آغا خان یونیورسٹی ایگزامینیشن بورڈ

AGA KHAN UNIVERSITY EXAMINATION BOARD

Notes from E-Marking Centre on HSSC-I Mathematics Annual Examinations 2025

Introduction

This document has been produced for the teachers and candidates of Higher Secondary School Certificate (HSSC) Part I Mathematics. It contains comments on candidates' responses to the 2025 HSSC-I Examination indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E-Marking Notes

This includes overall comments on candidates' performance on every question and *some* specific examples of candidates' responses that support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that requires candidates to respond by integrating knowledge, understanding and application skills they have developed during study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfill the demand of the question.

Candidates need to be aware that the marks allocated to the questions are related to the answer space provided on the examination paper as a guide to the length of the required response. A longer response will not in itself lead to higher marks. Candidates need to be familiar with the command words in the SLOs which contain terms commonly used in examination questions. However, candidates should also be aware that not all questions will start with or contain one of the command words. Words such as 'how', 'why' or 'what' may also be used. It is imperative to refer to the command word guide available on AKU-EB website for understanding the expectation of the command word.

General Observations

Most candidates were able to construct good responses across a range of topics. Better performance was seen particularly in the areas of **solving matrix equations, working with combinations, applying fundamental laws of trigonometry, using cube root identities, roots of quadratic equations and their nature, and handling polynomial expressions**. These topics were generally well understood.

However, there are still several areas that require greater focus and improvement. To build a more solid understanding, teachers are encouraged to provide more structured practice and targeted exercises in the following key areas: **completing the square, cofactors of matrices, problems involving geometric mean, sum of geometric series, counting techniques, mathematical induction, trigonometric identities, and trigonometric equations.**

Note: Candidates' responses shown in this report have not been corrected for grammar, spelling, format, or information.

DETAILED COMMENTS
Constructed Response Questions (CRQs)

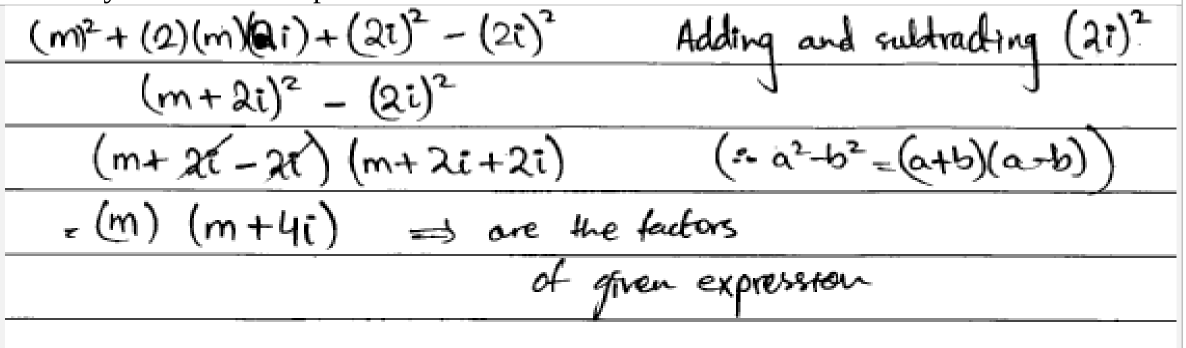
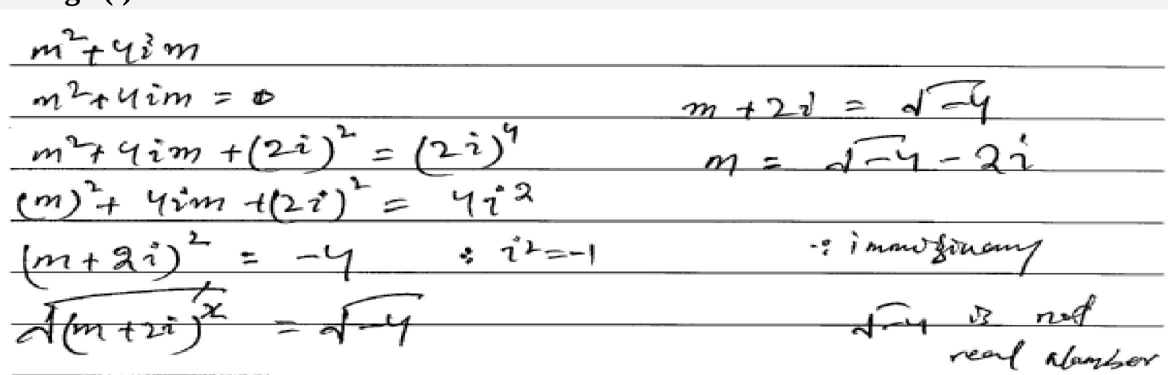
<table>
<tr><th colspan="2">Question No. 1</th></tr>
<tr><td>Question Text</td><td>Factorise the expression $m^2 + 4im$ by using completing square method.</td></tr>
<tr><td>SLO No.</td><td>1.3.3</td></tr>
<tr><td>SLO Text</td><td>Solve quadratic equation $pz^2 + qz + r = 0$; $p \neq 0$ by completing square method, where p, q, r are real numbers and z is a complex number.</td></tr>
<tr><td>Max Marks</td><td>4</td></tr>
<tr><td>Cognitive Level</td><td>*A</td></tr>
<tr><td>Checking Hints</td><td>1 mark for adding and subtracting $(2i)^2$ or $4i^2$
1 mark for writing $(m+2i)^2-(2i)^2$
1 mark for the correct factorisation $(m+2i+2i)(m+2i-2i)$
1 mark for the final answer $(m+4i)(m)$</td></tr>
<tr><td>Overall Performance</td><td>The majority of candidates struggled with applying the method of completing the square. While some followed the correct process, many either used the wrong method or made mistakes when applying the correct technique. Overall, performance was mixed, with clear differences between stronger and weaker attempts.</td></tr>
<tr><td>Description of Better Responses</td><td>In better responses, candidates correctly recognised the need to use the completing square method as instructed. They followed the proper steps by rearranging the expression, adding and subtracting the right values, and then factorising the completed square. These candidates showed that they understood the process and could apply algebraic rules correctly to solve the expression.</td></tr>
<tr><td>Image of Better Response</td><td>
</td></tr>
<tr><td>Description of Weaker Responses</td><td>In weaker responses, some candidates did not use the completing square method at all. Instead, they tried to factorise by taking common terms or used the quadratic formula, which was not asked for. Others attempted the correct method but made mistakes when adjusting terms, especially when adding or subtracting values to complete the square. These errors suggest that more practice is needed with this specific method.</td></tr>
<tr><td>Images of Weaker Responses</td><td>Image (i)

</td></tr>
</table>

	Image (ii) $(m)^2 + 2(m)(2i) + (2i)^2 - 2i$ $(m+2i)^2 - 2i$ $\because a^2-b^2 = (a+b)(a-b)$ $(m+2i+\sqrt{2i})(m+2i-\sqrt{2i}) \rightarrow$ can also be written as : $(m+2i+\sqrt{2}\sqrt{-1})(m+2i-\sqrt{2}\sqrt{-1})$ $(m+2i+\sqrt{-2})(m+2i-\sqrt{-2})$

Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	**Preferred Pedagogy** Used for this SLO**	**Assessment Strategies**
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration ** For description of each Pedagogy, refer to Annexure A	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login
Any Additional Suggestion: The following are some additional simple and classroom-friendly strategies to help improve students' understanding of the completing square method **Guided Fill-in-the-Blank Exercises** Provide the completing square method with blank spaces for key steps (e.g., missing term to complete the square, the factorised form). Students fill in the missing parts, focusing on the structure. **Game-Based Revision** Create a matching game where students pair quadratic expressions with their completed square forms. Use it as a warm-up or revision activity. **Formative Quiz and Feedback** Use a short quiz with a mix of correct and incorrect worked examples. Students identify which are correct and explain why. Follow up with instant feedback and corrections.		

*K = Knowledge U = Understanding A = Application and other higher-order cognitive skills

Question No. 2	
Question Text	If $\begin{bmatrix} p & p & p \\ 3p & 4p & p \\ 4p & 4p & 4p \end{bmatrix} = 2X + \begin{bmatrix} p & 3p & p \\ p & 0 & p \\ 2p & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$, then i. find the matrix *X*. ii. hence, find the X_{12} (Cofactor) of matrix *X*.
SLO No.	2.2.1 and 2.3.2

SLO Text	Apply scalar multiplication, addition and subtraction of matrices. Find the minor and cofactor of elements of a square matrix of order 3 by 3.
Max Marks	6
Cognitive Level	A
Checking Hints	i. 1 mark for writing $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}^{-1}$ as $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ 1 mark for the multiplication of matrix with the identity matrix 1 mark for the addition of the matrices 1 mark for scalar multiplication by $\frac{1}{2}$ ii. 1 mark for writing $(-1)^{1+2}$ correctly 1 mark for writing $\begin{vmatrix}p&0\\p&2p\end{vmatrix}$
Overall Performance	Most candidates showed a reasonable understanding of how to solve for matrix X and calculate a cofactor such as X_{12}. Many were able to follow the correct method, starting with finding the inverse of the identity matrix and proceeding step by step. However, some candidates misunderstood the process, especially in the first part, which affected the accuracy of their results in both parts of the question. Overall, performance was mixed, with better responses standing out due to clear method and correct working.
Description of Better Responses	In *better responses*, candidates correctly began by identifying that they needed to use the inverse of the identity matrix. They followed the correct sequence of steps: multiplying by the inverse, moving the matrix to the left-hand side, and subtracting as required. Finally, they divided each element of the resulting matrix by the scalar 2 to find matrix X. Their method was logical and their calculations were accurate. In the second part, they calculated the cofactor X_{12} correctly using the standard process, showing familiarity with cofactor expansion.
Images of Better Response	**Part (i)** $\begin{bmatrix}p&p&p\\3p&4p&p\\4p&4p&4p\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}^{+1} = 2X + \begin{bmatrix}p&3p&p\\p&0&p\\2p&0&0\end{bmatrix}$ $\because AI_3 = A$ $\begin{bmatrix}p&p&p\\3p&4p&p\\4p&4p&4p\end{bmatrix} - \begin{bmatrix}p&3p&p\\p&0&p\\2p&0&0\end{bmatrix} = 2X$ $2X = \begin{bmatrix}p-p&p-3p&p-p\\3p-p&4p-0&p-p\\4p-2p&4p-0&4p-0\end{bmatrix}$ $2X = \begin{bmatrix}0&-2p&0\\2p&4p&0\\2p&4p&4p\end{bmatrix} \Rightarrow X = \frac{1}{2}\begin{bmatrix}0&-2p&0\\2p&4p&0\\2p&4p&4p\end{bmatrix}$ $X = \begin{bmatrix}0&-p&0\\p&2p&0\\p&2p&2p\end{bmatrix}$ **Part (ii)** $X_{12} = (-1)^{1+2}\begin{vmatrix}p&0\\p&2p\end{vmatrix}$ $\because \{(ad-bc)\}$ $X_{12} = (-1)(2p^2 - 0)$ $X_{12} = -2p^2$

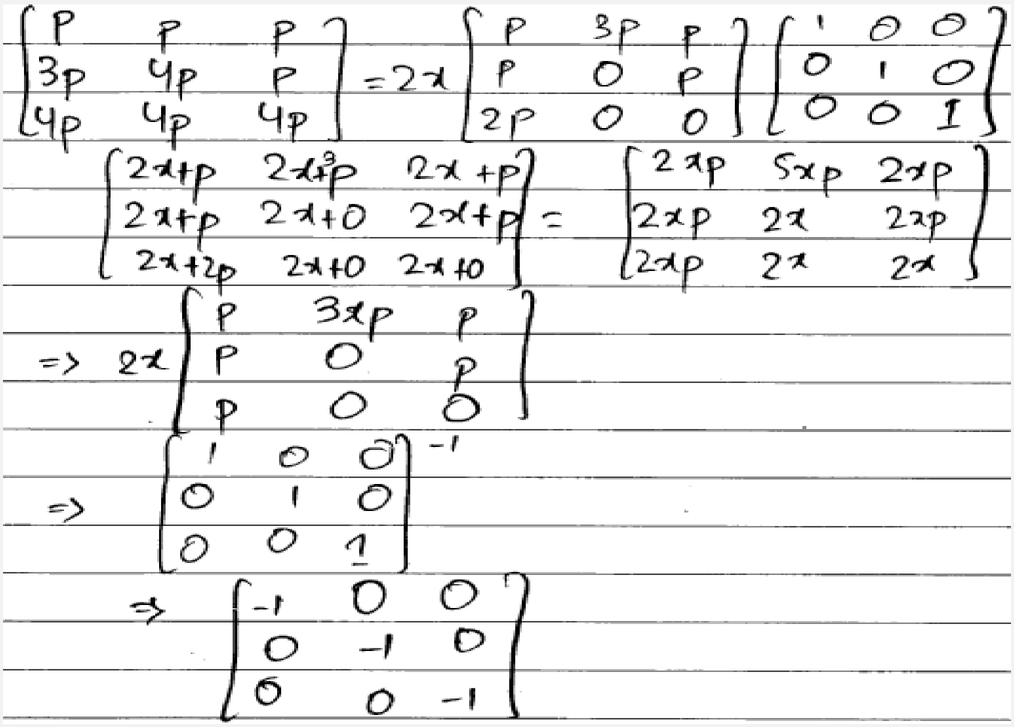
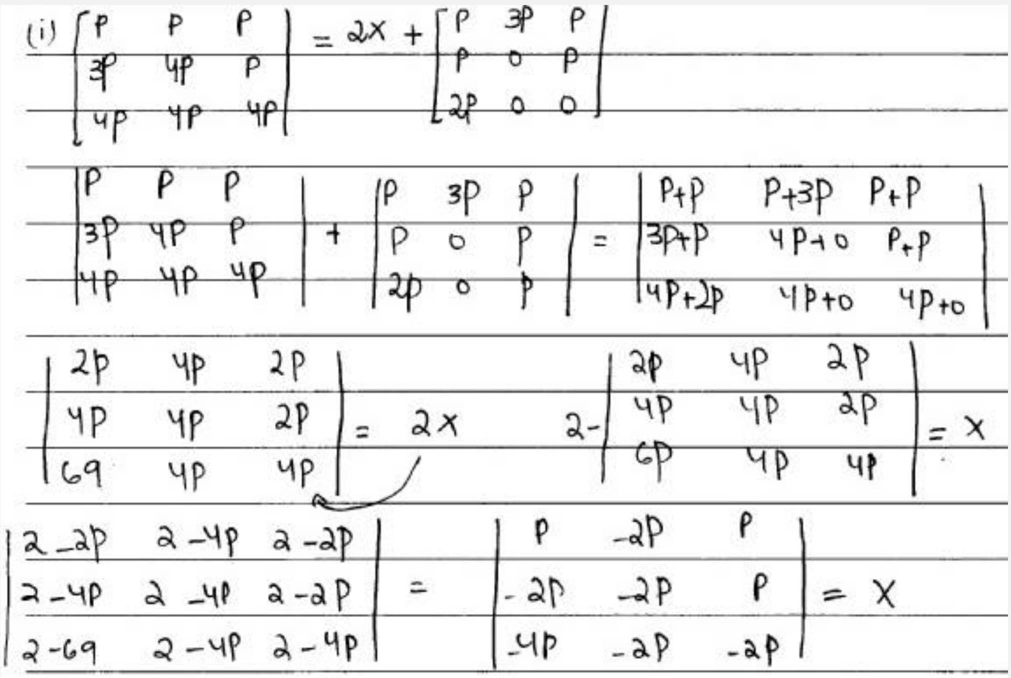
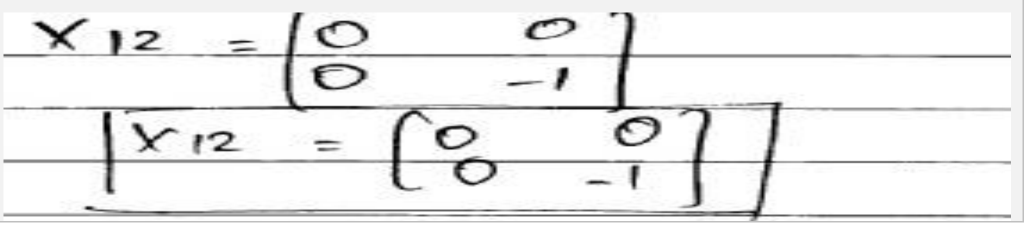
Description of Weaker Responses	In *weaker responses*, candidates showed confusion in the first part of the question. Instead of starting with the inverse of the identity matrix, some attempted addition and multiplication at the same time, which led to errors in the order of operations. This confused the working, and many could not correctly determine matrix *X*. In the second part, while some candidates attempted the cofactor calculation, the earlier mistakes often affected the accuracy of this part as well. Responses were often incomplete or based on incorrect matrices.
Images of Weaker Response	**Part (i)** **Image (i)**  **Image (ii)**  **Part (ii)** 

Image (i)

$$\begin{bmatrix} p & p & p \\ 3p & 4p & p \\ 4p & 4p & 4p \end{bmatrix} = 2x \begin{bmatrix} p & 3p & p \\ p & 0 & p \\ 2p & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$\begin{bmatrix} 2x+p & 2x+3p & 2x+p \\ 2x+p & 2x+0 & 2x+p \\ 2x+2p & 2x+0 & 2x+0 \end{bmatrix} = \begin{bmatrix} 2xp & 5xp & 2xp \\ 2xp & 2x & 2xp \\ 2xp & 2x & 2x \end{bmatrix}$$

$$\Rightarrow 2x \begin{bmatrix} p & 3xp & p \\ p & 0 & p \\ p & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Image (ii)

$$(i) \begin{bmatrix} p & p & p \\ 3p & 4p & p \\ 4p & 4p & 4p \end{bmatrix} = 2X + \begin{bmatrix} p & 3p & p \\ p & 0 & p \\ 2p & 0 & 0 \end{bmatrix}$$

$$\begin{vmatrix} p & p & p \\ 3p & 4p & p \\ 4p & 4p & 4p \end{vmatrix} + \begin{vmatrix} p & 3p & p \\ p & 0 & p \\ 2p & 0 & p \end{vmatrix} = \begin{vmatrix} p+p & p+3p & p+p \\ 3p+p & 4p+0 & p+p \\ 4p+2p & 4p+0 & 4p+0 \end{vmatrix}$$

$$\begin{vmatrix} 2p & 4p & 2p \\ 4p & 4p & 2p \\ 6q & 4p & 4p \end{vmatrix} = 2X \qquad 2 - \begin{vmatrix} 2p & 4p & 2p \\ 4p & 4p & 2p \\ 6p & 4p & 4p \end{vmatrix} = X$$

$$\begin{vmatrix} 2-2p & 2-4p & 2-2p \\ 2-4p & 2-4p & 2-2p \\ 2-6q & 2-4p & 2-4p \end{vmatrix} = \begin{vmatrix} p & -2p & p \\ -2p & -2p & p \\ -4p & -2p & -2p \end{vmatrix} = X$$

Part (ii)

$$X_{12} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\boxed{X_{12} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}}$$

Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	Preferred Pedagogy Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

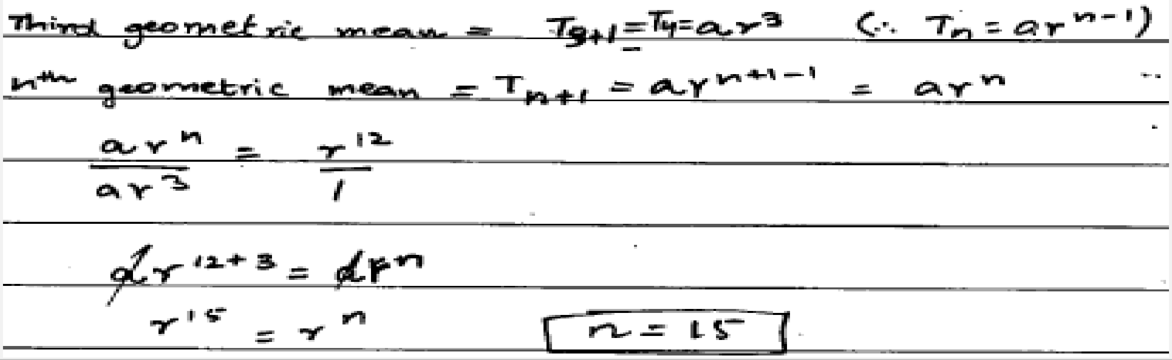
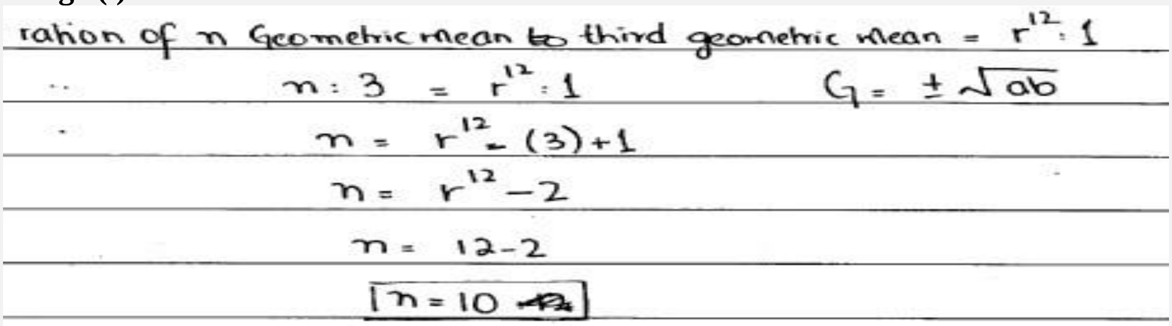
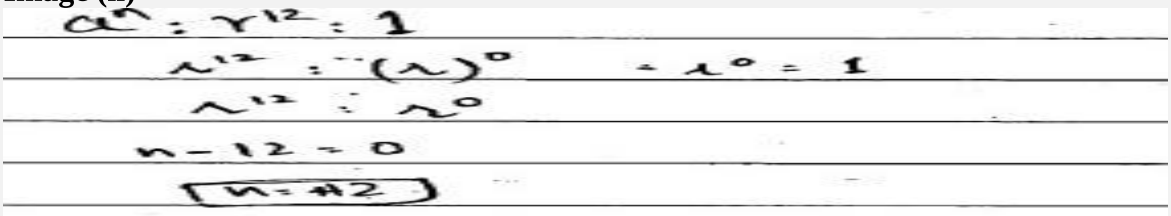
Any Additional Suggestion:
The following are some additional, simple and practical teaching methods to help improve students' understanding of matrix operations.

Use of Visual Aids for Determinants and Minors
Diagrams or grid overlays can help students see which rows and columns to exclude when calculating minors and cofactors.

Guided Scaffolded Worksheets
Create worksheets where the first few problems have structured hints or fill-in-the-blank steps (e.g., Write the matrix to be moved to the LHS, now subtract:). Later questions remove the scaffolding, encouraging independent thinking while reinforcing the process.

Error Analysis Discussion
Present a sample solution with 2–3 intentional errors. As a class or in groups, discuss and correct the mistakes. This builds critical thinking and reinforces correct steps.

Question No. 3i	
Question Text	The ratio of *n* geometric mean to the third geometric mean is given as $r^{12}:1$. Find the value of *n*.
SLO No.	3.6.2
SLO Text	Find '*n*' geometric means between two numbers.
Max Marks	2
Cognitive Level	A
Checking Hints	1 mark for writing $\frac{ar^n}{ar^3}=\frac{r^{12}}{1}$ 1 mark for the simplification to get the value of *n*
Overall Performance	Overall, only a few candidates showed a strong understanding and accuracy when solving this question. While some were able to apply the correct formula and simplify the expression properly, the majority of candidates lacked understanding in using the formulae for geometric means and simplifying ratios involving indices. Many did not fully understand the requirement of comparing the nth geometric mean with the third geometric mean, which led to errors in both formation and simplification of the expression.

Description of Better Responses	In *better responses*, candidates correctly identified that the question involved finding the ratio of the nth geometric mean to the third geometric mean. They used the general formula for the geometric mean, correctly formed the expression $\frac{G_n}{G_3}\frac{ar^n}{ar^3}$, and simplified it accurately using index laws. These candidates showed a clear understanding of geometric progression and how to work with powers when simplifying ratios. Their steps were logically presented, and they correctly found the value of *n*.
Image of Better Response	
Description of Weaker Responses	In *weaker responses*, many candidates misunderstood the question. Instead of forming the expression using the correct formula for the geometric mean in a sequence, several used the square root of *ab*, which is only valid when one mean is required between two terms. Others confused geometric and arithmetic concepts, used unrelated formulae, or made random substitutions without forming a proper ratio. These misunderstandings led to errors in simplification and incorrect answers.
Images of Weaker Responses	**Image (i)**  **Image (ii)** 

Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	Pedagogy Used for that SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

• Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	

Any Additional Suggestion:

The following are some additional, achievable strategies to improve students' understanding of geometric means, formula use, and simplification with indices.

Formula Sorting Activity

Provide different formulae (arithmetic mean, geometric mean, nth term, etc.) and ask students to match them with the correct type of progression. This helps clarify when and where to use each formula.

Ratio Formation Practice

Give students pairs of expressions involving geometric means and ask them to form and simplify ratios. This builds fluency in working with expressions and powers.

Create Your Problem

After solving an example, ask students to create their version of a problem involving geometric mean ratios. Then swap with a partner and solve each other's problems. This encourages critical thinking and peer learning.

Question No. 3ii	
Question Text	Convert $0.06\overline{7}$ into the equivalent common fractions by using infinite geometric series. (**Note**: $0.06\overline{7} = 0.067777...$)
SLO No.	3.7.4
SLO Text	Convert the recurring decimal into an equivalent common fraction.
Max Marks	4
Cognitive Level	A
Checking Hints	1 mark for writing $0.06\overline{7} = 0.06 + \{.007 + 0.0007 + 0.00007 + ...\}$ 1 mark for identification of $a = 0.007$ and $r = \dfrac{0.0007}{0.007} = 0.1$ 1 mark application of the correct formula to get $S_\infty = \dfrac{0.007}{1-0.1} = \dfrac{0.007}{0.9} \times \dfrac{1000}{1000} = \dfrac{7}{900}$ 1 mark for the final Answer ($\dfrac{61}{900}$)
Overall Performance	The question tested candidates' understanding of converting recurring decimals into fractions using geometric series. Overall performance was not as per the grade level/ as per the expectations. While some candidates showed a clear grasp of the process and applied it correctly, the majority found it difficult to identify and separate the non-repeating and repeating parts of the decimal. This confusion often led to errors in forming the geometric series and applying the correct formula.
Description of Better Responses	In *better responses*, candidates successfully identified the non-repeating and repeating parts of the decimal. They correctly broke down the decimal and recognised the repeating part as an infinite geometric series. Candidates accurately identified the first term (a) and common ratio (r) and applied the sum formula for an infinite geometric series. Finally, they combined the result with the non-repeating part to form the correct fractional representation. Their steps showed understanding of both the structure of recurring decimals and how they relate to series.

Image of Better Response	$0.06 + (0.007 + 0.0007 + 0.00007 + \cdots)$ $a_1 = 0.007$ $r = \frac{0.0007}{0.007} = 0.1$ $S_\infty = \frac{a}{1-r} = \frac{0.007}{1-0.1} = \frac{0.007}{0.9}$ $S_\infty = \frac{7}{900} + 0.06$ $S_\infty = \frac{61}{900}$
Description of Weaker Responses	In *weaker responses*, many candidates struggled to separate the recurring part from the non-recurring part of the decimal. This led to incorrect breakdowns of the number. Some did not identify the first term correctly or used the wrong common ratio. Others used the wrong formula, often applying the finite sum formula rather than the correct infinite sum formula, which resulted in incorrect fractions. In some cases, responses were incomplete or skipped steps, suggesting uncertainty about the method.
Image of Weaker Response	$S_\infty = \frac{a}{1-r}$ $\lvert r \rvert < 1$ $a_1 = 0.067$ $S_\infty = \frac{0.067}{1-r}$ $S_\infty = \frac{0.067}{1-0.067}$ $S_\infty \ \frac{0.067}{0.933}$ $S_\infty = \frac{0.067}{\frac{0.933}{100}} = \frac{0.067}{\frac{933}{100000}}$ $S_\infty = 0.067 \times \frac{100000}{933}$ $S_\infty = \frac{6700}{933}$ Ans

Suggestions for improvement (Highlight all that apply)

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• Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	

Any Additional Suggestion:
The following are more additional and practical teaching methods specifically designed to strengthen students' understanding of converting recurring decimals into fractions using geometric series.

Compare Both Methods
Teach the standard algebraic method and the geometric series method side-by-side. Then, ask students to solve the same decimal using both approaches to compare and understand why they lead to the same result.

Scaffolded Worksheets
Use worksheets with partial steps provided. For example, the repeating part is already identified, and students must complete the remaining steps. Gradually remove scaffolding as confidence builds.

Use of Decimal Grids
Show recurring decimals on place value grids to visually explain which parts repeat and where the geometric structure starts.

Highlight and Label Technique
Ask students to highlight or underline the repeating part of a decimal in one colour and the non-repeating part in another. Then label the place value of each digit. This helps make the structure of the decimal visible and easier to split into parts.

True or False Sorting
Provide examples of recurring decimal conversions (some correct, some intentionally incorrect). Students sort them into "Correct" and "Incorrect" piles, explaining why. This helps them recognise common errors (e.g., using the wrong ratio or formula).

Question No. 4i	
Question Text	The number of outcomes in which at least one of the five dice show '6' is x. Find x, if five dice are rolled once.
SLO No.	5.2.4
SLO Text	Solve problems related to ${}^{n}P_{r}$.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for finding total number of outcomes 6^5 1 mark for writing $x = 7776 - 3125$ or writing in words 1 mark for the correct answer $x = 4651$
Overall Performance	This question was challenging for candidates. Although the method relied on the standard complement rule, only a small number of candidates solved it correctly. While many understood the basic concept, several struggled with interpreting the phrase "at least one of the five dice show a 6" and selecting an efficient approach. As a result, responses showed a wide range of understanding.
Description of Better Responses	In *better responses*, candidates correctly used the complement rule. They realised that it was easier to calculate the total number of outcomes where no dice shows a 6 and subtract this from the total number of all outcomes. The steps are shown in the image. These candidates showed a sound understanding of the use of complementary counting to solve probability problems efficiently.

Images of Better Responses	**Image (i)** Total number of outcomes $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \Rightarrow 7776$ atleast one dice is rolled = Total – no 6 rolled $x = 6^5 - (5 \cdot 5 \cdot 5 \cdot 5 \cdot 5)$ ← no 6th choice thus only choice of 5 faces $= 7776 - 3125$ $x = 4651$ numbers of possible outcomes. **Image (ii)** Total outcomes of 6 dices $= 6^5 = 7776$ Outcomes in which no 6 is there $= 5^5 = 3125$ x = Outcomes in which atleast one dice show x = Total – Outcomes in which no 6 is there $= 7776 - 3125$ $\boxed{x = 4651}$
Description of Weaker Responses	In *weaker responses*, many candidates misinterpreted the phrase "at least one of the 5 dice show a 6" and attempted to manually count cases where exactly 1, 2, 3, 4, or 5 sixes appeared. This approach was unnecessarily long, increased the chance of miscounting, and often resulted in overlapping cases. Some candidates calculated only 6^5 or 5^5 and assumed these alone were the answer, without applying the logic behind the complement rule. These mistakes showed a lack of clarity in strategy and understanding of the structure of probability problems involving "at least one".
Images of Weaker Responses	$= {}^5C_5 + {}^5C_4 + {}^5C_3 + {}^5C_2 + {}^5C_1$ $= 1 + 5 + 10 + 10 + 5$ $\boxed{= 31 \text{ outcomes}}$

Suggestions for improvement (Highlight all that apply)

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• Utilise the resource guide for additional materials	• Practical Demonstration	

Any Additional Suggestion:
The following are more classroom-friendly and achievable activities to improve students' understanding of the complement rule in probability.
Concept Clarification Using Venn Diagrams
Use visual aids like Venn diagrams or block models to show the difference between "at least one", "none", and "exactly one", helping students grasp the idea of complements in probability.
Complement Rule Practice Tasks
Provide students with a set of problems that all use the complement rule (e.g., "At least one head in 4 coin tosses"). This repeated use reinforces pattern recognition.
Worked Examples with Parallel Cases
Solve simpler examples in class (e.g., 2 dice or 3 coins), then gradually build up to 5-dice problems. This helps students see how the method scales up.
Try Both Ways Activity
Allow students to attempt both the direct counting and complement method for a 3-dice problem, then compare which is easier and why. This builds reasoning skills.
Interactive Simulation
Use a dice-rolling simulator to show outcomes in real-time. Ask students to count how many times "at least one 6" appears in a set of trials. This connects theory with practice.

Question No. 4ii	
Question Text	Using combinations, prove that ${}^nC_r = {}^nC_{n-r}$.
SLO No.	5.2.7
SLO Text	Prove that, for any positive integers *n* and *r*, where $n > r$: a. $\binom{n}{n} = \binom{n}{0} = 1$ b. $\binom{n}{r} = \binom{n}{n-r}, \binom{n}{1} = \binom{n}{n-1} = n$ c. $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$
Max Marks	3
Cognitive Level	U
Checking Hints	1 mark for substituting ${}^nC_{n-r} = \dfrac{n!}{(n-(n-r))!(n-r)!}$ 1 mark for expanding ${}^nC_{n-r} = \dfrac{n!}{(n-n+r))!(n-r)!}$
Overall Performance	This question was well-attempted by the majority of candidates. Most were able to recall the definition of combinations using factorials and recognised that the identity involved the symmetry property of combinations, ${}^nC_r = {}^nC_{n-r}$. However, the quality of performance varied depending on the accuracy of factorial manipulation and the clarity of reasoning during simplification.
Description of Better Responses	In *better responses*, candidates showed a good understanding of the combination formula. They correctly substituted $(n - r)$ in place of *r*, leading to equivalent expressions. These responses demonstrated that both forms are mathematically the same. Candidates simplified the factorials properly and reached the correct conclusion. They used accurate notation and followed logical steps throughout the solution, effectively justifying the identity.

Image of Better Response	R.H.S: ${}^{n}C_{n-r} = \frac{n!}{(n-(n-r))!(n-r)!}$ $= \frac{n!}{(n-n+r)!(n-r)!}$ $= \frac{n!}{(r)!(n-r)!}$ $= \frac{n!}{(n-r)!r!}$ $= {}^{n}C_r$ L.H.S: ${}^{n}C_r = \frac{n!}{(n-r)!r!}$ L.H.S = R.H.S hence proved!
Description of Weaker Responses	In *weaker responses*, although most candidates attempted to use the factorial form, many made calculation or cancellation errors. Some candidates cancelled factorial terms incorrectly or confused the order of operations in the numerator and denominator. Others failed to demonstrate the symmetry of the expressions. A few candidates only wrote the identity without any algebraic justification or left the solution incomplete. These responses showed gaps in understanding of how factorials behave in fraction form and the basic symmetry property of combinations.
Images of Weaker Responses	**Image (i)** $\Rightarrow {}^{n}C_r = \frac{n!}{(n-r)!r!}$ Taking R.H.S; $= {}^{n}C_{n-1}$ $= \frac{n!}{(n-r-r)!r!}$ $= \frac{n!}{(n-2r)!r!}$ $= \frac{n!}{(n-2r+r)!r!}$ $\Rightarrow \frac{n!}{(n-r)!r!} = {}^{n}C_r$ Hence Proved! **Image (ii)** $\frac{n!}{(n-r)!r!} = \frac{n!}{(n+n-r)!(n-r)!}$ $\frac{n!}{(n-r)!r!} = \frac{n!}{(2n-r)!(n-r)!\,r!}$ $\frac{n!}{(n-r)!r!} = \frac{n!}{(n-r)!r!}$

Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	Preferred Pedagogy Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

• Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	

Any Additional Suggestion:
The following are some additional teaching methods and classroom strategies to strengthen students' understanding of combinatory, specifically the symmetry property of combinations.
Visual Proof Using Pascal's Triangle
How to use: Construct Pascal's Triangle up to at least row 6 or 7.
Activity: Ask students to observe how entries are symmetrical across each row.
Card Sorting: True or False
Task: Students receive cards with different combinations identities (some correct, some incorrect).
Job: Sort them into "True", "False", or "Needs Working".

Question No. 5	
Question Text	Use mathematical induction to show that $n^2 - n$ is divisible by 2, for all integral values of n.
SLO No.	6.1.2
SLO Text	Prove the statements, identities and formulae using the principle of mathematical induction. (**Note:** Questions involving inequalities are not included for example: $n^2 > n + 3$ for integral values of $n \geq 3$);
Max Marks	6
Cognitive Level	A
Checking Hints	1 mark for proving for $n = 1$ 1 mark for writing the proposition for $n = k$ 1 mark for writing the proposition for $n = k + 1$ to get $\frac{k^2 + 2k + 1 - k - 1}{2}$ 1 mark for the simplification to get $\frac{k^2 - k + 2k}{2}$ 1 mark for writing $\frac{k^2 - k}{2} + \frac{(2k)}{2}$ 1 mark for writing a reason to show that it is true for $n = k + 1$
Overall Performance	The question on mathematical induction required candidates to follow a clear sequence of steps, including verifying the base case, forming the inductive hypothesis, and using it to prove the case for $n = k + 1$. Overall, performance was weak. While few candidates followed the correct structure and reasoning, many struggled to apply the method accurately, particularly in the inductive step.
Description of Better Responses	In *better responses*, candidates demonstrated a solid understanding of the induction process. They correctly verified the base case for $n = 1$, clearly stated the inductive hypothesis for $n = k$, and logically applied it to prove the result for $n = k + 1$. Their steps were well-organised, with each stage of the proof identified and justified. These responses showed familiarity with the method and confident algebraic handling.

Images of Better Responses

Image (i)

C – I we have to show that $n^2 - n$ is divisible by 2 if $n = 1$

$n^2 - n = (1)^2 - 1 = 1 - 1 = 0$ is divisible by 2

C-I is satisfied.

C – II We assume that eq is true when $n = k$

$k^2 - k$ is divisible by 2

C – III Now we have to prove it is also divisible by k+1

$(k+1)^2 - (k+1)$

$k^2 + 2k + \cancel{1} - k - \cancel{1}$

$\cancel{k^2 + k}$ $k^2 - k + 2k$

As we have assumed that $k^2 - k$ is divisible by 2 and it can be seen that $2k$ is also divisible by 2 this means it is true for all values of n.

Proved

Image (ii)

$\boxed{n=1}$ $(1)^2 - 1 = 0$

$\boxed{n=2}$ $(2)^2 - 2 = 2$

$\boxed{n=k}$ $k^2 - k = 2Q$ —①

$\boxed{n=k+1}$

$(k+1)^2 - (k+1)$

$k^2 + \cancel{1} + 2k - k - \cancel{1}$

$k^2 + k$

* Adding And substracting k

$k^2 + k - k + k$

$k^2 - k + 2k$

$2Q + 2k$

$2[Q + k]$ —②

Hence, Proved that given expression divisible by 2 for $n = k+1$.

Image (iii)

$S(1): n^2 - n$

$(1)^2 - (1)$

$1 - 1 = 0 \rightarrow$ Divisible by 2

$S(k): k^2 - k$

$= k(k-1)$ Divisible by 2

$k^2 - k = 2Q$ —①

$S(k+1) = (k+1)^2 - (k+1)$

$= k^2 + 2k + \cancel{1} - k - \cancel{1}$

$= k^2 + k$

$= k^2 + k + k - k$

$= k^2 - k + 2k$

$= 2Q + 2k$ By eq ①

$= 2(Q + k)$ Hence proved

$S(k+1)$ is true for $S(k)$, so $S(n)$ is true for all +ve integers

Description of Weaker Responses

In *weaker responses*, candidates often skipped the base case or assumed it without checking. Many did not clearly state the inductive hypothesis or struggled to apply it in the inductive step. Some candidates incorrectly attempted to prove the statement by testing various values of *n*, rather than using general reasoning. Others performed algebraic manipulations without providing proper explanation or justification, which resulted in incomplete or incorrect proofs.

Images of Weaker Responses

Image (i)

$\frac{n^2 - n}{2}$ $n \in \mathbb{R}$

$\frac{2^2 - 2}{2} = \frac{4 - 2}{2} = \frac{\cancel{2}}{\cancel{2}} = 1$

$\frac{4^2 - 4}{2} = \frac{16 - 4}{2} = \frac{\cancel{12}}{\cancel{2}} = 6$

$\frac{9^2 - 9}{2} = \frac{81 - 9}{2} = \frac{\cancel{72}}{\cancel{2}} = 36$

Image (ii)

$n^2 - n$

$2 \,\big|\, n^2 - n$

$+2 \quad 2n^2$

$4 \quad -2n^2 - n$

$+6 \quad +4n^2$

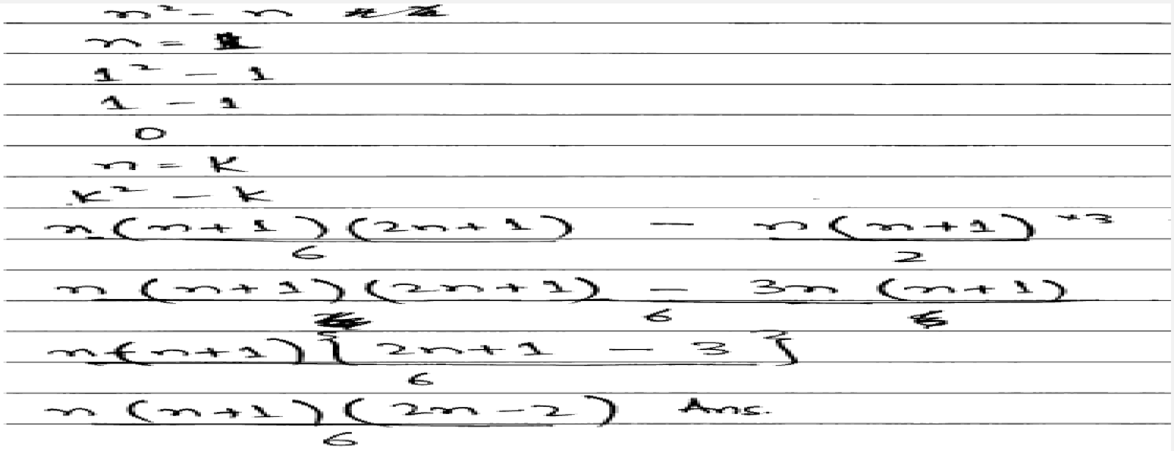
$6-4 \quad 2 - n$

$+4n \quad +2$

$-n \quad -n$

$+n$

0

	Image (iii)
	

Suggestions for improvement (Highlight all that apply)

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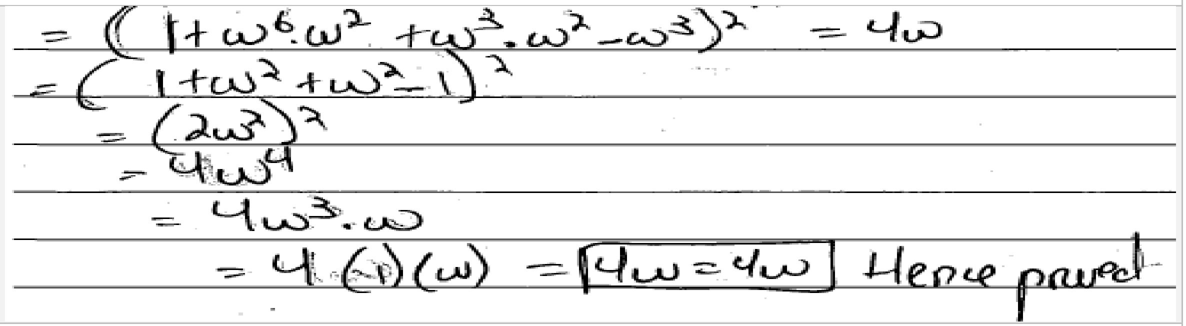
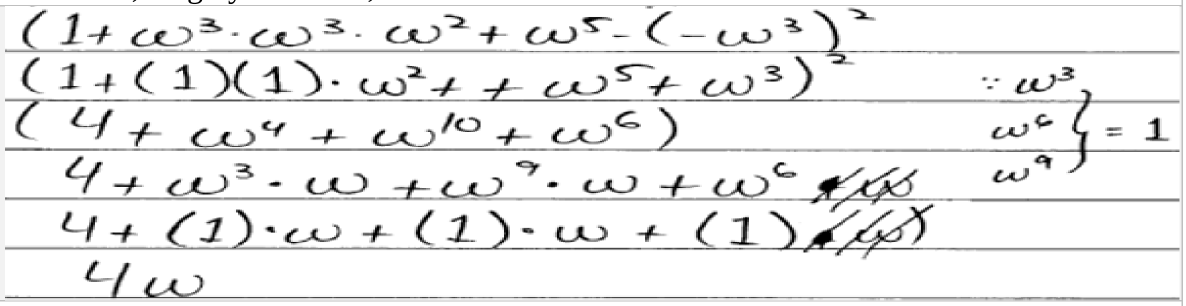
Any Additional Suggestion:
The following are several achievable and classroom-friendly strategies and methods to help students improve their understanding and application of mathematical induction.
Error Hunt Worksheets
Create a worksheet with intentionally flawed induction proofs. Let students:
Spot the errors in the base case, assumption, or inductive step.
Explain what went wrong and correct it.
Reverse Induction Problems
Give students a completed induction proof with steps jumbled. They must:
Reorder the steps.
Label each part (base case, assumption, step, conclusion).
Interactive Digital Tools (e.g., Desmos or GeoGebra)
For numerical or inequality-based induction, use tools to graph both sides of an equation to visually confirm validity for different values of *n*.

Question No. 6a
Candidates were given the choice to attempt any ONE out of the two questions: 6a and 6b.

Question Text	i. Prove that $(1+\omega^8+\omega^5-\omega^3)^2 = 4\omega$ (**Note**: Where ω is one of the cube roots of unity.)
SLO No.	7.4.4
SLO Text	Solve problems related to properties of cube roots of unity.

Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for writing $(1+\omega^8+\omega^5-\omega^3)^2 = [1+\omega^2.(\omega^3)^2+\omega^2.\omega^3-\omega^3]^2$ or equivalent 1 mark for application of the fact that $\omega^3=1$ 1 mark for the simplification to get the required prove.
Overall Performance	Many of the candidate opted for part a rather than part b. The question tested understanding of the cube roots of unity and their properties. While many candidates handled the simplification of powers correctly and applied the identities as required, some responses showed uncertainty, particularly when reducing higher powers of ω. Mistakes in index manipulation affected the accuracy of the final answers.
Description of Better Responses	Candidates who performed well showed a good understanding of the complex cube roots of unity. They correctly recalled the fundamental identity $\omega^3 = 1$ and used this to simplify higher powers by expressing them in terms of ω, ω^2, or 1. Their approach was systematic, and they simplified powers such as ω^5 by recognising that $\omega^5 = \omega^2$, and similarly for other powers. These candidates used these values effectively in the final expression and reached the correct result.
Image of Better Response	
Description of Weaker Responses	In weaker response, candidates struggled with applying the identity $\omega^3 = 1$. Common errors included simplifying powers like ω^5 or ω^8 incorrectly, either by leaving them as they were or by misapplying the reduction (e.g., stating $\omega^5 = \omega$ instead of ω^2). In other cases, candidates expanded and added terms without first simplifying the powers, which led to confusion, lengthy solutions, and incorrect final answers.
Image of Weaker Response	
Question Text	ii. For a quadratic equation $ax^2+bx+c=0$, the sum of roots is $\frac{13}{6}$, the product of roots is 1 and $c=6$. Find the values of *a* and *b*. Also, calculate discriminant of the equation.
SLO No.	7.5.3 and 7.3.2
SLO Text	Solve problems based on the sum and product of roots. Determine the nature of roots of a given quadratic equation.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for writing values of sum and product of roots in terms of *a*, *b* and *c* 1 mark finding the values of *a* and *b* 1 mark for finding the discriminant

Overall Performance	Overall, candidates performed reasonably well in this question. Many were able to recall and apply the correct concept related to the sum and product of the roots of a quadratic equation. Some candidates worked directly using the standard formulae, while others showed all the substitution steps in both the sum and product formulae before proceeding to calculate the discriminant. However, variations in approach and accuracy led to mixed levels of success.
Description of Better Responses	In *better responses*, candidates demonstrated a sound understanding of the relationship between the roots and the coefficients of a quadratic equation. They correctly recalled and applied the identities for the sum and product of the roots, typically using the formulae $\alpha + \beta = -b/a$ and $\alpha\times\beta = c/a$. Some candidates directly substituted into the formulae to find the values of *a* and *b*, while others clearly showed all the intermediate steps. Once the values were correctly found, they went on to substitute these into the discriminant formula $D = b^2 - 4ac$, arriving at the correct or near-correct answer. Their work was usually neat and logically presented, which often earned them full or high marks even if a minor arithmetic error was present.
Images of Better Responses	**Image (i)** Given; $\alpha+\beta = \frac{13}{6}$ $\alpha\cdot\beta = 1$, $c = 6$ $\alpha+\beta = \frac{-b}{a}$ so, $-b = 13$, and $a = 6$. , $C = 6$ Equation: $ax^2+bx+c=0$; to find Determinant : b^2-4ac. $D : (-13)^2 - 4(6)(6)$ {Result; Determinant= 25} $\boxed{D = 25}$ **Image (ii)** $\cancel{\alpha+\beta = \frac{-b}{a}}$ | $\alpha+\beta = \frac{-b}{a}$ | Disc $= b^2 - 4ac$ $\frac{13}{6} = \frac{-b}{a}$ $= (-13)^2 - 4(6)(6)$ $\alpha\beta = \frac{c}{a}$ $= 169 - 144$ $\boxed{-13 = b}$ $= 25$ $1 = \frac{6}{a}$ $\boxed{a = 6}$
Description of Weaker Responses	*Weaker responses* revealed several common misunderstandings and mistakes. Many candidates incorrectly assumed that the given values in the question represented the sum and product of the roots, labelling them directly as *a* and *b* without applying the appropriate formulae. This led them to calculate the discriminant based on incorrect assumptions, which affected their final answers. Some responses showed a lack of clarity in formula use, with confusion between the coefficients of the quadratic equation and the actual values of the roots. In other cases, candidates substituted values inconsistently, such as using incorrect signs or placing values in the wrong parts of the formula. A few did not show any working steps at all, making it difficult to award method marks, while others attempted to find the discriminant without determining the correct values of *a*, *b*, and *c*, resulting in entirely inaccurate conclusions.
Image of Weaker Response	In quadratic equation product of root is b and sum of root is a Hence $a = \frac{13}{6}$, $b = 1$ discriminant $\Delta = b^2 - 4ac$ $D = \Delta = -51$ not real

Suggestions for improvement (Highlight all that apply)

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Any Additional Suggestion:
To strengthen conceptual understanding and accuracy in such questions, the following teaching approaches are suggested:

Spot the Mistake
Show a wrongly simplified expression like: "$\omega^5 = \omega$", ask students to correct it, and explain the error.

Use Foldables or Flip Cards
Provide students with a foldable: One flap shows ω^n, the other flap shows its simplified form.

Concept Sorting Activities
Mix cards with correct and incorrect statements about ω (e.g., $\omega^3 = 0$, $\omega^6 = 1$, $\omega^2 = -\omega - 1$).
Students sort them into True/ False piles, then explain why.

Deliberate Practice of Common Errors
Include incorrect worked examples in your lessons. Ask students to identify and correct the mistake, such as swapping the sum and product, using wrong signs, or misapplying coefficients. This builds awareness of pitfalls and improves their attention to detail.

Visual Anchors and Formula Maps
Create and display a simple visual anchor chart in the classroom that shows the connection between the coefficients of a quadratic equation and its roots. Include the formulae for sum, product, and discriminant in a single diagram or table to support visual learners.

Digital Tools and Apps
Use online platforms like GeoGebra, Desmos, or other quadratic calculators to allow students to explore how changes in a, b, and c affect the sum/ product and the discriminant. Interactive tools can make abstract formulae more concrete.

Question No. 6b

Candidates were given the choice to attempt any ONE out of the two questions: 6a and 6b.

Question Text	The polynomial $Q(x)$ is given as $px^3 - 2x^2 + 7x + q$, and it is divisible by $x - 2$. i. Show that the equation connecting p and q is $8p + q = -6$. ii. Further, the polynomial $Q(x)$ is divided by $x + 1$ and leaves a remainder of -10. Find the relationship between p and q. iii. Use the answer of (i) and (ii), find the value of p and q.
SLO No.	7.7.1
SLO Text	Solve problems related to remainder and factor theorem.
Max Marks	6

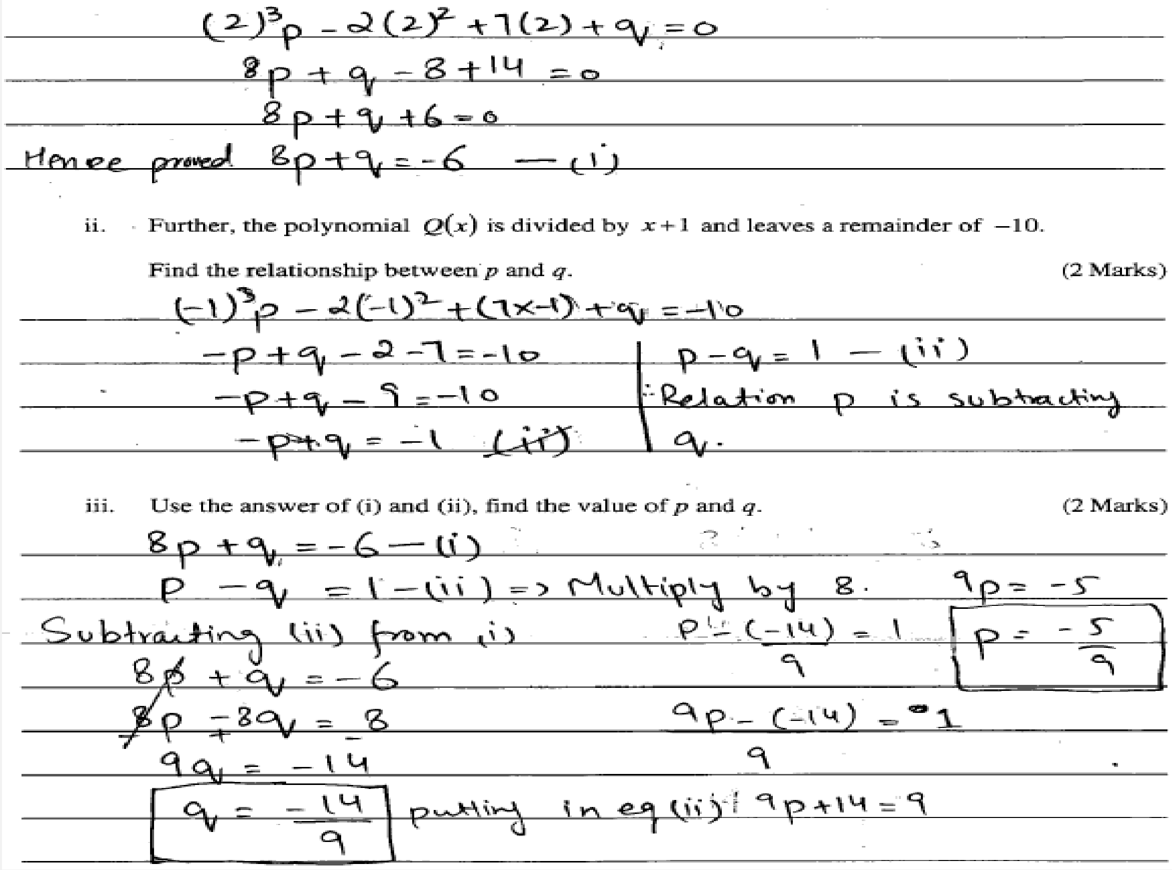
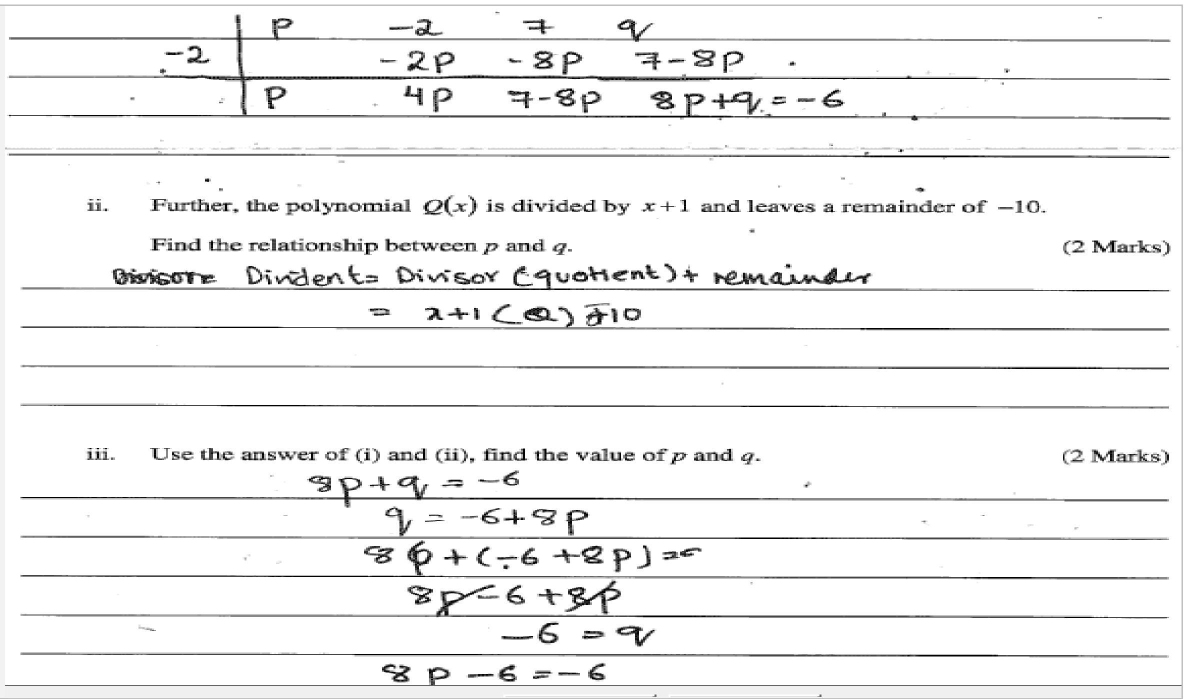
Cognitive Level	A
Checking Hints	i. 1 mark for substituting $x = 2$ in $Q(x) = px^3 - 2x^2 + 7x + q$ 1 mark for writing $Q(2) = 0$ to get the required proof ii. 1 mark for writing $Q(-1) = -10$ 1 mark for writing the relationship between p and q iii. 1 mark for finding the value of p 1 mark for finding the value of q
Overall Performance	In comparison, few candidates opted for part b. Overall, candidates showed a reasonable understanding of the topic. Many were able to attempt the question and apply the correct methods. In part (i), most candidates correctly used the Factor Theorem to form the required equation. However, in part (ii), some candidates found it difficult to apply the Remainder Theorem accurately. While several candidates successfully used both conditions to calculate the values of p and q, others made algebraic errors or misread the conditions, which affected their final answers.
Description of Better Responses	In *better responses*, candidates correctly recognised that divisibility by $x - 2$ meant substituting $x = 2x$ into $Q(x)$ and setting the expression equal to zero. They carried out the substitution properly, simplified the expression correctly, and formed the equation relating p and q. In part (ii), they applied the Remainder Theorem by substituting $x = -1$ into the polynomial and setting the result equal to –10. They then formed two clear equations and solved them using suitable methods such as substitution or elimination to find the correct values of p and q. Their work was clearly shown and followed a logical order.
Image of Better Response	 $(2)^3p - 2(2)^2 + 7(2) + q = 0$ $8p + q - 8 + 14 = 0$ $8p + q + 6 = 0$ Hence proved $8p + q = -6$ — (i) ii. Further, the polynomial $Q(x)$ is divided by $x+1$ and leaves a remainder of -10. Find the relationship between p and q. (2 Marks) $(-1)^3p - 2(-1)^2 + (7\times-1) + q = -10$ $-p + q - 2 - 7 = -10$ $-p + q - 9 = -10$ $-p + q = -1$ (ii) $p - q = 1$ — (ii) ∴ Relation p is subtracting q. iii. Use the answer of (i) and (ii), find the value of p and q. (2 Marks) $8p + q = -6$ — (i) $p - q = 1$ — (ii) => Multiply by 8. Subtracting (ii) from (i) $8p + q = -6$ $8p - 8q = 8$ $9q = -14$ $\boxed{q = \frac{-14}{9}}$ putting in eq (ii): $p - \frac{(-14)}{9} = 1$ $\frac{9p - (-14)}{9} = 1$ $9p + 14 = 9$ $9p = -5$ $\boxed{p = -\frac{5}{9}}$
Description of Weaker Responses	In *weaker responses,* some candidates did not apply the Factor Theorem correctly. They either substituted incorrectly or did not equate the result to zero. In part (ii), several candidates made errors with the Remainder Theorem, such as using the wrong value for x or making sign mistakes during substitution. This led to incorrect equations and final answers. A few candidates attempted synthetic division, but without setting it up properly, which caused further confusion. In many cases, the work was either missing or unclear, making it difficult to follow the method.

Image of Weaker Response



	p	-2	7	q
-2		$-2p$	$-8p$	$7-8p$
	p	$4p$	$7-8p$	$8p+q=-6$

ii. Further, the polynomial $Q(x)$ is divided by $x+1$ and leaves a remainder of -10.

Find the relationship between p and q. (2 Marks)

~~Divisore~~ Dividents= Divisor (quotient)+ remainder

= x+1 (Q) ∓10

iii. Use the answer of (i) and (ii), find the value of p and q. (2 Marks)

8p+q = −6

q = −6+8p

8p+(−6+8p) = −6

8p−6+8p

−6 = q

8p −6 = −6

Suggestions for improvement (Highlight all that apply)

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Any Additional Suggestion:

The following strategies, when used consistently and adapted to student needs, can significantly improve confidence, accuracy, and depth of understanding in algebraic reasoning and polynomial operations:

Introduce Error Spotting Exercises

Give students a worked-out solution with intentional errors. Ask them to identify and correct the mistake. This helps them review key steps, such as substitution, simplification, and equation formation.

Encourage Multiple Methods

Show more than one way to approach a question (e.g., substitution, synthetic division, factorisation). Let students compare the methods and choose the one they're most confident using.

Link to Graphical Understanding

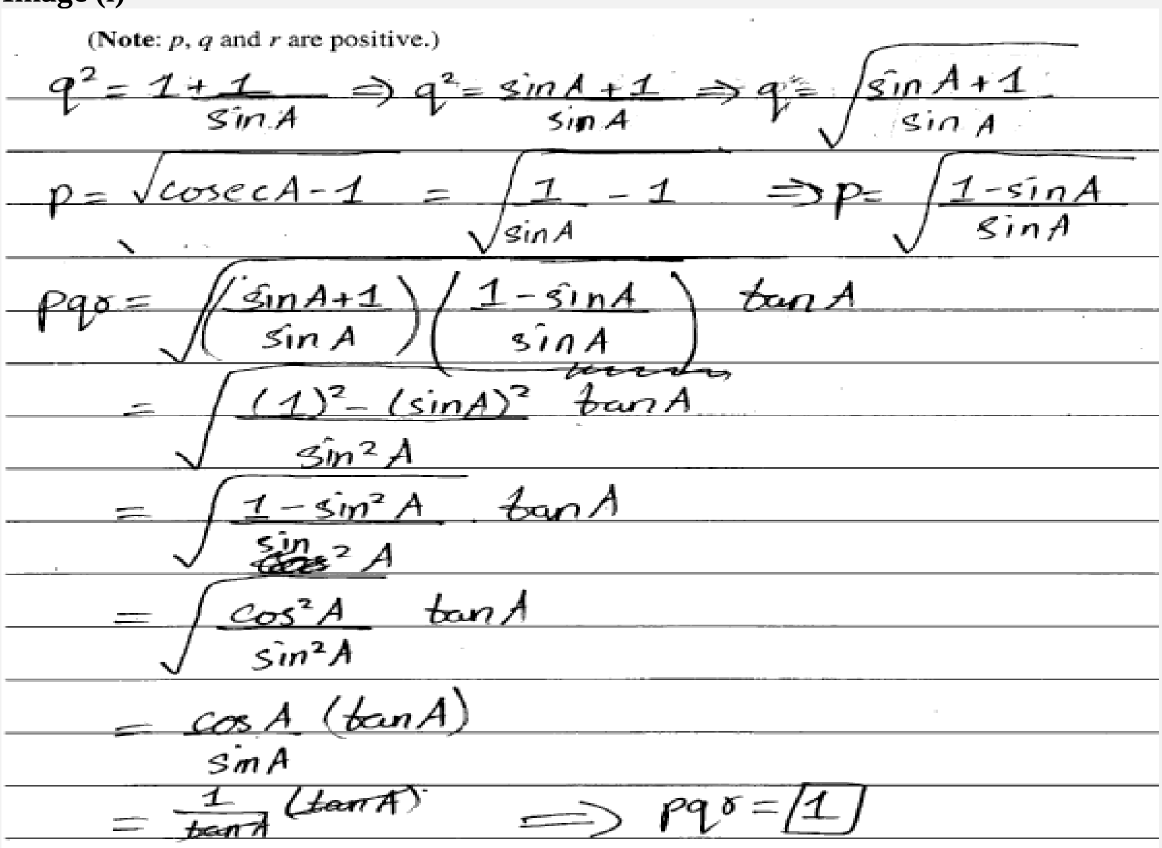
Where appropriate, connect algebraic outcomes to graphical features. For example, explain that if $x = a$ is a factor, then the graph of the polynomial crosses the x-axis at $x = a$.

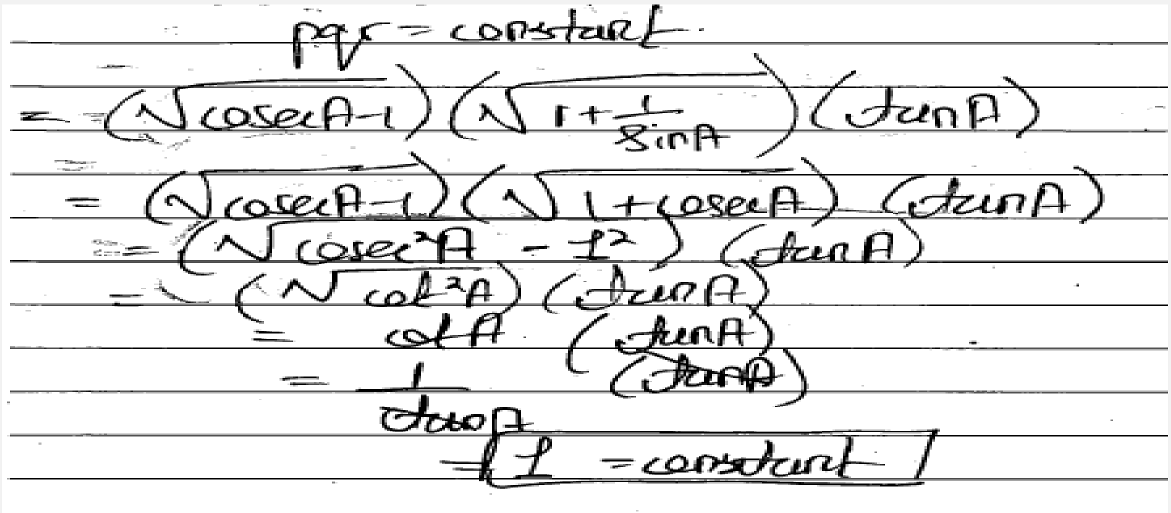
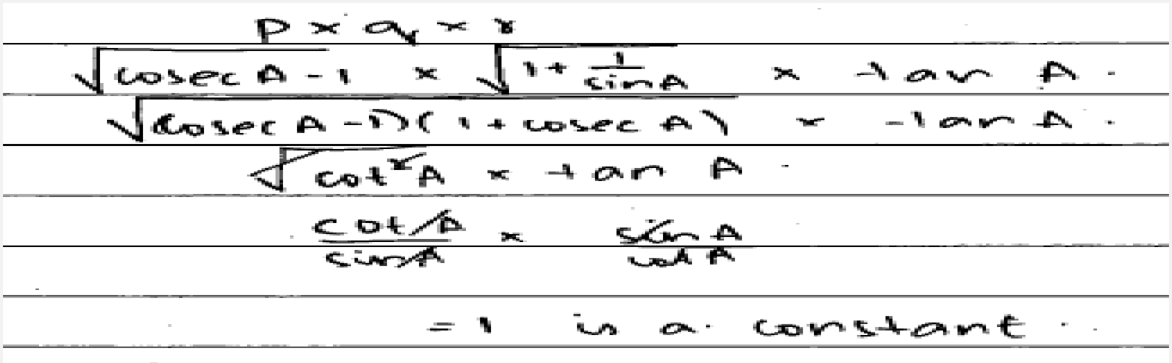
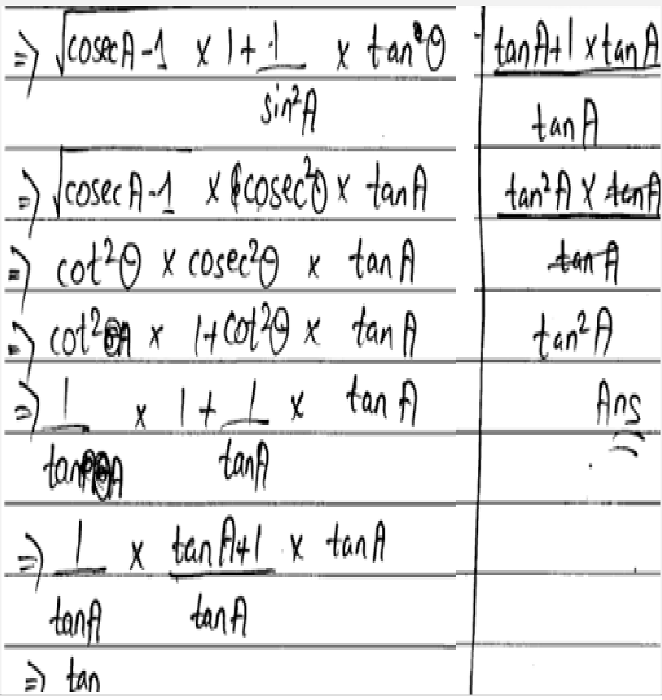
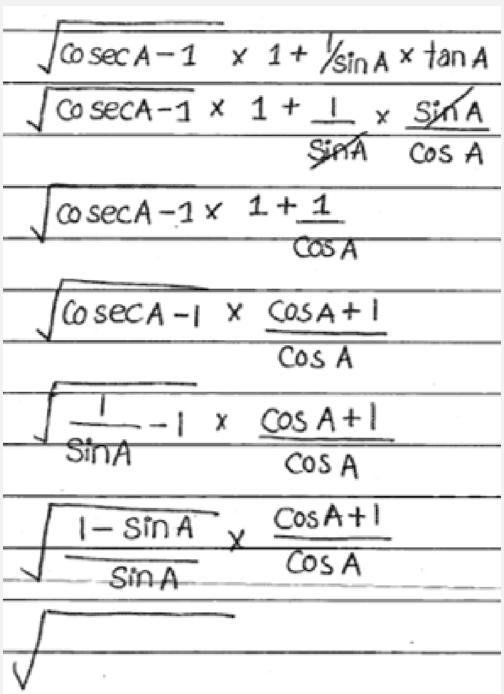
Practice Through Real-Time Quizzes

Use tools like Kahoot, Quizizz, or in-class flashcards for timed practice. Focus on quick recall of rules and properties (e.g. "What is the remainder when $x^2 + 5x + 6$ is divided by $x + 2$?").

Question No. 7a

Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b and 7c.

Question Text	If $p = \sqrt{\operatorname{cosec} A - 1}$, $q^2 = 1 + \dfrac{1}{\sin A}$ and $r = \tan A$, then show that pqr is constant. (**Note**: *p*, *q* and *r* are positive.)
SLO No.	8.2.2
SLO Text	Prove different trigonometric relations using trigonometric identities.
Max Marks	5
Cognitive Level	A
Checking Hints	1 mark for writing $q = \sqrt{1 + \operatorname{cosec} A}$ 1 mark for writing $pqr = \left(\sqrt{\operatorname{cosec} A - 1}\right)\left(\sqrt{1 + \operatorname{cosec} A}\right)(\tan A)$ 1 mark for writing $\left(\sqrt{\operatorname{cosec}^2 A - 1}\right)(\tan A)$ 1 mark for using the trigonometric identity to get $(\cot A)(\tan A)$ 1 mark for the required proof
Overall Performance	This question was selected by the students more than part b. Overall, candidates found this question difficult. While a few were able to complete the proof by applying the correct trigonometric identities, many struggled to choose and use the right identity. This made it hard for them to carry out the required steps and reach a correct answer. Some were able to find part of the solution but were not able to complete the full proof.
Description of Better Responses	In *better responses*, candidates applied the correct identities, such as $cot^2A = csc^2A - 1$ or $1 - sin^2A = \cos^2A$, at the right stage in their working. They substituted values carefully and followed each step in a clear order. These candidates correctly found the value of *q*, and then used it to calculate the product *pqr*, giving the correct result of 1. Their work showed that they understood the connection between the identities and the steps needed to complete the proof.
Images of Better Responses	**Image (i)**  (**Note**: *p*, *q* and *r* are positive.) $q^2 = 1 + \frac{1}{\sin A} \Rightarrow q^2 = \frac{\sin A + 1}{\sin A} \Rightarrow q = \sqrt{\frac{\sin A + 1}{\sin A}}$ $p = \sqrt{\operatorname{cosec} A - 1} = \sqrt{\frac{1}{\sin A} - 1} \Rightarrow p = \sqrt{\frac{1 - \sin A}{\sin A}}$ $pqr = \sqrt{\left(\frac{\sin A + 1}{\sin A}\right)\left(\frac{1 - \sin A}{\sin A}\right)} \tan A$ $= \sqrt{\frac{(1)^2 - (\sin A)^2}{\sin^2 A}} \tan A$ $= \sqrt{\frac{1 - \sin^2 A}{\sin^2 A}} \tan A$ $= \sqrt{\frac{\cos^2 A}{\sin^2 A}} \tan A$ $= \frac{\cos A}{\sin A} (\tan A)$ $= \frac{1}{\tan A} (\tan A) \Rightarrow pqr = \boxed{1}$

<table>
<tr><td></td><td>Image (ii)

$pqr = \text{constant}$
$= \left(\sqrt{\operatorname{cosec}A - 1}\right)\left(\sqrt{1 + \frac{1}{\sin A}}\right)(\tan A)$
$= \left(\sqrt{\operatorname{cosec}A - 1}\right)\left(\sqrt{1 + \operatorname{cosec}A}\right)(\tan A)$
$= \left(\sqrt{\operatorname{cosec}^2 A - 1^2}\right)(\tan A)$
$= \left(\sqrt{\cot^2 A}\right)(\tan A)$
$= \cot A \,(\tan A)$
$= \frac{1}{\tan A}(\tan A)$
$= \boxed{1 = \text{constant}}$
Image (iii)

$p \times q \times r$
$\sqrt{\operatorname{cosec}A - 1} \times \sqrt{1 + \frac{1}{\sin A}} \times \tan A$.
$\sqrt{(\operatorname{cosec}A - 1)(1 + \operatorname{cosec}A)} \times \tan A$.
$\sqrt{\cot^2 A} \times \tan A$
$\frac{\cot A}{\sin A} \times \frac{\sin A}{\cot A}$
$= 1$ is a constant.</td></tr>
<tr><td>Description of Weaker Responses</td><td>In weaker responses, candidates had difficulty choosing the correct identity or applying it properly. Some used incorrect substitutions, such as putting in q^2 instead of q. Others confused similar identities or left out key steps in the proof. In many cases, their work was not clear, and mistakes in algebra or simplification led to incorrect answers. These issues showed that the understanding of identities and how to apply them was not secure.</td></tr>
<tr><td>Images of Weaker Responses</td><td>Image (i)

$\Rightarrow \sqrt{\operatorname{cosec}A - 1} \times 1 + \frac{1}{\sin^2 A} \times \tan^2\theta$
$\Rightarrow \sqrt{\operatorname{cosec}A - 1} \times (\operatorname{cosec}^2\theta) \times \tan A$
$\Rightarrow \cot^2\theta \times \operatorname{cosec}^2\theta \times \tan A$
$\Rightarrow \cot^2\theta A \times 1 + \cot^2\theta \times \tan A$
$\Rightarrow \frac{1}{\tan A} \times 1 + \frac{1}{\tan A} \times \tan A$
$\Rightarrow \frac{1}{\tan A} \times \frac{\tan A + 1}{\tan A} \times \tan A$
$\Rightarrow \tan$
$\frac{\tan A + 1 \times \tan A}{\tan A}$
$\frac{\tan^2 A \times \cancel{\tan A}}{\cancel{\tan A}}$
$\tan^2 A$
Ans
Image (ii)

$\sqrt{\operatorname{Cosec}A - 1} \times 1 + 1/\sin A \times \tan A$
$\sqrt{\operatorname{Cosec}A - 1} \times 1 + \frac{1}{\cancel{\sin A}} \times \frac{\cancel{\sin A}}{\cos A}$
$\sqrt{\operatorname{Cosec}A - 1} \times 1 + \frac{1}{\cos A}$
$\sqrt{\operatorname{Cosec}A - 1} \times \frac{\cos A + 1}{\cos A}$
$\sqrt{\frac{1}{\sin A} - 1} \times \frac{\cos A + 1}{\cos A}$
$\sqrt{\frac{1 - \sin A}{\sin A}} \times \frac{\cos A + 1}{\cos A}$
$\sqrt{\ }$</td></tr>
</table>

Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	Preferred Pedagogy Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:

The following strategies can be used flexibly in class or during revision, helping candidates build better understanding and accuracy when using trigonometric identities in questions that require proof or simplification.

Matching Activities

Use cards or worksheets where students match identities to their equivalent forms, to help them recognise the correct one when needed.

Complete-the-Step Exercises

Provide partly completed solutions and ask students to fill in the missing steps. This guides them through the logic of the proof.

Mix with Basic Algebra Review

Make sure candidates are confident with expanding brackets, simplifying expressions, and solving equations. These algebra skills are needed to complete trigonometric proofs correctly.

Question No. 7b

Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b and 7c.

Question Text	Given that $\tan(\alpha+\beta)=\frac{1}{6}$, $\sec\alpha=\sqrt{\frac{13}{9}}$ and $\ell(\alpha)$ is in 1st quadrant. Find $\tan\beta$.
SLO No.	8.3.4
SLO Text	Solve problems related to fundamental law of trigonometry and its deductions.
Max Marks	5
Cognitive Level	A
Checking Hints	1 mark for converting the identity into $\tan\alpha=\sqrt{\sec^2\alpha-1}$ / $\alpha=\cos^{-1}\left(\sqrt{\frac{9}{13}}\right)$ 1 mark for finding $\tan\alpha$ / $\alpha=33.6°\ or\ 0.5\ rad$ 1 mark for getting $\frac{1}{6}\left(1-\frac{2}{3}\tan\beta\right)=\frac{2}{3}+\tan\beta$ 1 mark for getting $\frac{1}{6}-\frac{2}{3}=\frac{1}{9}\tan\beta+\tan\beta$ 1 mark for getting $\tan\beta$

<table>
<tr><td>Overall Performance</td><td>Many candidates did not select this question and preferred to answer parts (a) and (c). Those who attempted it mostly applied the correct method to find the required value. The most common error was the incorrect identification of tan(α + β) and mistakes in simplification.</td></tr>
<tr><td>Description of Better Responses</td><td>In better responses, candidates were correct in identifying the value of tanα from the given information. They substituted this value correctly into the identity for tan(α+β) and rearranged the formula carefully to solve for tanβ. Their work was shown in a logical sequence, and they used brackets properly to avoid errors during simplification. Some candidates also approached the question differently by finding the angle α using the inverse tangent function, then using angle addition to calculate tan(α+β), and finally solving for tanβ. These candidates demonstrated a good understanding of the relationship between angles and trigonometric ratios and were able to manage the algebra involved in the process with reasonable accuracy.</td></tr>
<tr><td>Images of Better Responses</td><td>

Image (i)

$\tan(\alpha+\beta) = \frac{1}{6}$ $\quad \sec\alpha = \sqrt{\frac{13}{9}}$ $\quad \alpha$ (I Quadrant)

$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$

$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\frac{1}{6} = \frac{\frac{2}{3} + \tan\beta}{1 - \frac{2}{3}\tan\beta}$

$\frac{1}{6} = \frac{\frac{2+3\tan\beta}{\cancel{3}}}{\frac{3-2\tan\beta}{\cancel{3}}}$

$\frac{1}{6} = \frac{2+3\tan\beta}{3-2\tan\beta}$

$3 - 2\tan\beta = 12 + 18\tan\beta$

$20\tan\beta = 3 - 12$

$20\tan\beta = -9$

$\boxed{\tan\beta = -9/20}$

$\sec\alpha = \frac{\sqrt{13}}{3}$

$\cos\alpha = \frac{3}{\sqrt{13}}$

$\sin\alpha = \sqrt{1 - \left(\frac{3}{\sqrt{13}}\right)^2}$

$\sin\alpha = \sqrt{1 - \frac{9}{13}}$

$= \sqrt{4/13} = \frac{2}{\sqrt{13}}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{2}{\sqrt{13}} \times \frac{\sqrt{13}}{3}$

$\tan\alpha = \frac{2}{3}$

Image (ii)

$\Rightarrow \sec\alpha = \sqrt{\frac{13}{9}}$

$\Rightarrow \frac{1}{\cos\alpha} = \frac{\sqrt{13}}{3}$

$\Rightarrow \cos\alpha = \frac{3}{\sqrt{13}}$ — B, — H

$P = \sqrt{H^2 - B^2}$

$= \sqrt{13 - 9}$

$P = 2$

$\boxed{\tan\alpha = \frac{2}{3}}$

$\Rightarrow \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\frac{1}{6} = \frac{\frac{2}{3} + \tan\beta}{1 - \left(\frac{2}{3}\right)\tan\beta}$

$\frac{1}{6} = \frac{2+3\tan\beta}{\cancel{3}} \div \frac{3-2\tan\beta}{\cancel{3}}$

$\frac{1}{6} = \frac{2+3\tan\beta}{3-2\tan\beta}$

$3 - 2\tan\beta = 12 + 18\tan\beta$

$3 - 12 = 18\tan\beta + 2\tan\beta$

$-9 = 20\tan\beta$

$\boxed{\tan\beta = \frac{-9}{20}}$

</td></tr>
</table>

	Image (iii) $\sec\alpha = \frac{1}{\cos\alpha}$ $\qquad \cos\alpha = \frac{\sqrt{9}}{\sqrt{13}} \frac{\to B}{\to H}$ for P $\quad P^2 = H^2 - B^2$ $= (\sqrt{13})^2 - (\sqrt{9})^2$ $\sqrt{P^2} = \sqrt{4}$ $P = 2$ $\tan\alpha = \frac{P}{B} = \frac{2}{\sqrt{9}} = \frac{2}{3}$ $\tan(\alpha+\beta) = 1/6$ $\quad\vert\quad \tan\alpha = \frac{2}{3}$ $(\alpha+\beta) = \tan^{-1}(1/6)$ $\quad\vert\quad \alpha = \tan^{-1}(\frac{2}{3}) = 33.69$ $\beta = 9.46 - \alpha$ $\beta = 9.46 - 33.69$ $\beta = -24.23$ $\tan(-24.23) = -0.45$ $\tan\beta = -0.45$ $\qquad$ S.Set = $\{-0.45\}$
Description of Weaker Responses	In *weaker responses*, many candidates had difficulty identifying or calculating tanα correctly. Some confused it with sinα or cosα, while others made errors in dividing the given values. Even when they found tanα correctly, they often substituted it into the formula for tan(α+β) incorrectly, sometimes placing values in the wrong position or omitting brackets, which led to mistakes in simplification. A few candidates attempted to rearrange the formula but struggled with the algebra, especially when solving for tanβ. In some cases, key steps were missing or not clearly shown, making it difficult to follow the method. These issues suggest that more practice is needed in applying trigonometric identities and rearranging expressions accurately.
Images of Weaker Responses	**Image (i)** $\sec\alpha = \sqrt{\frac{13}{9}} = \frac{H}{B}$ $\qquad \tan(\alpha+\beta) = 1/6$ $\tan\alpha + \tan\beta = \frac{1}{6}$ $\cos\alpha = \sqrt{\frac{9}{13}} = \frac{B}{H}$ $\qquad \tan\beta = \frac{1}{6} - \cancel{\frac{2}{\sqrt{13}}}\ \frac{2}{\sqrt{9}}$ $P = \sqrt{H^2 - B^2}$ $\qquad \boxed{\tan\beta = 1/2}$ $P = \sqrt{(\sqrt{13})^2 - (\sqrt{9})^2}$ $\qquad \Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $P = \sqrt{13 - 9}$ $\qquad \frac{\tan(\frac{2}{\sqrt{9}}) + \tan(\frac{1}{2})}{1 - \tan(\frac{2}{\sqrt{9}})\tan(\frac{1}{2})}$ $P = \sqrt{4}$ $\boxed{P = \pm 2}$ $\tan\alpha = \frac{P}{B} = \cancel{\frac{2}{\sqrt{13}}}\ \frac{2}{\sqrt{9}}$ $\Rightarrow 1/6$ answer! **Image (ii)** $\tan(\alpha+\beta) = \frac{1}{6}$ $\tan\alpha + \beta = \frac{\tan\alpha + \tan\beta}{1 + \tan\alpha \tan\beta}$ $\qquad$ taking tan common $\tan\alpha + \beta = \frac{\tan(\alpha + \beta)}{1 + \tan(\alpha\beta)}$ $\frac{1}{6} = \frac{\frac{1}{6}}{1 + \tan(\alpha\beta)}$ $\frac{1}{36} = \frac{1}{\tan\alpha\beta}$ $\quad\vert\quad$ add α in numerator $\frac{1}{36} = \frac{\alpha}{\tan\alpha\beta}$ $\quad\vert\quad \frac{1}{36} = \frac{1}{\tan\beta} = \boxed{\tan\beta = 36}$ 1^{st} Quadrant

Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	**Preferred Pedagogy Used for this SLO**	**Assessment Strategies**
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Any Additional Suggestion:
The following are the additional simple and practical teaching strategies to support students in learning and applying tangent addition identities and related trigonometric concepts:

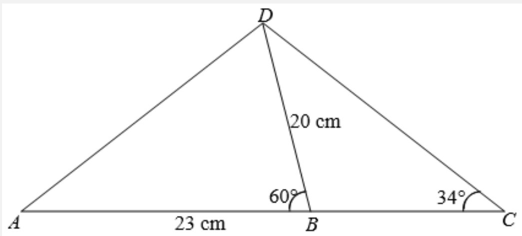
Formula Breakdown Activities
Break the formula into parts and guide students through each component (numerator and denominator), so they understand how each value fits in.

Error-Checking Practice
Give students completed questions with small mistakes in calculation or substitution and ask them to find and correct the errors.

Support Use of Scientific Calculators
Provide guidance on using calculators properly for inverse trigonometric functions and decimal approximations, particularly for checking final answers.

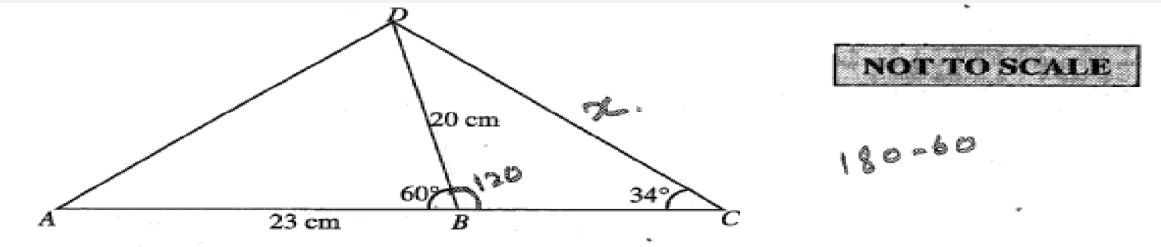
Question No. 7c

Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b and 7c.

Question Text	In the given diagram, $AB = 23\,\text{cm}$, $BD = 20\,\text{cm}$, $\angle ABD = 60°$ and $\angle BCD = 34°$. **NOT TO SCALE**  Calculate i. the length of CD. ii. the length of AD.
SLO No.	9.1.5
SLO Text	Solve problems related to SLOs from 9.1.1 to 9.1.4. (9.1.2. Prove the: a. law of cosines, b. law of sines, c. law of tangents.)
Max Marks	5
Cognitive Level	A
Checking Hints	i. 1 mark for finding $\angle DBC$ 1 mark for applying sine law correctly 1 mark for funding CD ii. 1 mark for applying cosine law correctly 1 mark for finding AD
Overall Performance	Many candidates were able to attempt this question successfully. A good number correctly identified the 120° angle, although a few found the diagram challenging due to the presence

<table>
<tr><td></td><td>of two triangles. While many managed to apply the correct methods, some struggled with choosing the appropriate rule or identifying the right angle from the diagram.</td></tr>
<tr><td>Description of Better Responses</td><td>In better responses, candidates accurately found angle DBC and correctly applied the sine rule to calculate the length of CD. They then used the cosine rule effectively to find the length of AD. Their work was logical and steps were shown clearly. Some candidates also checked their answers using alternative approaches, showing proficiency in using both rules where required.</td></tr>
<tr><td>Images of Better Responses</td><td>

Image (i)



Calculate

i. the length of *CD*. (3 Marks)

let CD = a then by using formula $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

and DB = C = 20 ∵ $\alpha = 120°$ $\gamma = 34°$

$\frac{a}{\sin 120} = \frac{20}{\sin 34}$

$a\sin 34 = 20 \times \sin 120$

$a \times 0.56 = 17.32$

$\Rightarrow a = \frac{17.32}{0.56}$

$\boxed{a = 30.9 \approx 31 = CD}$

ii. the length of *AD*. (2 Marks)

let AD = b for △ADB

AB = b = 23cm

DB = A = 20 cm

using formula $c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = (20)^2 + (23)^2 - 2(20)(23) \times 0.5$ ∵ $\cos\gamma = 0.5$

$c^2 = 400 + 529 - 920 \times 0.5$

$c^2 = 469$, $\boxed{c = 21.6 \approx 22} = DA$

Image (ii)

$= \angle CBD = 180° - \angle ABD = 180° - 60° = 120°$

$\frac{CD}{\angle CBD} = \frac{BD}{\angle BCD}$

$= \frac{CD}{\sin(120°)} = \frac{20}{\sin(34°)}$

$= CD = 30.96$ cm

ii. the length of *AD*.

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = 23^2 + 20^2 - 2(23)(20)\cos(60°)$

$c^2 = 469$

$\sqrt{c^2} = \sqrt{469} = c = 21.65$ cm.

</td></tr>
</table>

Description of Weaker Responses	In *weaker responses*, some candidates gave an incorrect value for angle *DBC*, often writing 60° instead of 120°, which affected the rest of their calculations. Others did not apply the correct rule to the appropriate triangle. Some used the cosine rule where the sine rule was required, or vice versa. A few tried to calculate the area of the triangle first, which was a longer and less efficient method and did not lead directly to the required lengths. Errors in selecting or applying the correct law were common among these responses.
Images of Weaker Responses	**Image (i)** $\cancel{CD = a^2 + b^2 + c^2 - 2ac\cos}$ $\Rightarrow c^2 = a^2 + b^2 - 2ab\cos\gamma$ $\Rightarrow c^2 = (23)^2 + (20)^2 - 2(23)\cos(34)^\circ$ $c^2 = 529 + 400 - 46(0.829)$ $c^2 = 929 - 38.134$ $\sqrt{c^2} = \sqrt{890.866}$ $C = 29.84 = CD$ ii. the length of *AD*. (2 Marks) $\cancel{b^2 = a^2 + c^2 - 2ac\cos\beta}$ $a^2 = b^2 + c^2 - 2bc\cos\alpha \Rightarrow a^2 = (20)^2 + (29.84)^2 - 2(20)(29.84)\cos 60^\circ$ $a^2 = 400 + 890.4256 - 1193.6\ \cancel{2(23)\cos 60^\circ}\ (\frac{1}{2})$ $\sqrt{a^2} = \sqrt{96.8256(\frac{1}{2})}$ $\overline{AD} = \sqrt{48.4128} \Rightarrow AD = 6.957$. **Image (ii)** i. the length of *CD*. (3 Marks) $\Delta = \frac{(20)^2 \sin(26)\sin(120)}{2\sin 34}$ $\Delta = \frac{400(0.43837)(0.866)}{2(0.559)}$ $\Delta = \frac{151.85}{1.118} = 135.8\ cm^2$ area of triangle DAC. $\Delta = \frac{(CD)^2 \sin 26 \sin 34}{2\sin 120}$ $\frac{135.8 \times 2(0.866)}{0.4383 \times 0.559} = (CD)^2$ $\frac{235.212}{0.24509} = (CD)^2$ $\sqrt{959.69} = CD$ $30.98cm = CD$ ii. the length of *AD*. (2 Marks) $\Delta = \frac{1}{\cancel{2}}(23)(\cancel{20}\ 10)\sin 60$ $\Delta = 230(0.866)$ $\Delta = 199.185$ area of triangle ADB. $\Delta = \frac{1}{2}(23)(AD)\sin$ $\frac{199.185 \times 2}{23\sin(\ )} = AD$ $\frac{398.37}{23\sin(\ )} = AD$ $\frac{17.320}{\sin(\ )} = AD$

Suggestions for improvement (Highlight all that apply)

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Any Additional Suggestion:

The following are the strategies aim to develop a deeper, more flexible understanding of trigonometric methods for solving triangle problems.

Focus on Angle Identification

Use specific exercises to practise identifying angles within larger shapes or overlapping triangles, to avoid common errors like mislabeling 60° for 120°.

Encourage Neat Working and Labelling

Teach students to label diagrams neatly and write all steps clearly. This helps reduce errors and makes it easier to follow their own work.

Build a Rule Selection Flowchart

Create or provide a simple flowchart that asks yes/no questions to help students decide whether to use the sine rule or the cosine rule. Display it on the wall or give each student a copy.

Provide Partial Diagrams for Completion

Give students partially labelled diagrams and ask them to complete missing values or identify steps before doing full calculations. This encourages planning and understanding of the triangle structure.

Encourage Use of Calculator Check Steps

Teach students to check their calculator steps after each stage (e.g., computing cosA, squaring lengths) to avoid typing errors, especially in multi-step cosine rule problems.

Offer Mixed Practice Sets

Use worksheets that mix sine rule, cosine rule, and basic trigonometry questions so that students must choose the correct approach based on the given information.

Revisit Common Errors Regularly

Use a "common errors" board in class where you regularly post a sample mistake (e.g., using the wrong triangle, forgetting to square a side, misreading an angle) and ask students to spot and correct it.

Question No. 8	
Question Text	Find all the possible values of x for which $\dfrac{4}{\operatorname{cosec}^2 x} - 3 = \sin x$, where $0^\circ \le x \le 360^\circ$.
SLO No.	10.4.2
SLO Text	Find general solution of a trigonometric equation and discard extraneous roots taking into account the period of the trigonometric function.
Max Marks	6
Cognitive Level	A

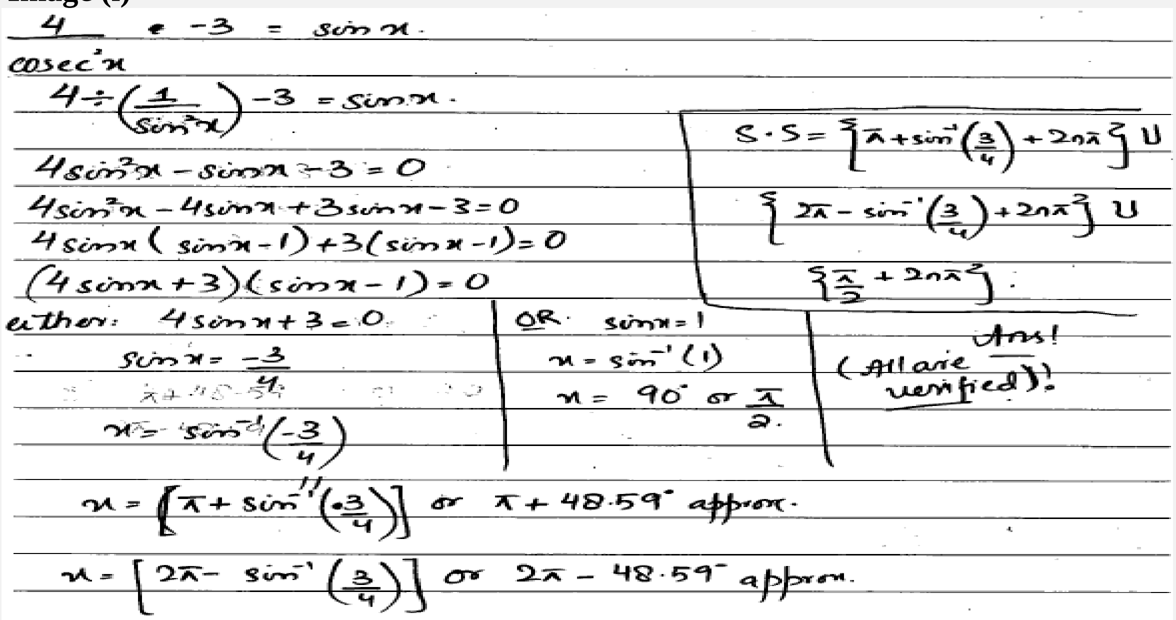
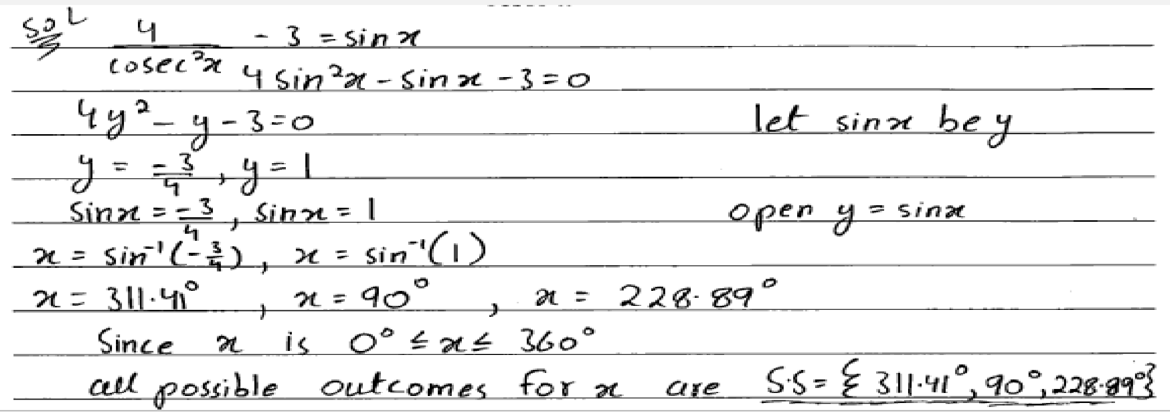
Checking Hints	1 mark for writing $4\sin^2 x - \sin x - 3 = 0$ 1 mark for breaking the middle term or alternate method 1 mark for writing $\sin x = -\frac{3}{4}$; $\sin x = 1$ 1 mark each for finding the basic angle of $\sin x = -\frac{3}{4}$ *and* $\sin x = 1$ 1 mark for finding the values of *x*, i.e. *x* = 228.59 or 311.40 1 mark for finding the value of *x*, i.e. $x = 90^{\circ}$
Overall Performance	This question produced a range of responses. While some candidates showed a clear understanding, many struggled with key parts of the process. Common difficulties included factorising trigonometric expressions, applying identities, and finding all possible angle solutions within the given domain of 0° to 360°. In some cases, candidates gave angles beyond 360°, or only identified a single solution, such as 90°, missing others that satisfied the equation. These issues suggest that further practice is needed in solving trigonometric equations and understanding the importance of working within a full rotation.
Description of Better Responses	In *better responses,* candidates showed confidence in solving trigonometric equations. They correctly simplified the given expressions, applied appropriate identities where needed, and factorised accurately. These candidates were also careful to find all solutions within the correct domain, identifying the correct angles in each relevant quadrant. Their answers were clearly laid out, with each step shown logically, from solving the equation to selecting the correct angles within 0° to 360°. These responses demonstrated a full understanding of both algebraic and trigonometric methods.
Images of Better Responses	**Image (i)**  $\frac{4}{\operatorname{cosec}^2 x} - 3 = \sin x$. $4 \div \left(\frac{1}{\sin^2 x}\right) - 3 = \sin x$. $4\sin^2 x - \sin x - 3 = 0$ $4\sin^2 x - 4\sin x + 3\sin x - 3 = 0$ $4\sin x(\sin x - 1) + 3(\sin x - 1) = 0$ $(4\sin x + 3)(\sin x - 1) = 0$ either: $4\sin x + 3 = 0$ OR $\sin x = 1$ $\sin x = \frac{-3}{4}$ $x = \sin^{-1}(1)$ $x = \sin^{-1}\left(\frac{-3}{4}\right)$ $x = 90^{\circ}$ or $\frac{\pi}{2}$. $x = \left[\pi + \sin^{-1}\left(\frac{3}{4}\right)\right]$ or $\pi + 48.59^{\circ}$ approx. $x = \left[2\pi - \sin^{-1}\left(\frac{3}{4}\right)\right]$ or $2\pi - 48.59^{\circ}$ approx. $S.S = \left\{\pi + \sin^{-1}\left(\frac{3}{4}\right) + 2n\pi\right\} \cup \left\{2\pi - \sin^{-1}\left(\frac{3}{4}\right) + 2n\pi\right\} \cup \left\{\frac{\pi}{2} + 2n\pi\right\}$. Ans! (All are verified)! **Image (ii)**  Sol: $\frac{4}{\operatorname{cosec}^2 x} - 3 = \sin x$ $4\sin^2 x - \sin x - 3 = 0$ $4y^2 - y - 3 = 0$ let $\sin x$ be y $y = \frac{-3}{4}$, $y = 1$ $\sin x = \frac{-3}{4}$, $\sin x = 1$ open $y = \sin x$ $x = \sin^{-1}\left(-\frac{3}{4}\right)$, $x = \sin^{-1}(1)$ $x = 311.41^{\circ}$, $x = 90^{\circ}$, $x = 228.89^{\circ}$ Since x is $0^{\circ} \le x \le 360^{\circ}$ all possible outcomes for x are $S.S = \{311.41^{\circ}, 90^{\circ}, 228.89^{\circ}\}$

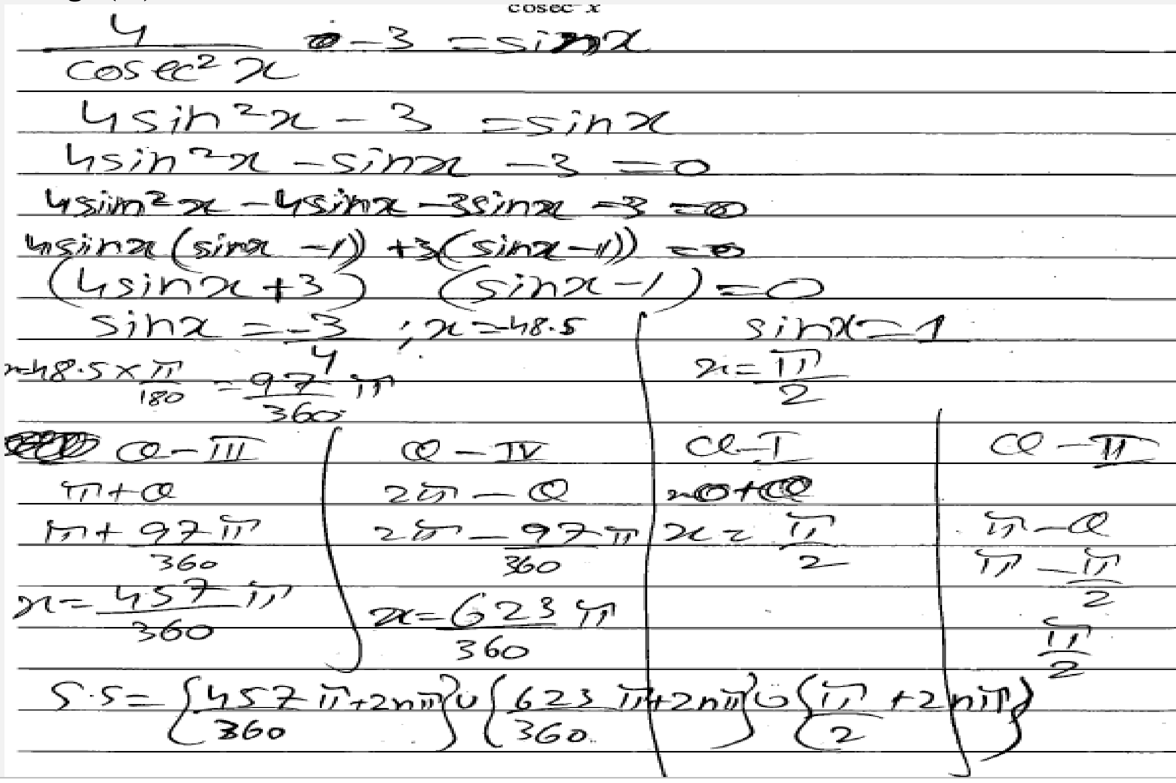
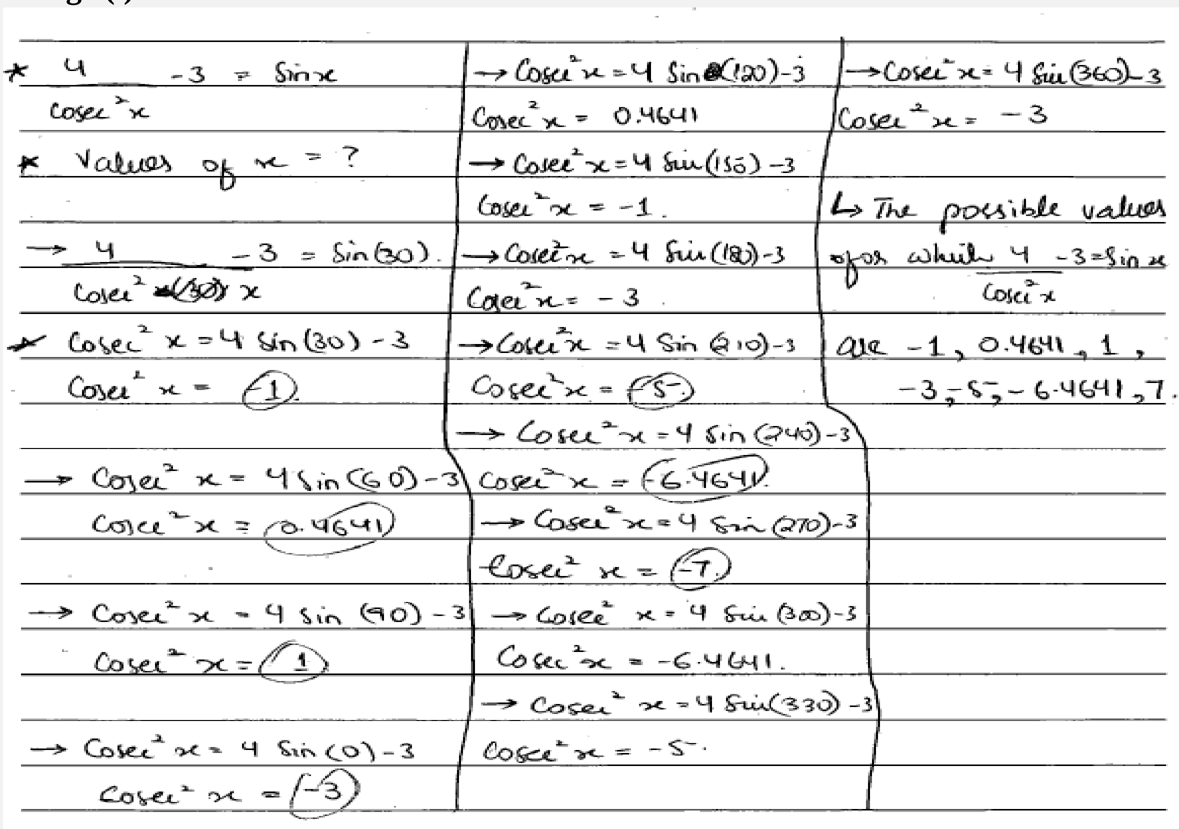
	Image (iii)  $\frac{4}{\operatorname{cosec}^2 x} - 3 = \sin x$ $4\sin^2 x - 3 = \sin x$ $4\sin^2 x - \sin x - 3 = 0$ $4\sin^2 x - 4\sin x - 3\sin x - 3 = 0$ $4\sin x(\sin x - 1) + 3(\sin x - 1) = 0$ $(4\sin x + 3)(\sin x - 1) = 0$ $\sin x = -\frac{3}{4}$; $x = 48.5$ | $\sin x = 1$ $x = 48.5 \times \frac{\pi}{180} = \frac{97}{360}\pi$ | $x = \frac{\pi}{2}$ $\alpha - \text{III}$: $\pi + \alpha$, $\pi + \frac{97\pi}{360}$, $x = \frac{457\pi}{360}$ $\alpha - \text{IV}$: $2\pi - \alpha$, $2\pi - \frac{97\pi}{360}$, $x = \frac{623\pi}{360}$ $\alpha - \text{I}$: $0 + \alpha$, $x = \frac{\pi}{2}$ $\alpha - \text{II}$: $\pi - \alpha$, $\pi - \frac{\pi}{2}$, $\frac{\pi}{2}$ $S.S = \left\{\frac{457\pi}{360} + 2n\pi\right\} \cup \left\{\frac{623\pi}{360} + 2n\pi\right\} \cup \left\{\frac{\pi}{2} + 2n\pi\right\}$
Description of Weaker Responses	*Weaker responses* revealed several areas of misunderstanding. Some candidates struggled with simplifying the given equation or factorising it correctly. Others found only one solution, often 90° and, did not identify the other angles that also satisfied the equation. In several cases, answers included angles beyond the 0° to 360° range, suggesting confusion about the domain or about how many solutions were required. Mistakes were also seen in selecting the correct quadrants for the solutions or in misapplying basic trigonometric identities.
Images of Weaker Responses	**Image (i)**  * $\frac{4}{\operatorname{cosec}^2 x} - 3 = \sin x$ * Values of x = ? → $\frac{4}{\operatorname{cosec}^2 x} - 3 = \sin(30)$. * $\operatorname{cosec}^2 x = 4\sin(30) - 3$ $\operatorname{cosec}^2 x = -1$. → $\operatorname{cosec}^2 x = 4\sin(60) - 3$ $\operatorname{cosec}^2 x = 0.4641$ → $\operatorname{cosec}^2 x = 4\sin(90) - 3$ $\operatorname{cosec}^2 x = 1$ → $\operatorname{cosec}^2 x = 4\sin(0) - 3$ $\operatorname{cosec}^2 x = -3$ → $\operatorname{cosec}^2 x = 4\sin(120) - 3$ $\operatorname{cosec}^2 x = 0.4641$ → $\operatorname{cosec}^2 x = 4\sin(150) - 3$ $\operatorname{cosec}^2 x = -1$. → $\operatorname{cosec}^2 x = 4\sin(180) - 3$ $\operatorname{cosec}^2 x = -3$ → $\operatorname{cosec}^2 x = 4\sin(210) - 3$ $\operatorname{cosec}^2 x = -5$ → $\operatorname{cosec}^2 x = 4\sin(240) - 3$ $\operatorname{cosec}^2 x = -6.4641$. → $\operatorname{cosec}^2 x = 4\sin(270) - 3$ $\operatorname{cosec}^2 x = -7$. → $\operatorname{cosec}^2 x = 4\sin(300) - 3$ $\operatorname{cosec}^2 x = -6.4641$. → $\operatorname{cosec}^2 x = 4\sin(330) - 3$ $\operatorname{cosec}^2 x = -5$. → $\operatorname{cosec}^2 x = 4\sin(360) - 3$ $\operatorname{cosec}^2 x = -3$ ↳ The possible values for which $\frac{4}{\operatorname{cosec}^2 x} - 3 = \sin x$ are $-1, 0.4641, 1, -3, -5, -6.4641, 7$.

	Image (ii)	Image (iii)
	Ans $\frac{4}{\text{cosec}^2 x} = \sin x - 3$ $\sin^2 x \; 4 = \sin x - 3$ $4\sin^2 x - \sin x + 3 = 0$ Solve by Quadratic eq $a = 4\sin^2 \quad b = -\sin \quad c = 3$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{\sin \pm \sqrt{\sin^2 - 48\sin^2}}{8\sin^2}$ $x = \frac{\sin \pm \sqrt{-47\sin^2}}{8\sin^2}$ $x = \frac{\sin \pm \sin\sqrt{-47}}{8\sin^2}$ $x = \frac{\sin - \sin\sqrt{47}\,i}{8\sin^2}$ OR $x = \frac{1}{8\sin} - \frac{\sin\sqrt{47}\,i}{8\sin^2}$ | $x = \frac{\sin - \sin\sqrt{47}\,i}{8\sin^2}$	$\frac{4}{\text{cosec}^2 x} - 3 = \sin(x)$ $= \frac{4}{1/\sin^2 x} - 3 = \sin x$ $= \frac{4}{\text{cosec}^2 x} - \frac{3(\text{cosec}^2 x)}{\text{cosec}^2 x}$ $= \frac{4 - 3(\text{cosec}^2 x)}{(1/\sin^2 x)(1/\sin^2 \theta)}$ $= \frac{1(\text{cosec}^2 x)}{(1/\sin^2 x)^2}$ $= \frac{1(1/\sin^2 x)}{(1/\sin^2 x)^2}$ $= \frac{1}{1/\sin^2 x}$ $= \frac{1}{\text{cosec}^2 x}$

Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	Preferred Pedagogy** Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:
The following strategies will help build confidence and accuracy in solving trigonometric equations and recognising the importance of domains and multiple solutions.

Emphasise Domain Awareness
Always underline or box the required domain in the question to help students stay focused on finding answers within the correct range.

Use Visual Aids for Angle Solutions
Use unit circle or quadrant diagrams to support students in locating correct angle values and understanding their symmetry.

Correct Use of Calculators and Inverse Functions
Ensure students understand how to use calculators for inverse trigonometric functions and how to adjust answers for other quadrants.

Annexure A: Pedagogies Used for Teaching the SLOs

Pedagogy: Storyboard
Description: A visual pedagogy that uses a series of illustrated panels to present a narrative, encouraging creativity and critical thinking. It helps learners organise ideas, sequence events, and comprehend complex concepts through storytelling.
Example: In a Literature class, students are tasked with creating storyboards to visually retell a novel. They draw key scenes, write captions, and present their stories to the class, enhancing their reading comprehension and fostering their imagination.

Pedagogy: Cause and Effect
Description: This pedagogy explores the relationships between actions and consequences. By analysing cause-and-effect relationships, learners develop a deeper understanding of how events are interconnected and how one action can lead to various outcomes.
Example: In a History class, students study the causes and effects of the Industrial Revolution. They research and discuss how technological advancements in manufacturing led to significant societal changes, such as urbanisation and labour reform movements.

Pedagogy: Fish and Bone
Description: A method that breaks down complex topics into main ideas (the fish) and supporting details (the bones). This visual approach enhances comprehension by highlighting essential concepts and their relevant explanations.
Example: During a Biology class on human anatomy, the teacher uses the fish and bone technique to teach about the human skeletal system. Teacher presents the main components of the human skeleton (fish) and elaborates on each bone's structure and function (bones).

Pedagogy: Concept Mapping
Description: An effective way to visually represent relationships between ideas. Learners create diagrams connecting key concepts, aiding in understanding the overall structure of a subject and fostering retention.
Example: In a Psychology assignment, students use concept mapping to explore the various theories of personality. They interlink different theories, such as Freud's psychoanalysis, Jung's analytical psychology, and Bandura's social-cognitive theory, to see how they relate to each other.

Pedagogy: Audio Visual Resources
Description: Incorporating multimedia elements like videos, images, and audio into lessons. This approach caters to different learning styles, making educational content more engaging and memorable.
Example: In a General Science class, the teacher uses a documentary-style video to teach about the solar system. The video includes stunning visual animations of the planets, interviews with astronomers, and background music, enhancing students' interest and understanding of space.

Pedagogy: Think, Pair, and Share
Description: A collaborative learning technique where students ponder a question or problem individually, then discuss their thoughts in pairs or small groups before sharing with the entire class. It fosters active participation, communication skills, and diverse perspectives.
Example: In a Literature in English class, the teacher poses a thought-provoking question about a novel's moral dilemma. Students first reflect individually, then pair up to exchange their opinions, and finally participate in a lively class discussion to explore different viewpoints.

Pedagogy: Questioning Technique (Socratic Approach)
Description: Based on Socratic dialogue, this method stimulates critical thinking by posing thought-provoking questions. It encourages learners to explore ideas, justify their reasoning, and discover knowledge through a process of inquiry.
Example: In an Ethics class, the instructor uses the Socratic approach to lead a discussion on the meaning of justice. By asking a series of probing questions, the students engage in a deeper exploration of ethical principles and societal values.

Pedagogy: Practical Demonstration
Description: A hands-on approach where learners observe real-life applications of theories or skills. Practical demonstrations enhance comprehension, skill acquisition, and problem-solving abilities by bridging theoretical concepts with real-world scenarios.
Example: In a Food and Nutrition class, the instructor demonstrates the proper technique for filleting a fish. Students observe and then practice the skill themselves, learning the practical application of knife skills and culinary precision.

(**Note:** The examples provided in this annexure serve as illustrations of various pedagogies. It is important to understand that these pedagogies are versatile and can be applied across subjects in numerous ways. Feel free to adapt and explore these techniques creatively to enhance learning outcomes in your specific context.)

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