$\boxed{x=\frac{15}{16}}$

Question 4b:

Find the equation of the line passing through the point of intersection of the lines $x+3y=6$ and $3x+2y=11$, which

ii. passes through the point (2, 3).

Better responses exhibited a good understanding of the concept of finding the equation of straight lines with the given conditions. Candidates found the point of intersection of the given two lines and applied two-point form to find the equation of the line.

Example:

Point of intersection $(3, 1)$ → found in above part.

Consider points $(2, 3)$ & $(3, 1)$

$$m = \frac{1-3}{3-2} = -2$$

Eq $y - y_1 = m(x - x_1)$

$$y - 3 = -2(x - 2)$$

$$y - 3 = -2x + 4$$

$$2x + y - 7 = 0$$

Weaker responses reflected that candidates were able to find the point of intersection of two lines but failed to find the required equation either due to the wrong selection of the form of equation, improper substitution of the values or calculation mistakes.

Example 1:

$\left(-\frac{45}{7}, 1\right)$ and $(2, 3)$ using two point form. $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

$$\frac{y-1}{3-1} = \frac{x+45}{2+45}$$

$$\frac{y-1}{2} = \frac{x+45}{47}$$

$$47y - 47 = 2x + 90$$

$47y - 2x - 137 = 0$ ans

Example 2:

$$(x, y) = (3, 1)$$

eq $= \cancel{3/x}$ $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

$\frac{y-3}{3-1} = \frac{x-1}{2-1}$, $\frac{y-3}{2} = \frac{x-1}{1}$, $y - 3 = 2x - 2$

$$2x - y = -1$$

Example 3:

$$(y - y_1) = m(x - x_1)$$

$$(y - 3) = \frac{1}{5}(x - 2)$$

$$5y - 15 = x - 2$$

$$5y - x - 15 + 2 = 0$$

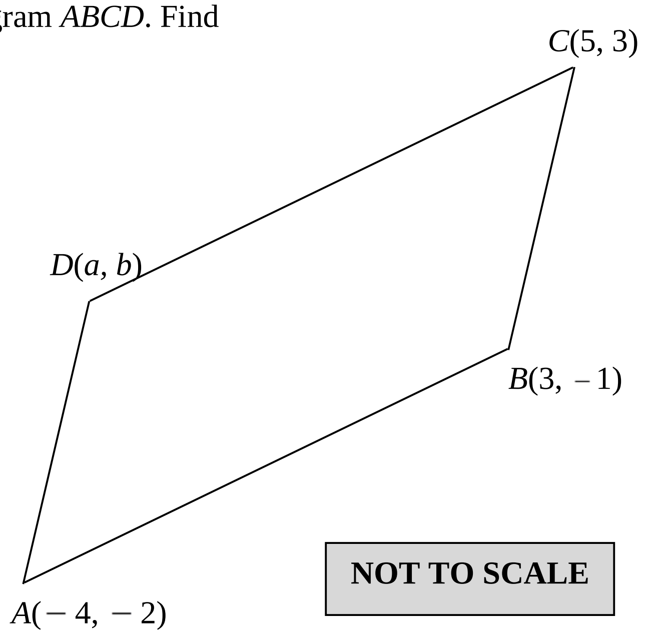
$$\boxed{5y - x - 13 = 0}$$

Question 4c:

The given figure shows a parallelogram *ABCD*. Find

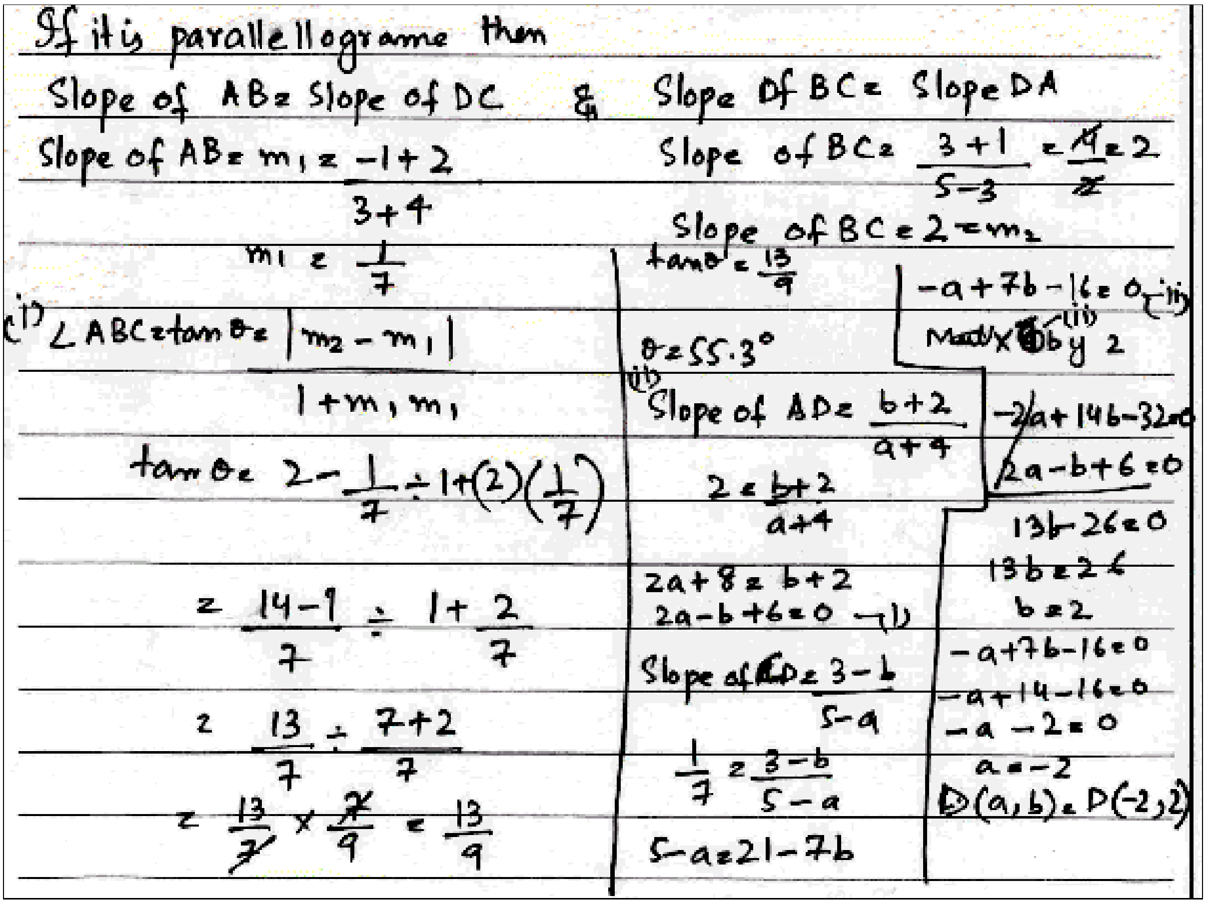
i. the angle *ABC*.

ii. the coordinates of point *D*



Better responses exhibited a good understanding of concept of angles formed at the vertex of a parallelogram. Candidates understood the question well and were able to find the slopes of the lines *AB* and *BC* and they correctly used the formula of angles between two lines to find the angle *ABC.* To find the coordinates of point *D*, they used the facts that slope of *AB*= slope of *CD* and slope of *BC*= slope of *DA* and performed arithmetical operations correctly.

Example:



If it is parallellograme then

Slope of AB = Slope of DC & Slope of BC = Slope DA

Slope of AB = $m_1 = \frac{-1+2}{3+4}$ Slope of BC = $\frac{3+1}{5-3} = \frac{\cancel{4}}{\cancel{2}} = 2$

$m_1 = \frac{1}{7}$ Slope of BC = $2 = m_2$

(i) $\angle ABC = \tan\theta = \frac{|m_2 - m_1|}{1 + m_1 m_1}$

$\tan\theta = 2 - \frac{1}{7} \div 1 + (2)(\frac{1}{7})$

$= \frac{14-1}{7} \div 1 + \frac{2}{7}$

$= \frac{13}{7} \div \frac{7+2}{7}$

$= \frac{13}{\cancel{7}} \times \frac{\cancel{7}}{9} = \frac{13}{9}$

$\tan\theta = \frac{13}{9}$

$\theta = 55.3°$

(ii) Slope of AD = $\frac{b+2}{a+4}$

$2 = \frac{b+2}{a+4}$

$2a + 8 = b + 2$

$2a - b + 6 = 0$ —(i)

Slope of CD = $\frac{3-b}{5-a}$

$\frac{1}{7} = \frac{3-b}{5-a}$

$5 - a = 21 - 7b$

$-a + 7b - 16 = 0$ (ii)

Mult (ii) by 2

$-2a + 14b - 32 = 0$

$2a - b + 6 = 0$

$13b - 26 = 0$

$13b = 26$

$b = 2$

$-a + 7b - 16 = 0$

$-a + 14 - 16 = 0$

$-a - 2 = 0$

$a = -2$

$D(a, b) = D(-2, 2)$

Weaker responses reflected that candidates were able to write the formula of angles between two straight lines but failed to apply the formula properly. Other weaker responses exhibited the candidates' confusions in finding the slope of the line. Instead of using correct formula they wrote $m = \frac{y_2 + y_1}{x_2 + x_1}$, $m = \frac{x_2 - x_1}{y_2 - y_1}$ or $m = \frac{x_1 - y_1}{x_2 - y2}$ etc. The weaker responses also exhibited the errors in writing the formula of angles between lines such as $\tan\theta = \frac{m_2 - m_1}{1 - m_1 m_2}$, $\tan\theta = \frac{m_2 + m_1}{1 - m_1 m_2}$ or $\tan\theta = \frac{m_2 + m_1}{1 + m_1 m_2}$. Therefore, they failed to find the required angle *ABC* and coordinates of the point *D.*

Example 1:

i). $\tan\theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

$= \dfrac{\frac{2}{1} - \frac{4}{3}}{1 + (2)\left(\frac{4}{3}\right)}$ $\qquad \theta = 28.5^\circ$

$= \dfrac{\frac{6-4}{3}}{\frac{3+8}{3}}$

$= \dfrac{2}{\frac{11}{3}}$

$\tan\theta = \dfrac{6}{11}$

$\theta = \tan^{-1}\left(\dfrac{6}{11}\right)$

Example 2:

$A(-4, -2)$

Slope of $AB = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1+2}{3+4} = \dfrac{1}{7} \Rightarrow m_1$

Slope of $BC = \dfrac{y_3 - y_2}{x_3 - x_2} = \dfrac{3+1}{5-3} = \dfrac{4}{2} = 2 = m_2$

Slope of $CA = \dfrac{y_3 - y_1}{x_3 - x_1} = \dfrac{3+2}{5+4} = \dfrac{5}{9} = m_3$

$\theta_1 = \tan^{-1} \dfrac{m_3 - m_1}{1 + m_3 m_1} = \dfrac{5/9 - \frac{1}{7}}{1 + 5/9 \times \frac{1}{7}} = \dfrac{\frac{35-9}{63} = \frac{26}{63}}{\frac{63+5}{63}} = \dfrac{26}{63} \times \dfrac{63}{68}$

$\theta_1 = 20.9$

$\theta_2 = \tan^{-1} \dfrac{m_3 - m_2}{1 + m_3 m_2} = \dfrac{5/9 - 2}{1 + \frac{5}{9} \times 2} = \dfrac{\frac{5-18}{9}}{\frac{9+10}{9}} = \dfrac{-\frac{13}{9}}{\frac{19}{9}}$

$\theta_3 = -34.3$

$\theta_3 = \tan^{-1} \dfrac{m_1 - m_2}{1 + m_1 m_2} = \dfrac{\frac{1}{7} - 2}{1 + \frac{2}{7}} = \dfrac{\frac{1-14}{7}}{\frac{7+2}{7}} = -\dfrac{13}{7} \times \dfrac{7}{9} = -55.3$

Example 3:

i) $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

$A = (-4, -2)$

$mA = \frac{-(-\cancel{4})}{-\cancel{2}} = 2$

$C = (5, 3)$

$mC = \frac{-5}{3}$

$\tan\theta = \frac{\frac{-5}{3} - 2}{1 + (2)\left(\frac{-5}{3}\right)}$

$= \frac{-11/\cancel{3}}{-10/\cancel{3}} = \tan\theta = \frac{11}{10}$

$\theta = \tan^{-1}\left(\frac{11}{10}\right)$

$\theta = 47.72°$

$180 - 47.72 = 132.28°$

(ii) $\frac{-4+3}{2}, \frac{-2-1}{2}$

$\left(\frac{-1}{2}, \frac{-3}{2}\right)$

$\left(5 + \left(\frac{-1}{2}\right), 3 + (-3)\right)$

$\left(5 - \frac{1}{2}, 3 - 3\right)$

$\left(\frac{10-1}{2}, 0\right)$

$D = \left(9/2, 0\right)$

Question 5:

A burger shop sells chicken and beef burgers. The profit on chicken burger and beef burger is Rs 12 and Rs 10 respectively.

Due to existing cooking facilities, it cannot cook

- more than 200 chicken burgers.
- more than 250 beef burgers.
- and sell more than 400 burgers altogether.

For the given linear programming problem,

i. state the constraints.

ii. state the profit function.

iii. draw constraints on the given graph to find feasible region and corner points.

iv. find the maximum profit.

This was a problem based on the linear programming. Every year it is noted that the problems of linear programming are not performing well in the examination.

Better responses indicated that candidates systematically followed the steps of the given problem to solve it. The better responses indicated that they stated constraint and profit function correctly. It was followed by skillful drawing of constraints to find the feasible region. Finally they calculated the maximum profit.

Example 1:

For the given linear programming problem, let x be chicken and y be beef burgers

i. state the constraints (1 Mark)

i) ~~$x \geq 200$~~ , ~~$y \geq 250$~~, (i) $x \leq 200$, (ii) $y \leq 250$

~~$x+y \geq$~~ (iii) $x+y \leq 400$.

ii. state the profit function (1 Mark)

$f(x,y) = 12x + 10y$.

iii. draw constraints on the given graph to find feasible region and corner points. (3 Marks)

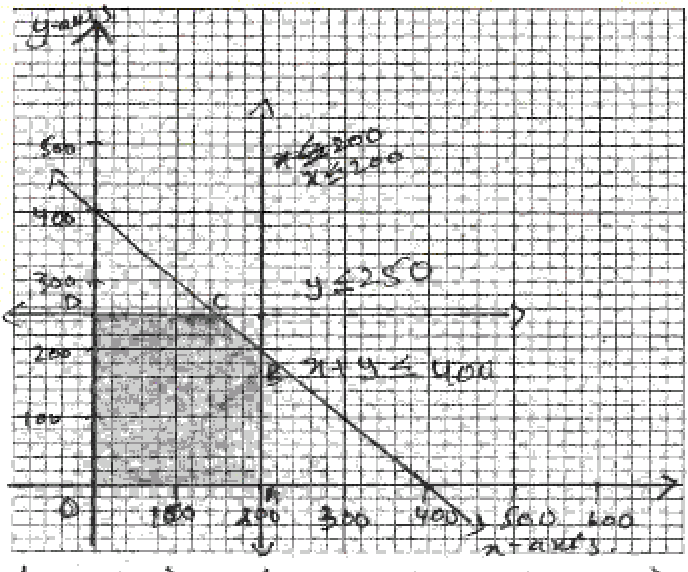
$5 = 50$

$12(200) + 10(200)$

$12(150) + (10)(250)$

4300

y-axis & x-axis let 1 box = 20 units



$O(0,0)$, $B(200,200)$, $C(150,250)$, $D(0,250)$, $A(200,0)$

iv. find the maximum profit. (1 Mark)

$f(200,200) = 12(200) + 10(200)$

$= 2400 + 2000$

Example 2:

i. state the constraints

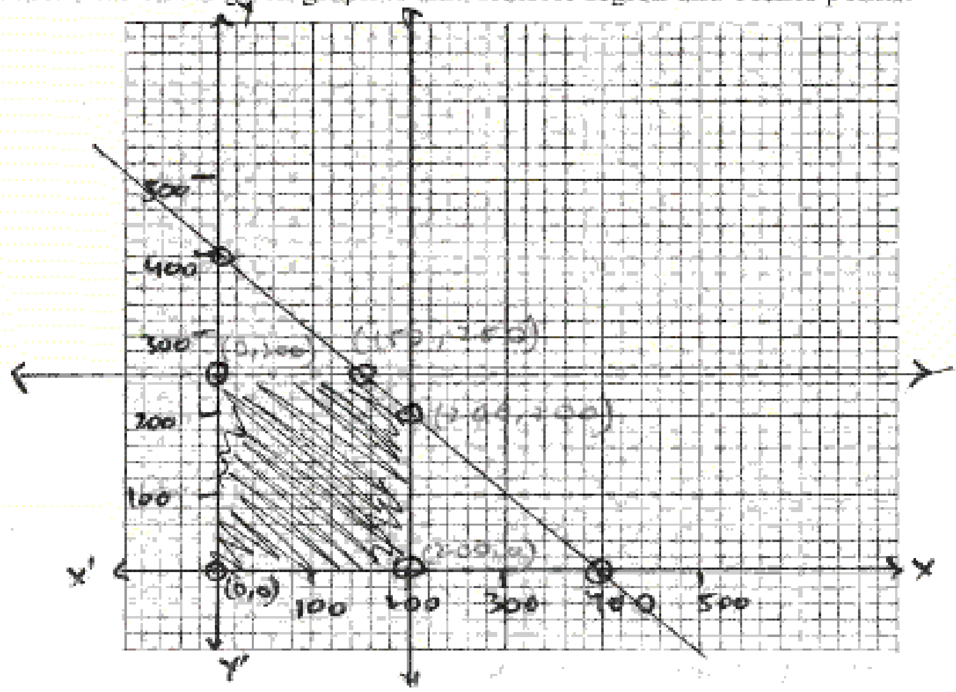
$x \leq 200$, $y \leq 250$, $x + y \leq 400$

$x \geq 0$ $y \geq 0$

ii. state the profit function

$f(x, y) = 12x + 10y$

iii. draw constraints on the given graph to find feasible region and corner points.



iv. find the maximum profit.

$(200, 200)$

$f(x, y) = 12(200) + 10(200)$

$= 4400$.

Weaker responses reflected a lack of understanding of application of linear programming. The weaker responses reported that candidates felt difficulty in writing constrains, particularly in writing the correct sign of equalities. Mostly, it was observed that candidates were able to write the constraints but they failed to draw these constraints on the given graph paper and consequently, they failed to find the feasible region and its corner points.

Example 1:

i. state the constraints

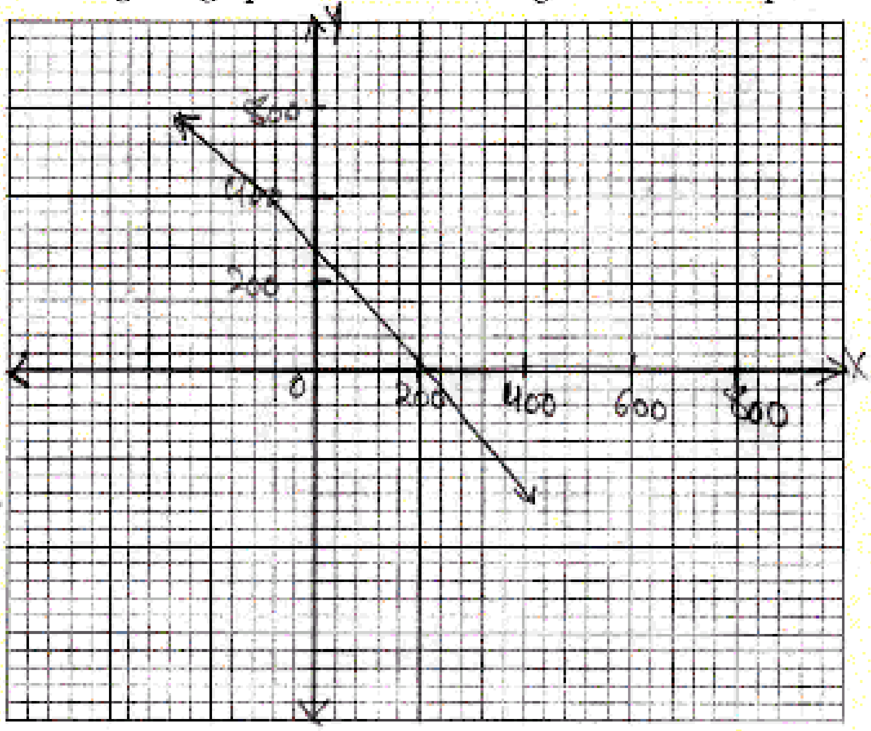
$$\frac{x}{200} + \frac{y}{250} \leq 400$$

$$x \geq 0 \ , \ y \geq 0$$

ii. state the profit function

$$P(x,y) = 12x + 10y$$

iii. draw constraints on the given graph to find feasible region and corner points.



corner points are $(0,0)$, $(200,0)$, $(0,250)$

iv. find the maximum profit.

$$P(250, 150) = 12(250) + 10(150)$$

$$= 4500$$

Example 2:

$x + y \geq 200$

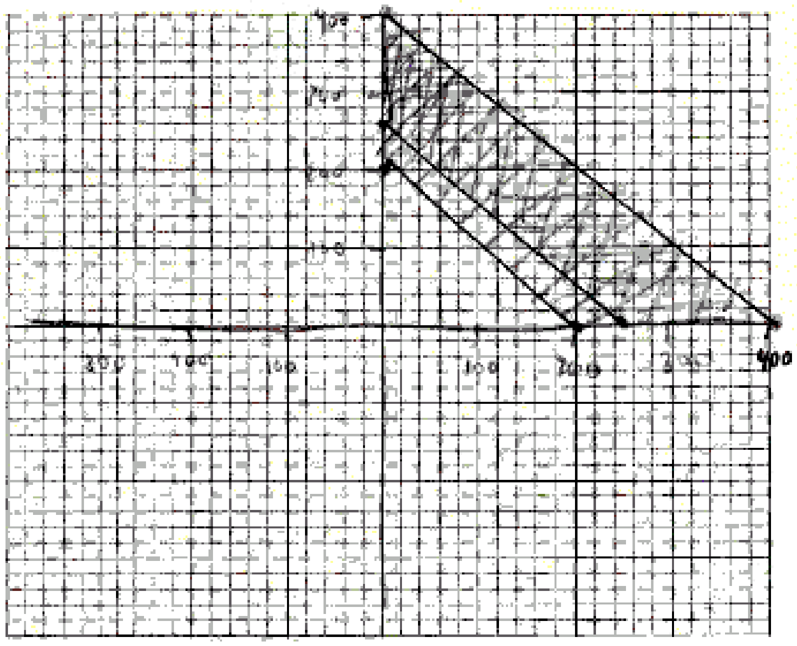
$x + y \geq 250$

$x + y \leq 400$

i. state the profit function

$P(x,y) = 12X + 10Y$

ii. draw constraints on the given graph to find feasible region and corner points. (



v. find the maximum profit.

$12(400) + 8(0) = 4800$

Example 3:

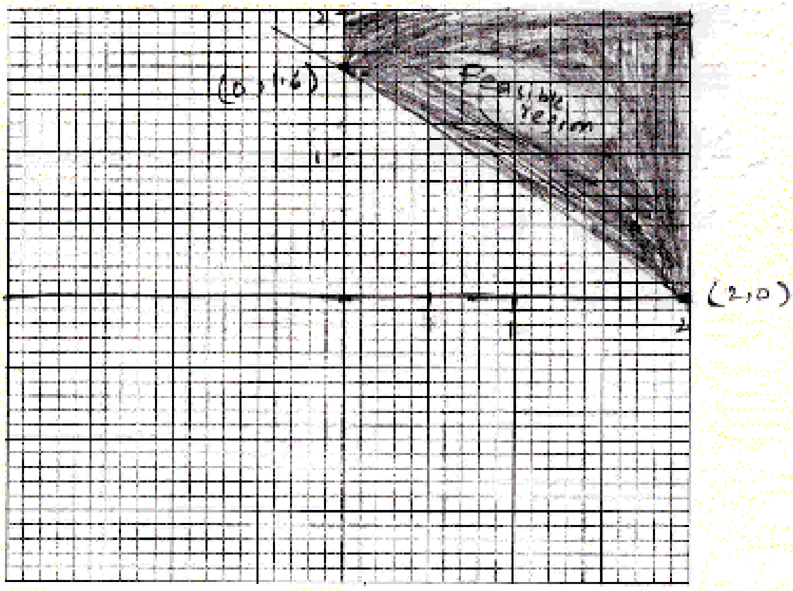
$200x + 250y \geq 400.$

$x \geq 0 \quad , \quad y \geq 0$

ii. state the profit function (1 Mark

$f(x, y) = 12x + 10y$. A function which is maximized or minimized is called profit function.

iii. draw constraints on the given graph to find feasible region and corner points. (3 Marks



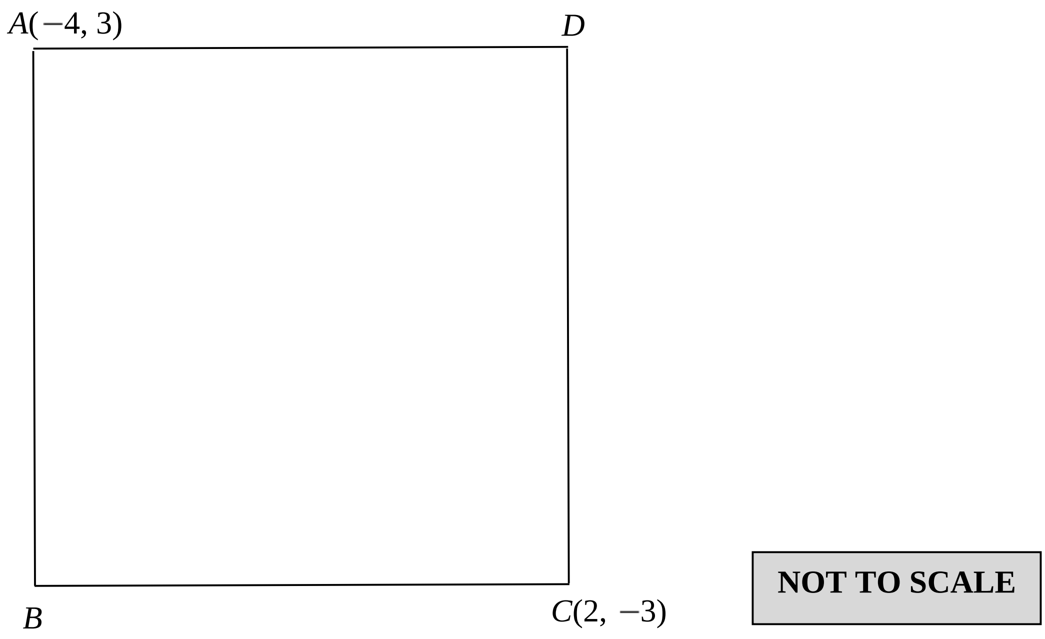
(2,0) and (0, 1.6).

iv. find the maximum profit. (1 Mark

The maximum profit come at (2,0).

Question 6a:

In the diagram, a square *ABCD* is given. Find the equation and radius of the circle passing through the vertices of the given square.



Better responses showed that candidates were clear about the concepts of analytical treatment of a circle and they found the equation and radius of the circle passing through the vertices of the given square. Mostly candidates considered AC as the diameter and they used this information to find the centre of the circle by midpoint formula and radius of the circle by distance formula. Then, they substituted the values of radius and coordinates of the centre in the standard equation to find the required equation of the circle. Few other responses exhibited the use of general equation and found the values of g, f and C to get the required equation of the circle.

Example 1:

• Diameter of Circle

$|AC| = \sqrt{(2+4)^2 + (-3-3)^2}$

$= \sqrt{6^2 + (-6)^2}$

$= \sqrt{36 + 36}$

$|AC| = \sqrt{72}$

$|AC| = 6\sqrt{2}$

radius of circle $= 6\sqrt{2}$

$= 3\sqrt{2}$

Center of the circle

$mp(AC) = \left(\frac{-2}{2}, \frac{-3+3}{2}\right)$

$mp\ AC = (-1, 0)$

equation of the circle

$(x-h)^2 + (y-k)^2 = r^2$

$(x+1)^2 + (y)^2 = (3\sqrt{2})^2$

$(x+1)^2 + y^2 = 18$

Example 2:

mid point of AC $= \left(\frac{-4+2}{2}, \frac{3-3}{2}\right) = (-1, 0)$

$P(-1, 0) = (h, k)$

$\overline{AP}$ is radius.

$\overline{AP} = \sqrt{(-3)^2 + 3^2} = 3\sqrt{2}$ = radius.

as centre $(x-h)^2 + (y-k)^2 = r^2$

$(x+1)^2 + (y)^2 = 18$ Ans → eqn of circle.

Weaker responses reflected that candidates had the lack of understanding of the concept of analytical treatment of a circle. They applied distance formula and midpoint formula inappropriately. In few other weaker responses, it was noted that candidates applied the general form of the equation of a circle. They substituted the given points in the general equation to get simultaneous linear equation in terms of *g, f* and *C* but, failed to find the values of *g, f* and *C*.

Example 1:

$|AO|$ $O(x, y)$

$|AO| = \sqrt{(x+4)^2 + (y-3)^2}$ $|CO| = \sqrt{(x-2)^2 + (y+3)^2}$

$r = \sqrt{x^2 + 8x + 16 + y^2 - 6y + 9}$ $r = \sqrt{x^2 - 4x + 4 + y^2 + 6y + 9}$

$x^2 + 8x + 16 + y^2 - 6y + 9 = x^2 - 4x + 4 + y^2 + 6y + 9$

$12x - 12y + 12 = 0$

$x - y + 12 = 0$

by solving simultaneously

$x = \frac{-19}{2}, y = \frac{-17}{2}$ $C\left(\frac{19}{2}, \frac{17}{2}\right)$

$|CA| = \sqrt{\left(\frac{19}{2} + 4\right)^2 + \left(\frac{17}{2} - 3\right)^2}$

r

$r = 5\sqrt{34}/2$

Example 2:

$|AO| = |OC|$

$(x+4)^2+(y-3)^2 = (x-2)^2+(y+3)^2 = r^2$

$x^2+8x+16+y^2-6y+9 = x^2-4x+4+y^2+6y+9 =$

$4x-12y+12=0$

$x = \frac{-4+2}{2},\ y = \frac{3-3}{2}$

$x = \frac{-2}{2},\ y = 0$

equation of circle

$(x+1)^2+(y-0)^2 = 117$

centre $(x,y) = (-1,0)$

Example 3:

C and D are the points from where the circle passes. So

M.P of C D will be the centre of circle

Diameter $= \overline{AC}$

$\overline{AC} = \sqrt{(4-2)^2+(3+3)^2}$

$= \sqrt{(6)^2+(6)^2}$

$= \sqrt{36+36}$

$= \sqrt{72}$

$AC = 6\sqrt{2}$

$r = \frac{D}{2}$

$r = \frac{6\sqrt{2}}{2}$

$\boxed{r = 3\sqrt{2}}$

$\left(\frac{-4+2}{2}, \frac{3+3}{2}\right)$

$= \left(\frac{-6}{2}, \frac{6}{2}\right) = (-3, 3)$

$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$

$\frac{x^2}{-3}+\frac{y^2}{3} = 1$

$\frac{x^2}{-9}+\frac{y^2}{9} = 1$

$\frac{y^2}{9}-\frac{x^2}{-9} = 1$

Example 4:

$A = (-4, 3)$, $C = (2, -3)$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$(x-h)^2 + (y-k)^2 = r^2$

$A = (-4, 3)$

$B = (2, -3$

for $A = (-4, 3)$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$(-4)^2 + (3)^2 + 2g(-4) + 2f(3) + c = 0$

$16 + 9 - 8g + 6f + c = 0$

$-8g + 6f + c + 25 = 0$ — 1

for $B = (2, -3)$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$(2)^2 + (-3)^2 + 2g(2) + 2f(-3) + c = 0$

$4 + 9 + 4g - 6f + c = 0$

$4g - 6f + c + 13 = 0$

Question 6b:

i. Find the equations of tangent and normal to the circle $7x^2 + 7y^2 - 28x + 42y - 84 = 0$ at point $(-3, -3)$.

ii. If $y = bx$ is a tangent to the circle $x^2 + y^2 = a^2$, then show that $4a^2(1 + b^2) = 0$.

Better responses of part **i** showed that candidates found the derivative of the given equation $7x^2 + 7y^2 - 28x + 42y - 84 = 0$ and put the value of point $(-3, -3)$ in the derivative to find the slope of tangent to the given circle. Finally, they applied the point slope forms to find the required equation of tangent and normal.

In part **ii**, better responses reflected that candidates were well versed with the condition of tangency of a line to a circle. They put $y = bx$ in the equation of circle $x^2 + y^2 = a^2$, simplified the resulting equation then compared its slope to zero to get the required condition.

Example 1:

i) $7x^2+7y^2-28x+42y-84=0$

÷ by 7 on b/s

$x^2+y^2-4x+6y-12=0$

diff w.r.t x.

$2x+2y\frac{dy}{dx}-4+6\frac{dy}{dx}=0$

$\frac{dy}{dx}(2y+6)=-2x+4$

$\frac{dy}{dx}=\frac{-\cancel{2}(x-2)}{\cancel{2}(y+3)}=\frac{-(x-2)}{(y+3)}$

for slope: At $(-3,-3)$

$m=\frac{dy}{dx}=\frac{-(x-2)}{y+3}=\frac{-(-3-2)}{-3+3}=\infty$

for Equation of tangent: $(y-y_1)=m(x-x_1)$

$(y+3)=\frac{1}{0}(x+3)$

$0=x+3 \Rightarrow x+3=0$

for Equation of normal: $m=\frac{-1}{m}=\frac{-1}{1/0}=0$

$y+3=0(x+3) \Rightarrow y+3=0$

ii) $y=bx$; put this in circle.

$x^2+y^2=a^2$

$\Rightarrow x^2+(bx)^2=a^2$

$x^2+x^2b^2=a^2$

$x^2(1+b^2)-a^2=0$

tangent: $b^2-4ac=0$

$a=(1+b^2),\ b=0,\ c=-a^2$

$b^2-4ac=0$

$0^2-4(1+b^2)(-a^2)=0$

$4a^2(1+b^2)=0$

proved.

Example 2:

(i) $P(x_1, y_1) = (-3, -3)$

$7x^2+7y^2-28x+42y-84=0$

Dividing both sides by 7:

$= x^2+y^2-4x+6y-12=0$ —(i)

general equation of circle:

$x^2+y^2+2gx+2fy+c=0$

$2g=-4$, $2f=6$, $c=-12$

$g=-2$, $f=3$

for tangent: $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

$(x)(-3)+(y)(-3)-2(x-3)+3(y-3)-12=0$

$-3x-3y-2x+6+3y-9=0$

$-5x-15=0$ —(i)

$x+3=0$

$x=-3$

for normal:

$(x-x_1)(y_1+f)=(y-y_1)(x_1+g)$

$(x+3)(-3+3)=$ $(x_1+g$

$(y+3)(-3-2)$

$(y+3)(-5)=0$

$y+3=0$ —(ii)

$y=-3$

(ii) $x^2+y^2=a^2$ —(i)

$y=bx$ —(ii)

put $y=bx$ in (i):

$x^2+(bx)^2=a^2$

$x^2+b^2x^2-a^2=0$

$(1+b^2)(x^2)-a^2=0$

Discriminant: $b^2-4ac=0$

$0-4(1+b^2)(-a^2)=0$

$4a^2(1+b^2)=0$

Weaker responses showed that in part **i**, the candidates failed to find the correct derivative of the given equation of a circle. They also made mistakes in calculating the value of the slopes and made wrong selection of the equation to find the equations of tangents and normal.

They also made mistakes in the process of calculations to get the required results.

Example 1:

$14x + 14y\frac{dy}{dx} - 28 + 42\frac{dy}{dx} = 0$

$14x - 28 = -\frac{dy}{dx}(42 + 14)$

$\frac{dy}{dx} = \frac{14x - 28}{42 + 14}$

$(y - y_1) = -\frac{1}{m}(x - x_1)$

$(y+3) = \frac{-14x+28}{56}(x+3)$

$-14x+28(y+3) = 56x+168$

$98x - 28y + 14xy + 84 = 0$

(ii) $(y - y_1) = m(x - x_1)$

$(y+3) = \frac{-14x+28}{56}(x+3)$

$56y + 168 = -14x^2 - 42x + 28x + 8$

$56y + 168 + 14x^2 + 42x - 28x - 84 = 0$

$14x^2 + 14x - 56y + 84 = 0$

Example 2:

(i) $7x^2 + 7y^2 - 28x + 42y - 84 = 0$, $(-3, -3)$

Sol: $7(x^2+y^2-4x+6y-12)=0$	put $x=-3$ and $y=-3$ $\frac{dy}{dx} = \frac{-(-3)+4}{2(-3)+6} = \frac{6+4}{-6+6} = \frac{10}{0}$
$x^2+y^2-4x+6y-12=0$ — (1)	
Diff: w.r.t x $2x + 2y \cdot \frac{dy}{dx} - 4 + 6 \cdot \frac{dy}{dx} - 0 = 0$	$\frac{dy}{dx} = \infty$ Slope is undefined~~, no tangent~~
$2x + 2y\frac{dy}{dx} - 4 + 6\frac{dy}{dx} = 0$	Ans
$2x - 4 + 2y\frac{dy}{dx} + 6\frac{dy}{dx} = 0$	
$2y\frac{dy}{dx} + 6\frac{dy}{dx} = -2x+4$ $\frac{dy}{dx}(2y+6) = -2x+4$	
$\frac{dy}{dx} = \frac{-2x+4}{2y+6}$	

Example 3:

i. $7x^2 + 7y^2 - 28x + 42y - 84 = 0$

$\frac{dy}{dx}(7x^2 + 7y^2 - 28x + 42y - 84) = 0$

$14x + 14y - 28 + 42 = 0 \implies$ (i)

For slope; at point $(-3, -3)$ in equation (i)

$m = 14(-3) + 14(-3) - 28 + 42$

$m = -70$

For equation of tangent;

$y - y_1 = m(x - x_1)$

$y + 3 = -70(x + 3)$

$y + 3 = -70x - 210$

$70x + y + 3 + 210 = 0$

$70x + y + 213 = 0$ Ans

For equation of normal

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y + 3 = \frac{+1}{+70}(x + 3)$

$(y + 3)(70) = x + 3$

$70y + 210 = x + 3$

$0 = x - 70y - 207$ Ans

Question 7a:

Find the vertices, co-vertices and eccentricity of the ellipse represented by

$4x^2 + 49y^2 = 196$

Generally it was a well attempted question.

Better responses showed that the candidates have command over the analytical geometry of ellipse. They converted the given equation of ellipse into the form of $\frac{x^2}{49} + \frac{y^2}{4} = 1$ and got the values of *a* and *b*, which they used aptly to find the vertices, covertices and eccentricity of the given ellipse.

Example:

divide b/s with 196

$\frac{4x^2}{\cancel{196}\,49} + \frac{49y^2}{196} = \frac{\cancel{196}}{\cancel{196}\,1}$

$\frac{x^2}{49} + \frac{y^2}{4} = 1$ eq of Ellipse

$a^2 = 49 \quad 4 = b^2$

As we know $a=7$ and $b=2$

so vertices are $(\pm 7, 0)$ and covertices are $(0, \pm 2)$

we have given a and b so we find c

$c^2 = a^2 - b^2$

$c^2 = 49 - 4$

$c^2 = 45$

$c = \sqrt{45}$

to find eccentricity we know

$e = \frac{c}{a}$

$e = \frac{\sqrt{45}}{7}$

$\therefore c = \sqrt{45}$

$\because a = 7$

Weaker responses reflected that the candidates failed to find the vertices, covertices and eccentricity of the given equation of ellipse. They either failed to divide both sides of the equation by 196 or made calculation mistakes in the division process. It is also noted that they made wrong identifications of *a* and *b* or applied the incorrect formula to find eccentricity and thus, they failed to find the elements of the given ellipse.

Example 1:

Vertices of ellipse are

$(0, \pm 7)$

Coordinates of ellipse are

$(\pm 3, 0)$

eccentricity of ellipse is

$\frac{49}{4} = \frac{7}{2}$ Ans

Example 2:

$4x^2 + 49y^2 = 196$

~~multip~~ divide L/s by 196

$\frac{4x^2}{196} + \frac{49y^2}{196} = 1$

$\frac{x^2}{49} + \frac{y^2}{4} = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$49 = a^2$, $b^2 = 4$

$a = 7$, $b = 4$

$2a \Rightarrow 2(7) \Rightarrow \pm 14$

$2b \Rightarrow 2(4) \Rightarrow \pm 8$

$e > 1$ direct a/e

$c^2 = a^2 + b^2$

Example 3:

$$\frac{4x^2}{196} + \frac{49y^2}{196} = 1$$

$$\frac{x^2}{49} + \frac{y^2}{196} = 1$$

$$\frac{x^2}{(7)^2} + \frac{y^2}{(14)^2} = 1$$

Vertices $(0, \pm 14)$: $(0, 14)$ $(0, -14)$

Covertices $(\pm 7, 0)$: $(7, 0)$ $(-7, 0)$

2) $c^2 = a^2 - b^2$

$c^2 = 196 - 49$

$\boxed{c = 7\sqrt{3}}$

$e = \frac{c}{a}$

$e = \frac{\cancel{7}\sqrt{3}}{\cancel{14}_2} = \boxed{\frac{\sqrt{3}}{2} = e}$

Question 7b:

An ellipse with centre $(0,0)$ has vertices $(\pm 5,0)$ and foci $(\pm 4,0)$. Find

i. the equation of the ellipse.

ii. the eccentricity.

iii. the equation of directrices.

iv. the length of minor axis.

Better responses reported that the candidates have complete command over the analytical treatment of ellipse and they have the correct understanding of the elements of ellipse. The given data was used aptly to find the values of *a*, *b* and *c*. Therefore, the candidates were able to find the equation and different elements of the required ellipse.

Example 1:

Vertices $(\pm a,0) \to (\pm 5,0)$

$\boxed{a = \pm 5}$

Foci $(\pm c, 0) \to (\pm 4, 0)$

$\boxed{c = \pm 4}$

$b = ?$

$a^2 = c^2 + b^2$

$\cancel{b} = a^2 - c^2$

$b^2 = 9$

$\boxed{b = \pm 3}$

i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

(ii) $e = c/a$

$\boxed{e = 4/5}$

(iii) $x = \pm a/e$

$x = \pm 5/4/5 \Rightarrow \boxed{x = \pm 25/4}$

(iv) length of the minor axis $= 2b$

$2(3) = 6$

Example 2:

i. $b = \sqrt{a^2 - c^2}$. $a = 5$ $c = 4$.

$b = \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$.

$b = \sqrt{9} \Rightarrow \boxed{b = 3}$.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$

$\Rightarrow 9x^2 + 25y^2 = 225 \Rightarrow \boxed{9x^2 + 25y^2 - 225 = 0}$

ii. $e = \frac{c}{a} \Rightarrow \boxed{\frac{4}{5} = e}$

iii. eq. of directrices $\Rightarrow x = \pm \frac{a}{e}$.

$x = \pm \frac{5}{4/5} \Rightarrow \boxed{x = \pm 25/4}$.

iv. L of minor axis = $2b \Rightarrow 2(3)$.

$\boxed{L = 6 \text{ units}}$

Weaker responses informed lack of understanding of the concepts of equation, an ellipse and its elements. The candidates failed to find the equation of ellipse and its different elements. The main reason was their incorrect identification of *a*, *b* and *c* or inappropriate use of formulae. Therefore, they failed to find the equation and elements of the ellipse.

Example 1:

(i) equation $= \frac{x^2}{25^2} + \frac{y^2}{\sqrt{41}}$

$v(\pm a, 0) = (\pm 5, 0)$

$a = \pm 5$

(ii) eccentricity $= \frac{c}{a}$

$c(0,0)$

foci $(\pm c, 0) = (\pm 4, 0)$

eccentricity $= \frac{4}{5}$

(iii) equation of directrices $= \frac{\pm c}{e^2}$

directrices $= \frac{\pm 4}{(4/5)^2}$

(iv) length of minor axis $= 2b$

Example 2:

Here $a = 5$, $c = 4$

(ii) $e = \frac{c}{a} = \frac{4}{5}$

$c^2 = a^2 - b^2$

$16 = 25 - b^2$

$b^2 = 25 - 16$

$b^2 = 9$

(iv) $\boxed{b = \pm 3}$

~~(iii) $x = \frac{\pm}{c}$~~

(i) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

(iii) $x = \frac{e}{c^2} = \frac{4/5}{16} = \frac{4}{5 \times 16} = \frac{4}{80} = \frac{1}{20}$

~~$\frac{a}{c} =$~~

Question 7c:

The vertices of a hyperbola are $(15, 1)$ and $(-1, 1)$. If the length of its conjugate axis is 12 units, then find the equation of the hyperbola in standard form.

Better responses showed that the candidates understood the standard form of the equation of a hyperbola. They wrote the equation of a hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ and found the values of (h, k), a and b to find its equation.

Example:

Length of conjugate axis = $2b = 12 \Rightarrow \boxed{b = 6 \text{ units}}$

$A(15, 1)$, $A'(-1, 1)$

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow \sqrt{(15+1)^2 + (1-1)^2} \Rightarrow \boxed{16 \text{ units}}$

$|AA'| = 2a \Rightarrow 2a = 16 \therefore \boxed{a = 8 \text{ units}}$

for any hyperbola the midpoint of the vertices, covertices of foci is the centre (h, k)

$\therefore$ Using mid point formula; $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$(x_1, y_1) = (-1, 1)$, $(x_2, y_2) = (15, 1)$

$(h, K) = \left(\frac{15 - 1}{2}, \frac{1 + 1}{2}\right)$

$(h, K) = (7, 1)$

Since the hyperbola is lying on X axis $\therefore$ equation comes out to be: $\frac{(x-h)}{a^2} - \frac{(y-k)}{b^2} = 1$

$\frac{(x-7)}{64} - \frac{(y-1)}{36} = 1$

$\Rightarrow$ Answer.

Weaker responses exhibited confusion and candidates failed to find the equation of the hyperbola. They were unable to write the proper formula, its equation and elements. Few other weaker responses displayed usage of mid-point formula, which was not required.

Example 1:

Hyp (15, 1) and (-1, 1)

origin ⇒ $(x, y) = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

$x, y = \frac{15 - 1}{2}, \frac{1 + 1}{2}$

$x, y = (7, 1)$

Hyp (15, 1) and (-1, 1) = (7, 1) → origin

$\frac{(15, 1)(-1, 1)}{(7, 1)}$ = 12 units

(15, 1) = a, (-1, 1) = b, (7, 1) = c

Hpy $\frac{ab}{c}$ = 12 → axis

$\boxed{\text{Hpy } \frac{ab}{c} = x.y}$

Example 2:

$(15, 1) \quad (-1, 1)$

$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

$\frac{15 + (-1)}{2}, \frac{1 + 1}{2}$

$\frac{\cancel{14}^{7}}{\cancel{2}_{1}}, \frac{\cancel{2}^{1}}{\cancel{2}_{1}} = (7, 1)$

$(k - x)^2 + (h - y)^2 = r^2$

$(k - 7)^2 + (h - 1)^2 = (12)^2$

$k^2 - 2(k)(7) + (7)^2 + h^2 - 2(h)(1) + (1)^2 = 144$

$k^2 - 14k + 49 + h^2 - 2h + 1 = 144$

$k^2 + h^2 - 14k - 2h + 50 = 144$

$k^2 + h^2 - 14k - 2h + 50 - 144 = 0$

$\boxed{k^2 + h^2 - 14k - 2h - 94 = 0}$

Question 8a:

A triangle *ABC* located on *xy*–coordinate axes has vertices *A* (0, 3), *B* (– 3, 0) and *C* (3, – 3). The *x*–axis is translated through a distance of – 3 units while there is no change in the *y*-axis.

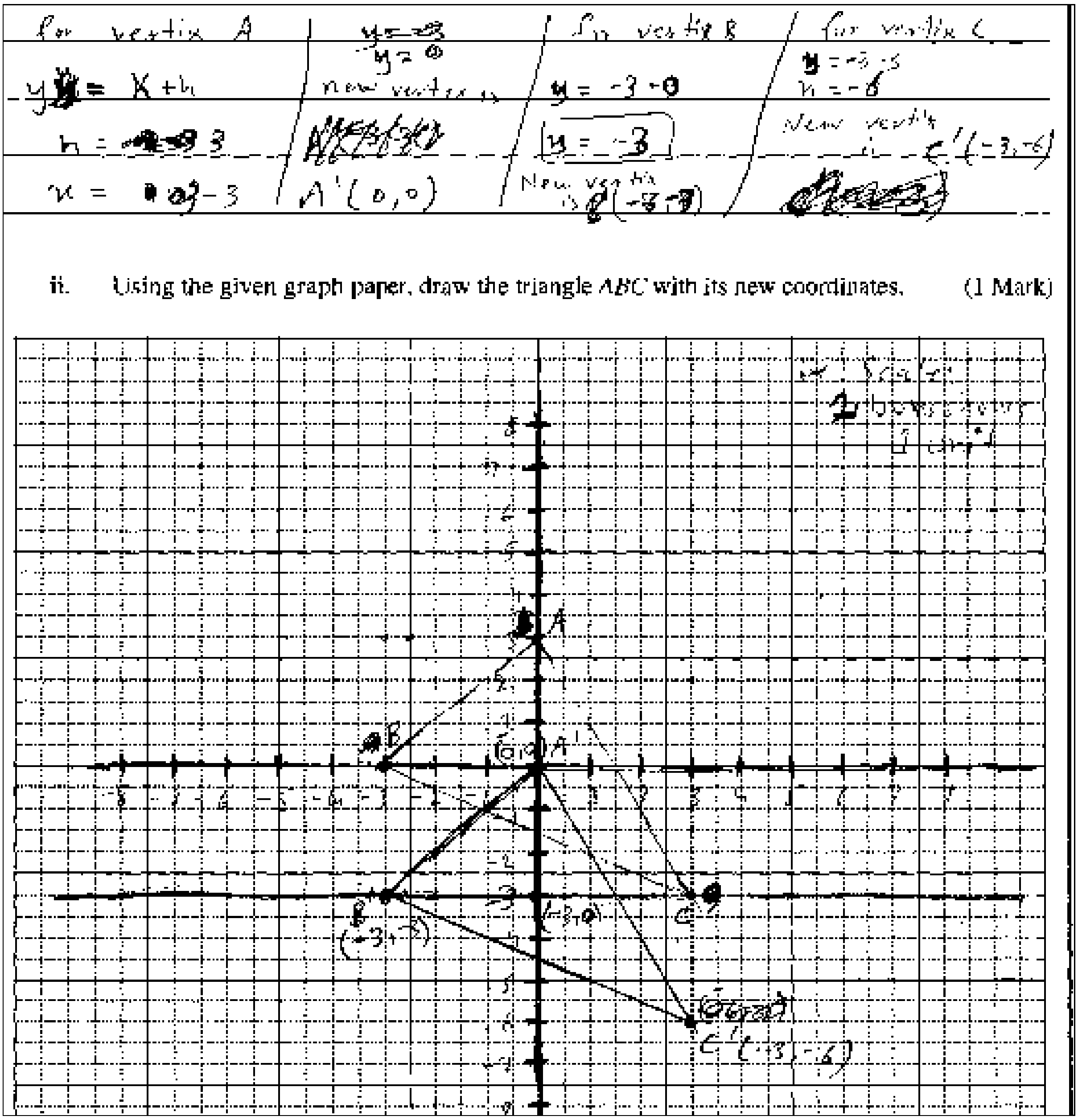
i. Find the new vertices of the triangle *ABC* with respect to the translated axis.

ii. Using the given graph paper, draw the triangle *ABC* with its new coordinates.

This question offered a choice between part **a** and part **b**. Majority of the candidates chose part **b.**

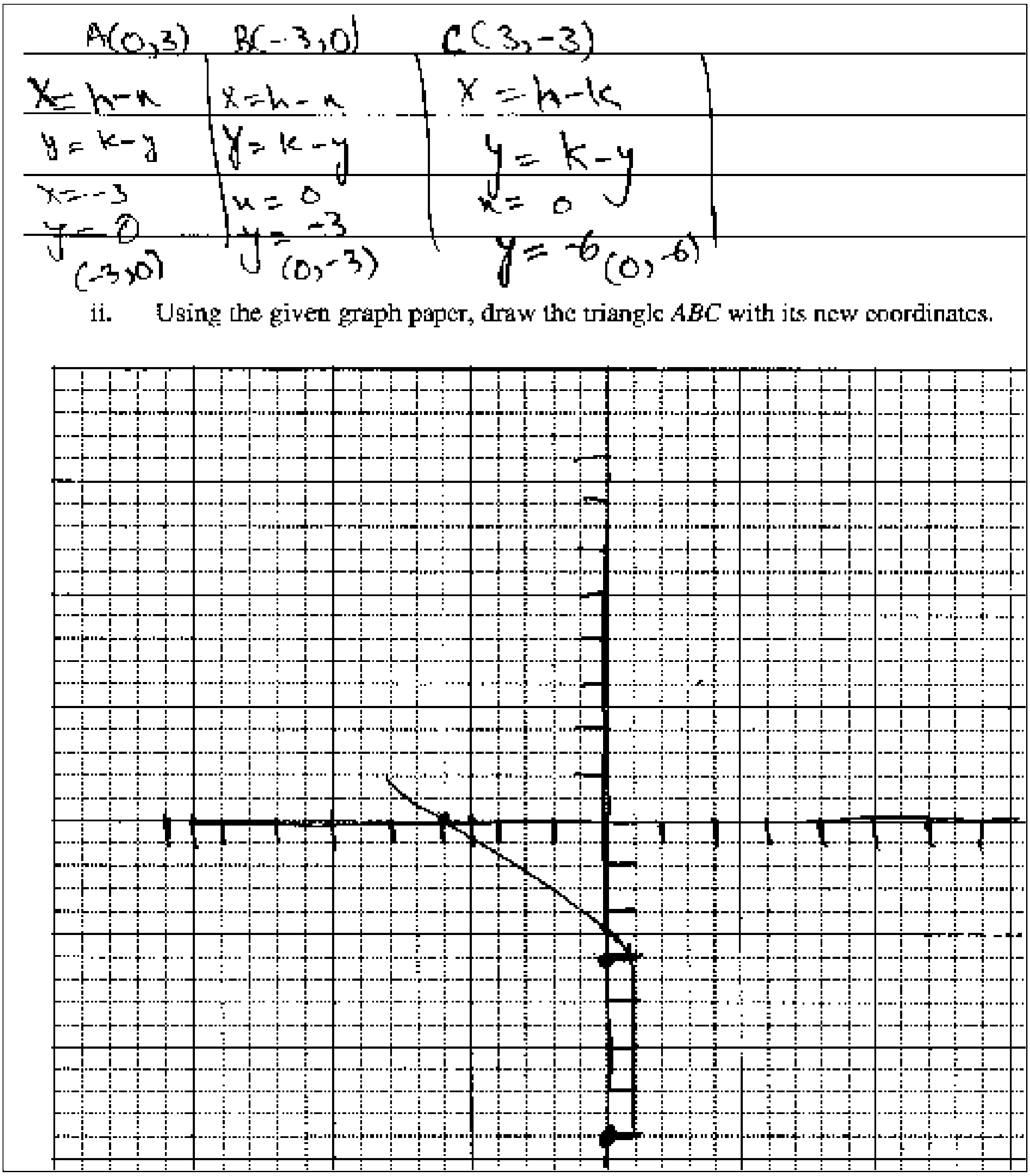
Better responses indicated that candidates applied the correct transformation and therefore, got the correct answer. They substituted $X = x$ and $Y = y + 3$ in the given equation and as a result, they were able to find the new vertices of the triangle *ABC*. They also drew the translated triangle on the given graph paper.

Example:



Weaker responses exhibited that candidates did not understand the concept of the transformation. It is observed that the candidates used incorrect transformation equations. Due to this, they were unable to get the required answer and hence, failed to plot the triangle after the transformation.

Example 1:

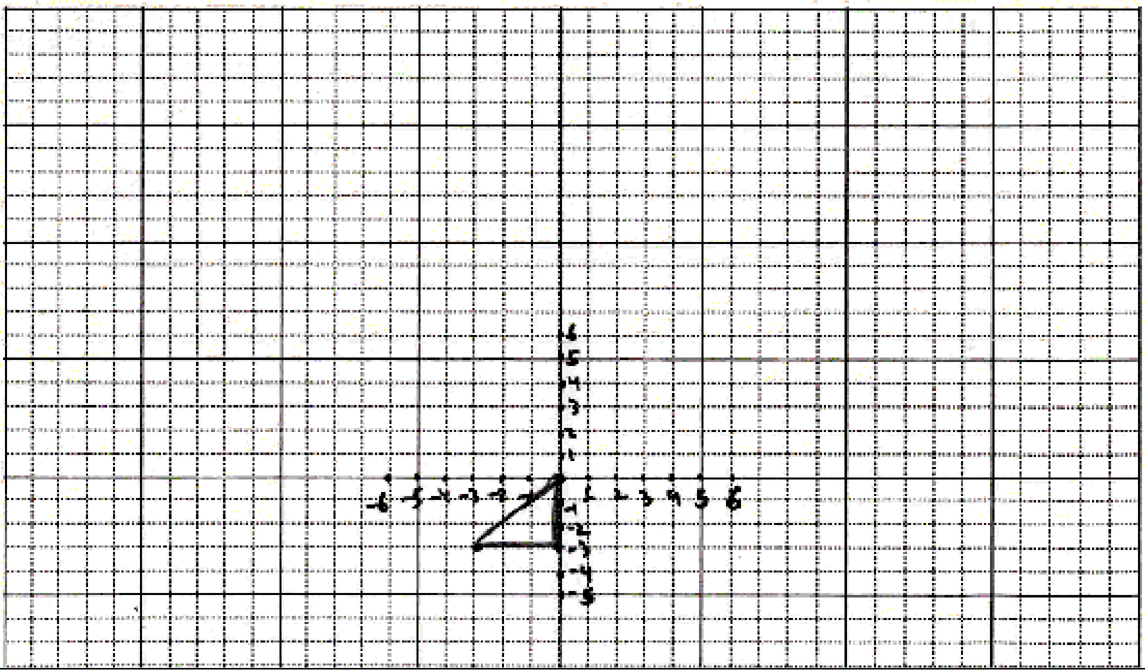


Example 2:

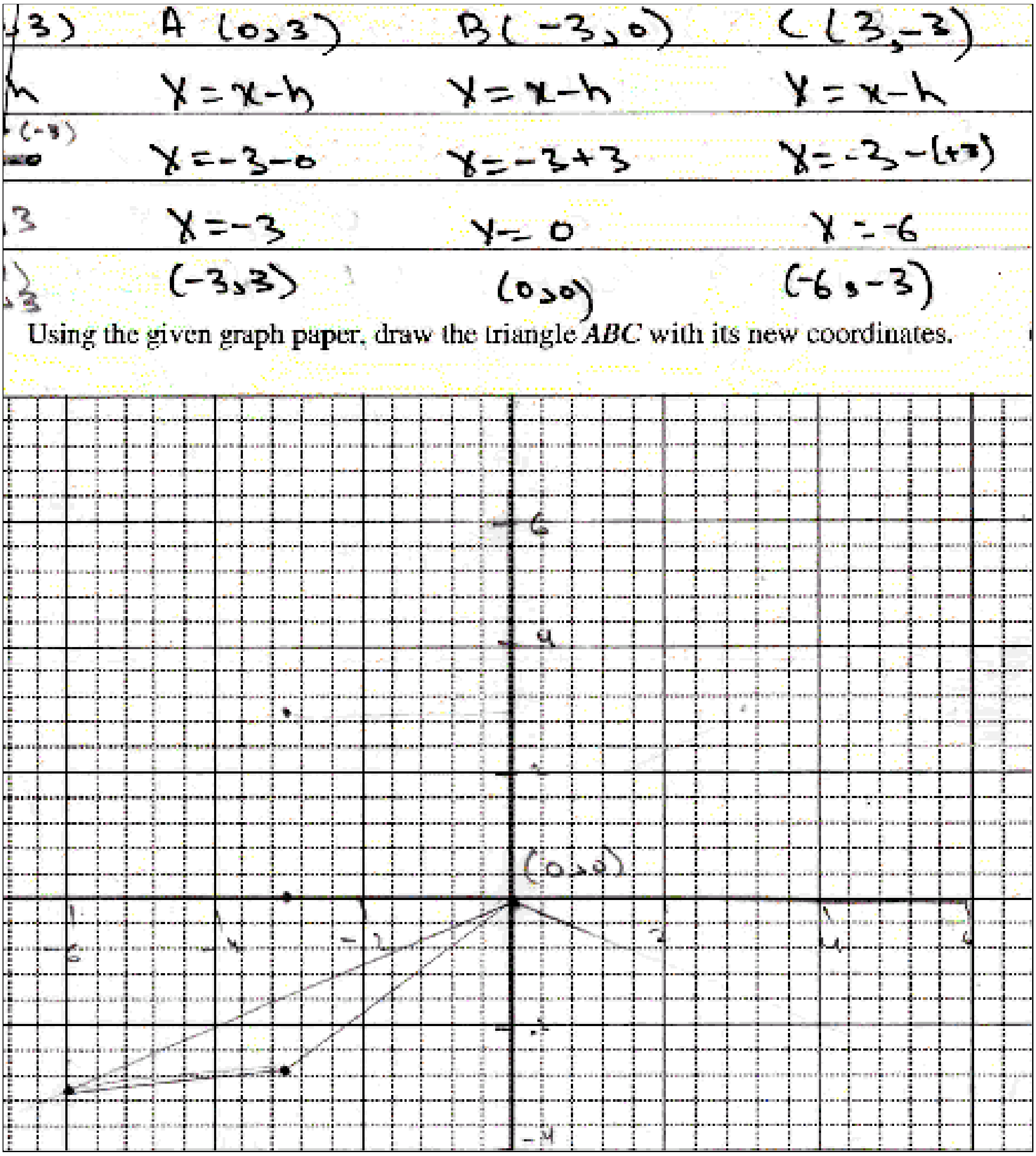
A i. Find the new vertices of the triangle ABC with respect to the translated axis. (3 Marks)

$x = X - h$	B: $X = x + h$	$X = x + h$
$0 - 3 = X \Rightarrow -3$	$X = -3 - 3 \Rightarrow 0$	$X = 3 - 3 \Rightarrow 0$
$y = Y - k$	$Y = y + k$	$Y = y + k$
$3 + 0 = 3$	$Y = 0$	$Y = -3$
$(-3, 3)$	$(0, 0)$	$(0, -3)$

ii. Using the given graph paper, draw the triangle ABC with its new coordinates. (1 Mark)



Example 3:



Question 8b:

The xy – coordinate axes are rotated about the origin through an angle of $60°$. The coordinates of a point A are (7, 11) with respect to xy – coordinate axis. Find coordinates of A with respect to rotated axes.

Better responses indicated the correct use of equations of transformation to get equations in terms of X and Y i.e., $X = x\cos\theta + y\sin\theta$ and $Y = y\cos\theta - x\sin\theta$. They correctly substituted the value of the angle and made right calculation to get the result.

Example 1:

$A(x,y) = 7, 11$

$x = 7, \quad y = 11$

$\theta = 60^\circ.$

$X = ??, \quad Y = ??$

$X = x\cos\theta + y\sin\theta.$

$X = 7\cos 60 + 11\sin 60^\circ$

$X = 7\left(\frac{1}{2}\right) + 11\left(\frac{\sqrt{3}}{2}\right)$

$X = \frac{7 + 11\sqrt{3}}{2}.$

$Y = y\cos\theta - x\sin\theta.$

$Y = 11\cos 60 - 7\sin 60$

$Y = 11\left(\frac{1}{2}\right) - 7\left(\frac{\sqrt{3}}{2}\right)$

$Y = \frac{11 - 7\sqrt{3}}{2}.$

$A(x,y) = \left(\frac{7 + 11\sqrt{3}}{2}, \frac{11 - 7\sqrt{3}}{2}\right)$

Example 2:

$x = X\cos\theta + Y\sin\theta \qquad (X,Y) = (7,11)$

$y = Y\cos\theta - X\sin\theta$

$x = 7\cos 60^\circ + 11\sin 60^\circ$

$x = \frac{7}{2} + \frac{11\sqrt{3}}{2} = \frac{7 + 11\sqrt{3}}{2}$

now $y = Y\cos\theta - X\sin\theta$

$= 11\cos 60 - 7\sin 60$

$y = \frac{11}{2} - \frac{7\sqrt{3}}{2}$

$y = \frac{11 - 7\sqrt{3}}{2}$

$(x,y) = \left(\frac{7 + 11\sqrt{3}}{2}, \frac{11 - 7\sqrt{3}}{2}\right)$ coordinates of A.

Weaker responses reflected that candidates mostly used incorrect equations of transformation, made wrong substitution of values and made mistakes in the calculations which led the candidates to incorrect answers.

Example 1:

Soln: let the new coordinates of A (x, y)

$\therefore \quad x = X\cos\theta - Y\sin\theta$

$y = Y\cos\theta + X\sin\theta$

$\therefore \quad X = 7$, and $Y = 11$

$x = (7)\cos 60 - (11)\sin 60$

$y = 11\cos(60) + 7\sin 60$

$x = 3.5 - 9.5 = -6$

$y = 5.5 + 6.5 = 12$

So the new x-y coordinates are

$A(x, y) = (-6, 12)$

Example 2:

Soln rotates axes are given by

$x = X\cos\theta - Y\sin\theta$ --- (1)

$y = X\sin\theta + Y\cos\theta$ --- (11)

Here

rotated angle $\theta = 60$

By Putting $\theta = 6$ equ (1) and (2) becomes

(1) $\Rightarrow$ $x = X\cos\theta - Y\sin\theta$ (11) $\Rightarrow$ $y = X\frac{\sqrt{3}}{2} + Y\frac{1}{2}$

$x = X\frac{1}{2} - Y\frac{\sqrt{3}}{2}$ $= \frac{\sqrt{3}X}{2} + \frac{Y}{2}$

$x = \frac{X}{2} - \frac{\sqrt{3}Y}{2}$