آغا خان یونیورسٹی ایگزامینیشن بورڈ

AGA KHAN UNIVERSITY EXAMINATION BOARD

Notes from E-Marking Centre on HSSC-I Mathematics Annual Examinations 2024

Introduction

This document has been produced for the teachers and candidates of Higher Secondary School Certificate (HSSC) Part I Mathematics. It contains comments on candidates' responses to the 2024 HSSC-I Examination indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E-Marking Notes

This includes overall comments on candidates' performance on every question and *some* specific examples of candidates' responses that support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that requires candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfill the demand of the question.

Candidates need to be aware that the marks allocated to the questions are related to the answer space provided on the examination paper as a guide to the length of the required response. A longer response will not in itself lead to higher marks. Candidates need to be familiar with the command words in the SLOs which contain terms commonly used in examination questions. However, candidates should also be aware that not all questions will start with or contain one of the command words. Words such as 'how', 'why' or 'what' may also be used.

General Observations

Most candidates achieved success in constructing good responses. Especially in the concepts of Complex factors by Completing the Square, Cramer's Rule, Solving Systems with Linear and Quadratic Equations, Sequence and Series, and Application of Sine and Cosine Laws.

Nonetheless, it is essential for teachers to concentrate on the following content and provide candidates with more practice to foster a strong understanding of the concepts.

- Conditional Probability and Tree Diagrams
- Proving Trigonometric Identities
- Graphing trigonometric function
- Solving word problem of Quadratic Equation

Note: Candidates' responses shown in this report have not been corrected for grammar, spelling, format or information.

DETAILED COMMENTS

Constructed Response Questions (CRQs)

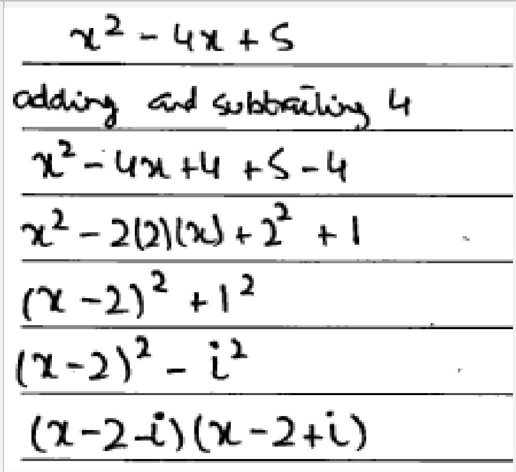
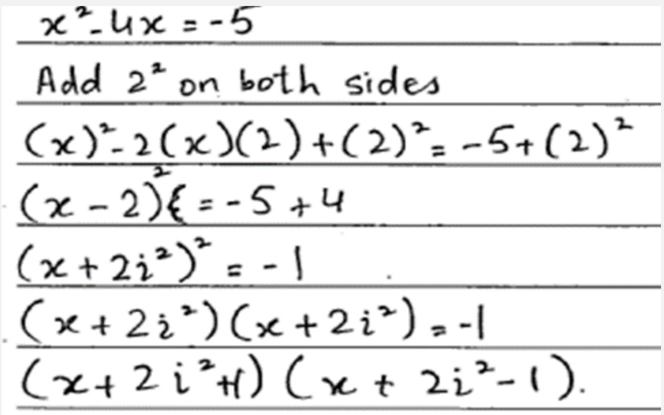
Question No. 1	
Question Text	Find the complex factors of $x^2 - 4x + 5$ by using completing square method.
SLO No.	1.3.3
SLO Text	Solve quadratic equation $pz^2 + qz + r =$; $p \neq 0$ by completing square method, where p, q and r are real numbers and z is a complex number.
Max Marks	4
Cognitive Level	A*
Checking Hints	1 mark for breaking 5 in the sum of 4 and 1 1 mark for application of formula of $a^2 - 2ab + b^2$ 1 mark for writing $(x-2)^2 - (-1)$ 1 mark for writing 1 as $-i^2$ and for application of formula of $a^2 - b^2$
Overall Performance	The overall performance in this question showed that some of the candidates correctly applied the method and found the complex factors, while others either used the wrong method or made mistakes in their calculations.
Description of Better Responses	Better responses effectively used the completing the square method. They transformed $x^2 - 4x + 5$ into $(x-2)^2 + 1$, and solved for x to find the complex factors $(x-2-i)$ and $(x-2+i)$. Their answers showed a clear understanding of the method.
Image of Better Response	 $x^2 - 4x + 5$ adding and subtracting 4 $x^2 - 4x + 4 + 5 - 4$ $x^2 - 2(2)(x) + 2^2 + 1$ $(x-2)^2 + 1^2$ $(x-2)^2 - i^2$ $(x-2-i)(x-2+i)$
Description of Weaker Responses	Weaker responses often involved using the quadratic formula instead of completing the square method. While some of the candidates applied the completing square method however, they made errors in applying it correctly such as added wrong terms or handling the imaginary unit i, leading to incorrect or incomplete results.
Images of Weaker Responses	**Image (i)**  $x^2 - 4x = -5$ Add 2^2 on both sides $(x)^2 - 2(x)(2) + (2)^2 = -5 + (2)^2$ $(x-2)^2 = -5 + 4$ $(x + 2i^2)^2 = -1$ $(x + 2i^2)(x + 2i^2) = -1$ $(x + 2i^2 + 1)(x + 2i^2 - 1).$

Image (ii)

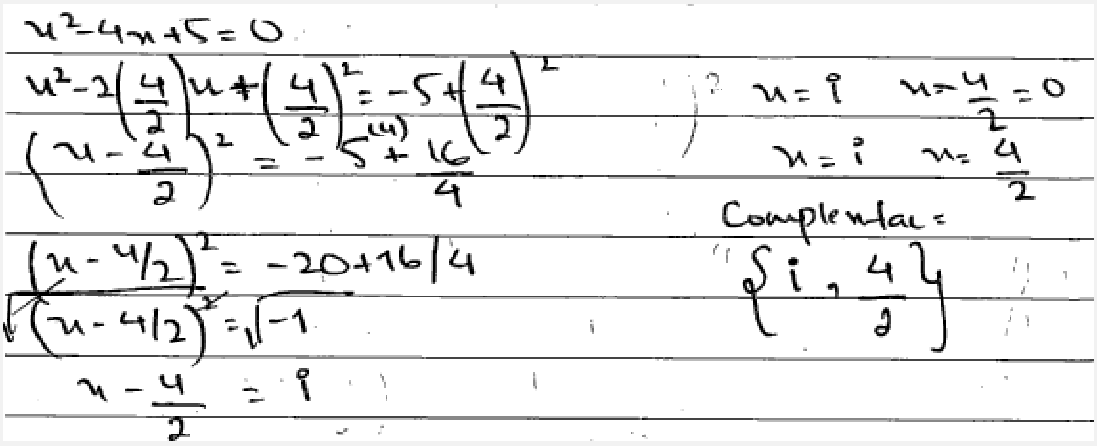
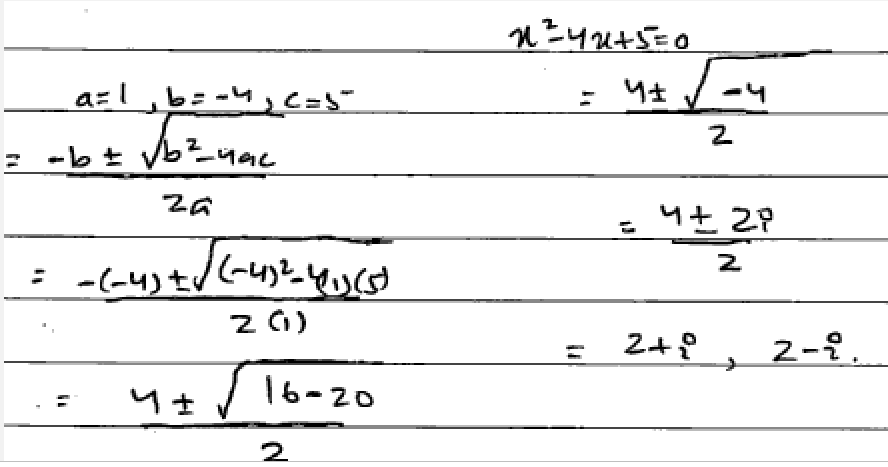


Image (iii)



Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	Preferred Pedagogy** Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration ** For description of each Pedagogy, refer to Annexure A	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:

To improve students' understanding and performance, teachers may consider the following strategies;

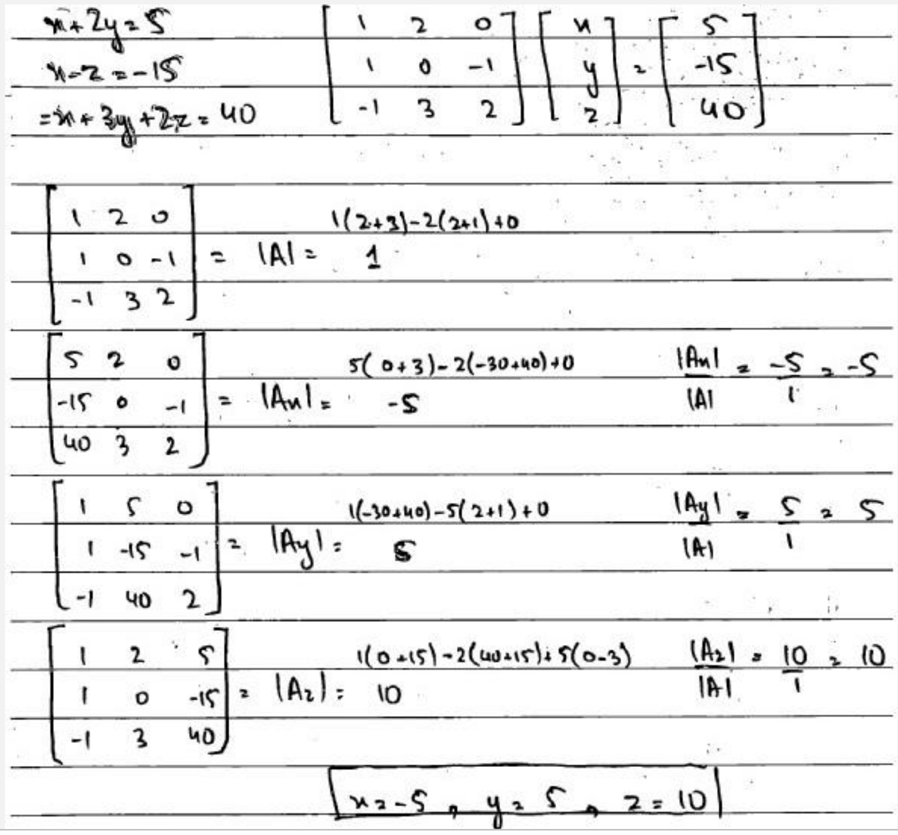
- **Visual Aids:** Use tools like interactive whiteboards to illustrate the steps of completing the square. GeoGebra can help students visualise the process.

- **Digital Tools:** Use platforms like Desmos for practice with completing the square. These tools offer instant feedback and interactive learning.
- **Online Practice:** Recommend platforms such as IXL for extra practice with quadratic equations and completing the square.
- **Real-Life Examples:** Show the use of completing the square in real-life situations, like in design optimisation. For example, illustrate its use for finding optimal dimensions.
- **Mathematical Puzzles:** Incorporate puzzles or games involving quadratic equations to make learning engaging and reinforce the concept in a fun way.

*K = Knowledge U = Understanding A = Application and other higher-order cognitive skills

Question No. 2	
Question Text	Consider the system of equations, $x + 2y = 5$, $x - z = -15$ and $-x + 3y + 2z = 40$. The determinant of the coefficient matrix is 1. Using Cramer's Rule, find the values of x, y and z that satisfy the given system of linear equations.
SLO No.	2.5.3 (b)
SLO Text	Solve a system of 3 by 3 non-homogeneous linear equations using (b)Cramer's rule.
Max Marks	6
Cognitive Level	A
Checking Hints	1 mark for determinant of x, y and z each (3 required) 1 mark for x, y and z (3 required)
Overall Performance	Candidates generally performed well on this question, which involved solving a system of three linear equations using Cramer's Rule. Given the simplicity of the concept, most candidates were able to approach the problem effectively. However, there were some issues related to incorrect application of the method and errors in matrix calculations.
Description of Better Responses	Candidates who answered correctly demonstrated a good understanding of Cramer's Rule. They correctly used the given determinant of the coefficient matrix (which was given as 1) and set up the matrices appropriately. These candidates accurately constructed the matrices for the variables and used them to find the values of x, y, and z. They also formed the matrices by substituting the constants from the right side of the equations into the coefficient matrix and solved for each variable correctly.

Image of Better Response



Description of Weaker Responses

Weaker responses showed several common errors. Some candidates made mistakes in calculating the determinant, which led to incorrect values for *x*, *y* and *z*. Despite the determinant being given as 1, incorrect determinant calculations by some candidates resulted in wrong final answers. Additionally, a few candidates incorrectly used the inverse method instead of Cramer's Rule, which was not appropriate for this question. Others struggled with forming the correct matrices, particularly where some variables had zero coefficients, leading to further calculation errors.

Images of Weaker Responses

Image (i)

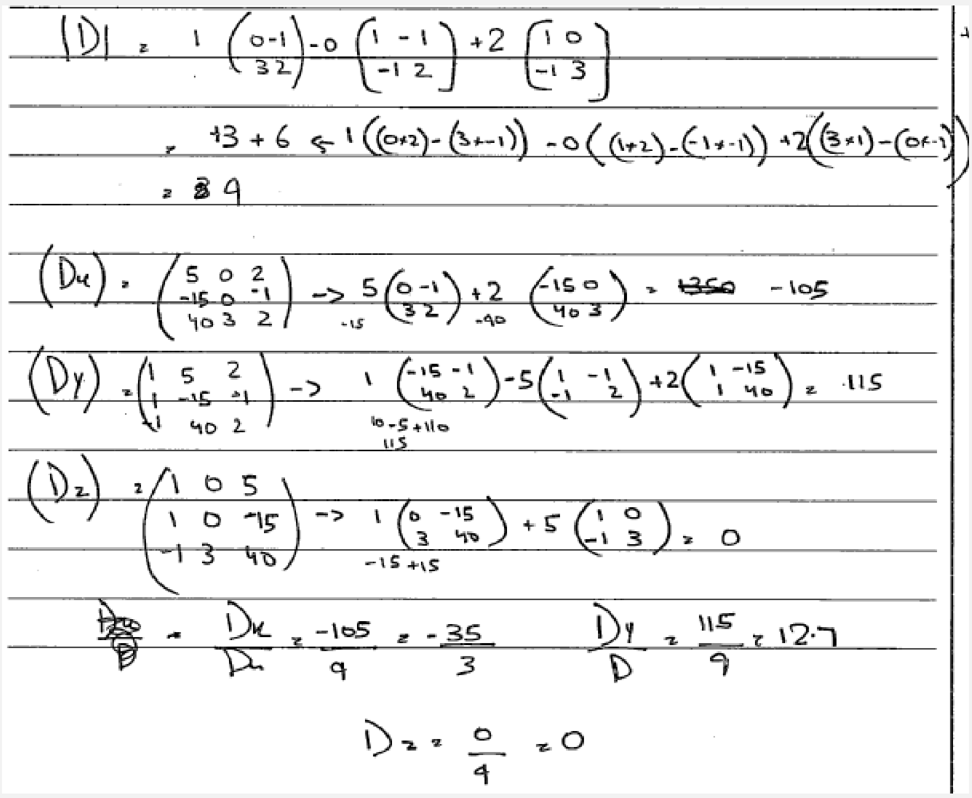
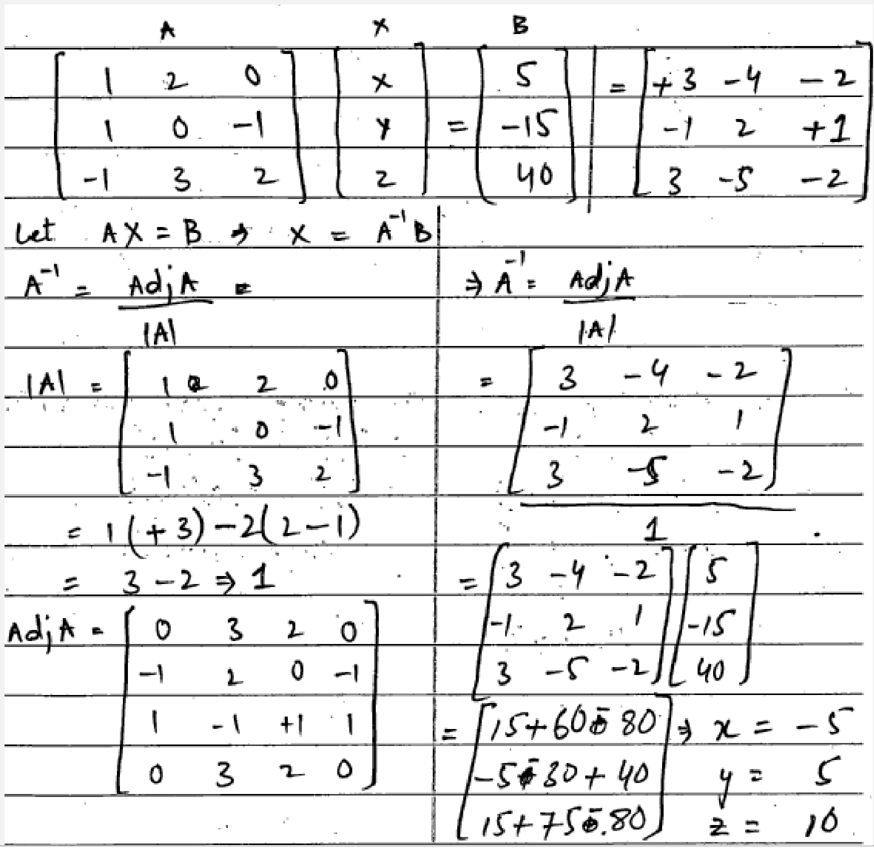


Image (ii)



$$\overset{A}{\begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & -1 \\ -1 & 3 & 2 \end{bmatrix}} \overset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \overset{B}{\begin{bmatrix} 5 \\ -15 \\ 40 \end{bmatrix}} \quad = \begin{bmatrix} +3 & -4 & -2 \\ -1 & 2 & +1 \\ 3 & -5 & -2 \end{bmatrix}$$

Let $AX = B \Rightarrow X = A^{-1}B$

$A^{-1} = \frac{Adj A}{|A|}$ $\quad \Rightarrow A^{-1} = \frac{Adj A}{|A|}$

$|A| = \begin{vmatrix} 1 & 2 & 0 \\ 1 & 0 & -1 \\ -1 & 3 & 2 \end{vmatrix}$ $\quad = \frac{\begin{bmatrix} 3 & -4 & -2 \\ -1 & 2 & 1 \\ 3 & -5 & -2 \end{bmatrix}}{1}$

$= 1(+3) - 2(2-1)$

$= 3 - 2 \Rightarrow 1$ $\quad = \begin{bmatrix} 3 & -4 & -2 \\ -1 & 2 & 1 \\ 3 & -5 & -2 \end{bmatrix} \begin{bmatrix} 5 \\ -15 \\ 40 \end{bmatrix}$

$Adj A = \begin{bmatrix} 0 & 3 & 2 & 0 \\ -1 & 2 & 0 & -1 \\ 1 & -1 & +1 & 1 \\ 0 & 3 & 2 & 0 \end{bmatrix}$ $\quad = \begin{bmatrix} 15+60-80 \\ -5-30+40 \\ 15+75-80 \end{bmatrix} \Rightarrow x = -5, \; y = 5, \; z = 10$

Suggestions for improvement (Highlight all that apply)

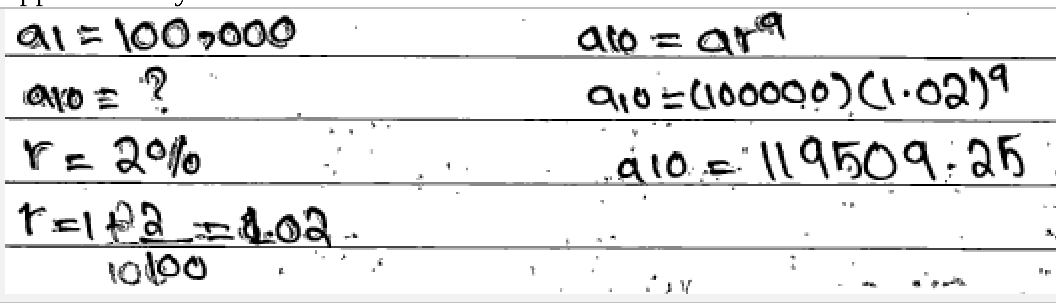
Maximising SLO Achievement	Preferred Pedagogy Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

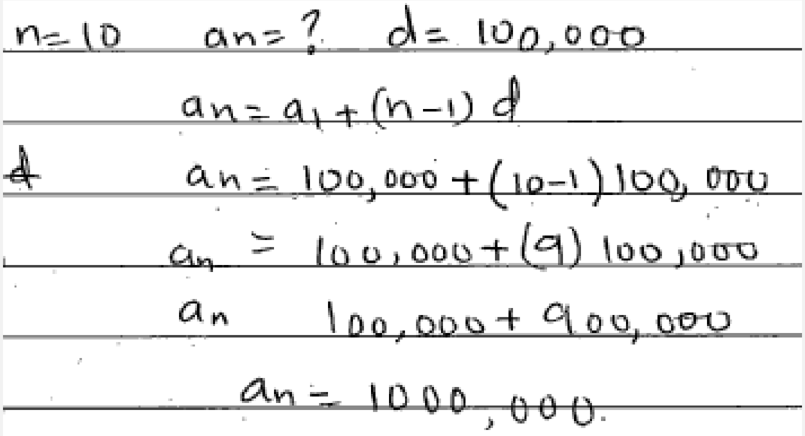
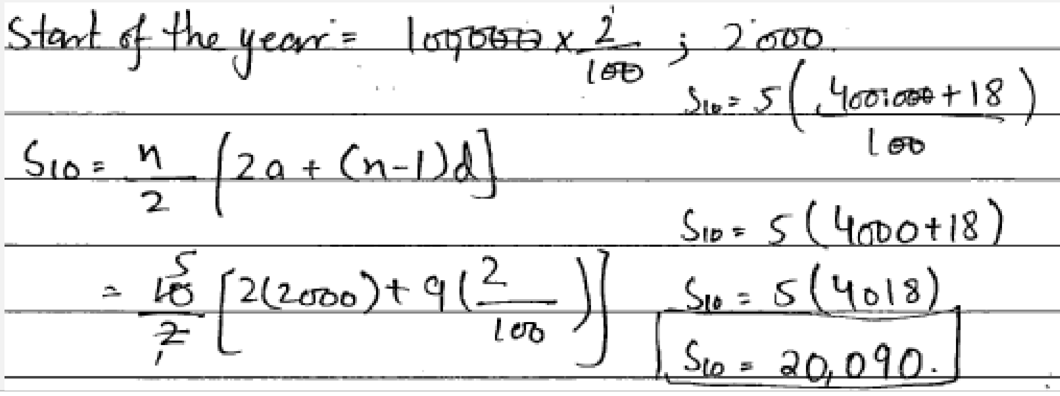
Any Additional Suggestion:

To enhance students' understanding and performance, teachers may consider the following strategies;

- **Step-by-Step Demonstrations:** Provide clear, step-by-step examples of applying Cramer's rule, including setting up matrices and calculating determinants. For instance, demonstrate the construction of matrices from given equations and use them to find variable values.
- **Matrix Calculation Practice:** Offer additional practice problems focusing on determinant calculation and matrix manipulation. This can help students become more comfortable with these concepts and reduce errors.

- **Digital Tools:** Utilise online tools such as matrix calculators and software like GeoGebra or Desmos to practice solving systems of equations and calculating determinants. These tools provide visual aids along with instant feedback.
- **Online Tutorials:** Recommend online resources or video tutorials that explain Cramer's rule and its application in solving systems of equations. Websites like Coursera often has relevant tutorials and exercises.
- **Real-Life Examples:** Show practical applications of solving systems of equations using Cramer's Rule, such as in engineering or economics. For example, use a real-world problem where different constraints need to be satisfied simultaneously.
- **Error Analysis:** Teach students to double-check their work and understand common errors in matrix calculations and determinant finding. Encourage them to verify their answers using different methods or by substituting the values back into the original equations.

Question No. 3i	
Question Text	A city's population is growing at a rate of 2% per year. If the population was 100,000 at the start of the year, then what will be the total population in 10 years?
SLO No.	3.5.2
SLO Text	Solve problems involving geometric sequence.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for correct value of a 1 mark for correct value of r 1 mark for correct substitution of a and r in T_n
Overall Performance	This question involved solving a problem related to geometric progression, specifically calculating the population growth over 10 years at a rate of 2% per year. Most candidates answered this question correctly, demonstrating a good understanding of geometric sequences.
Description of Better Responses	In good responses, candidates used the correct formula for geometric sequences, $a_n = a_1 r^{n-1}$, with r = 1 + 2%. They correctly calculated the population after 10 years. Some candidates also used a manual method, creating the geometric sequence year by year, starting from 100,000 and growing each year by 2%, leading to the final population of approximately.
Image of Better Response	
Description of Weaker Responses	Weaker responses included using inappropriate formulas, such as the arithmetic sequence formula $a_n = a_1 + (n-1)d$ or the sum formulas $S_n = \frac{n}{2}[a_1 + (n-1)d]$. These candidates struggled to differentiate between common ratio and common difference, leading to incorrect calculations and results.

Images of Weaker Responses	Image (i)  Image (ii) 

Suggestions for improvement (Highlight all that apply)

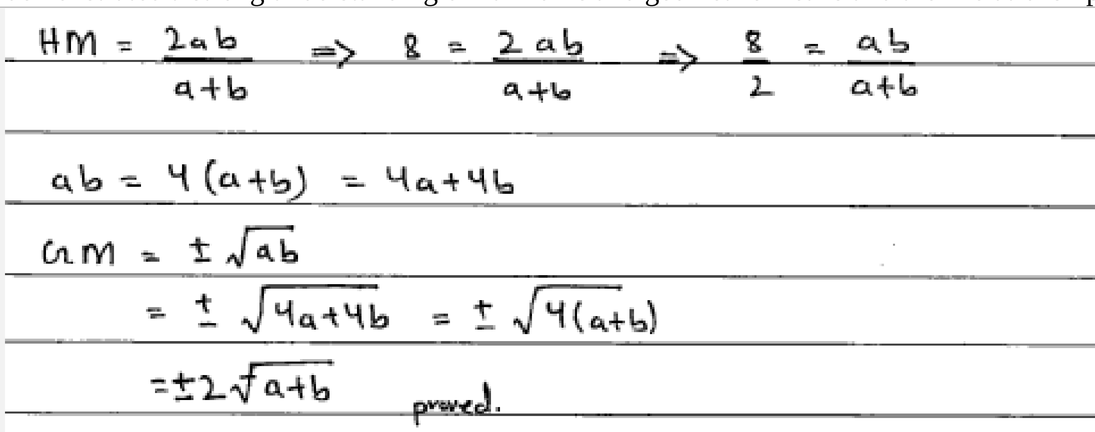
Maximising SLO Achievement	Preferred Pedagogy Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

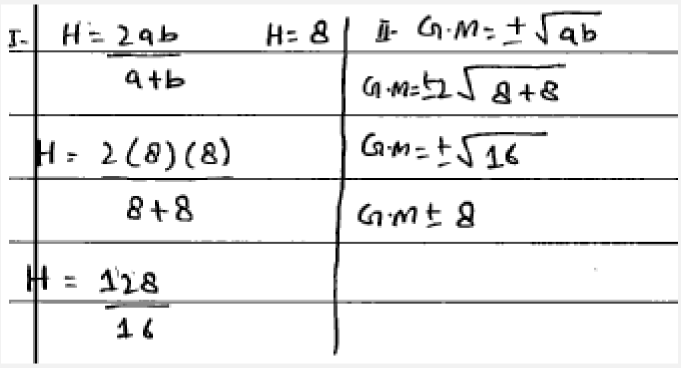
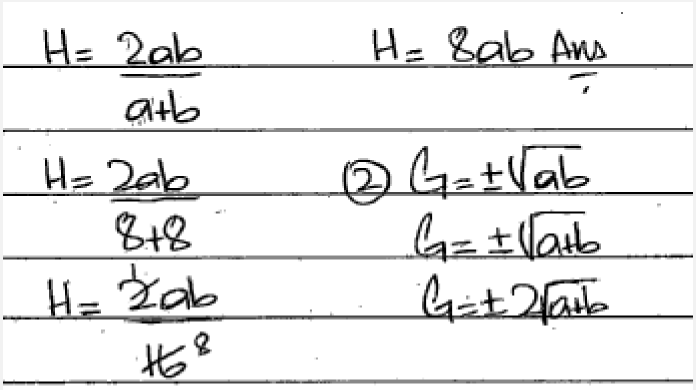
Any Additional Suggestion:

To enhance students' understanding and performance, teachers may consider the following strategies;

- **Formula Clarification:** Provide clear explanations and examples distinguishing between arithmetic and geometric sequences. Highlight the differences in formulas and their applications.
- **Practice Problems:** Provide additional practice problems focusing on geometric sequences. Encourage students to solve these problems using both formulaic and manual methods to reinforce their understanding.

- **Digital Tools:** Utilise online tools and calculators to help students visualise geometric sequences and understand their growth patterns. Websites like Desmos can graph the progression over time, offering a visual representation.
- **Online Resources:** Recommend online tutorials and resources, such as those available on Coursera, which offer step-by-step explanations and exercises on geometric sequences.
- **Real-Life Applications:** Show real-life examples of geometric sequences, such as population growth, compound interest or radioactive decay. For instance, illustrate the exponential growth of compound interest in savings accounts over time, similar to population growth.

Question No. 3ii	
Question Text	The Harmonic mean between two positive numbers, *a* and *b*, is 8. I. Find *ab*. II. Hence, show that the G.M between *a* and *b* is $\pm 2\sqrt{a+b}$.
SLO No.	3.9.3
SLO Text	Find the relationship between arithmetic, geometric and harmonic means.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for $8 = \frac{2ab}{a+b}$ 1 mark for $ab = 4(a+b)$ 1 mark for $2\sqrt{a+b}$
Overall Performance	This question involved finding the harmonic mean between two positive numbers *a* and *b*, calculating the product *ab* and showing the relationship between the harmonic mean and geometric mean. Overall, candidates performed excellently, showcasing the strong understanding of the concept and correct application of the formulas.
Description of Better Responses	In better responses, candidates correctly identified the harmonic mean *H* as 8 and used the appropriate formula $H = \frac{2ab}{a+b}$ to find *ab*. For part II, they correctly used the relationship derived from part I to show that the geometric mean (*G.M*) between *a* and *b* is $\pm 2\sqrt{a+b}$. These candidates demonstrated a strong understanding of harmonic and geometric means and their relationship.
Image of Better Response	 $HM = \frac{2ab}{a+b} \Rightarrow 8 = \frac{2ab}{a+b} \Rightarrow \frac{8}{2} = \frac{ab}{a+b}$ $ab = 4(a+b) = 4a+4b$ $GM = \pm\sqrt{ab}$ $= \pm\sqrt{4a+4b} = \pm\sqrt{4(a+b)}$ $= \pm 2\sqrt{a+b}$ proved.
Description of Weaker Responses	Weaker responses showed candidates' confusion related to the concept of harmonic mean. Some candidates could not recognise that *H* was given as 8. Instead of using the harmonic mean formula, a few candidates incorrectly used the arithmetic mean formula. Errors in part I led to mistakes in part II, as candidates could not correctly derive the geometric mean relationship without the correct value of *ab*.

Images of Weaker Responses	**Image (i)**  **Image (ii)** 

Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	**Pedagogy Used for that SLO**	**Assessment Strategies**
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

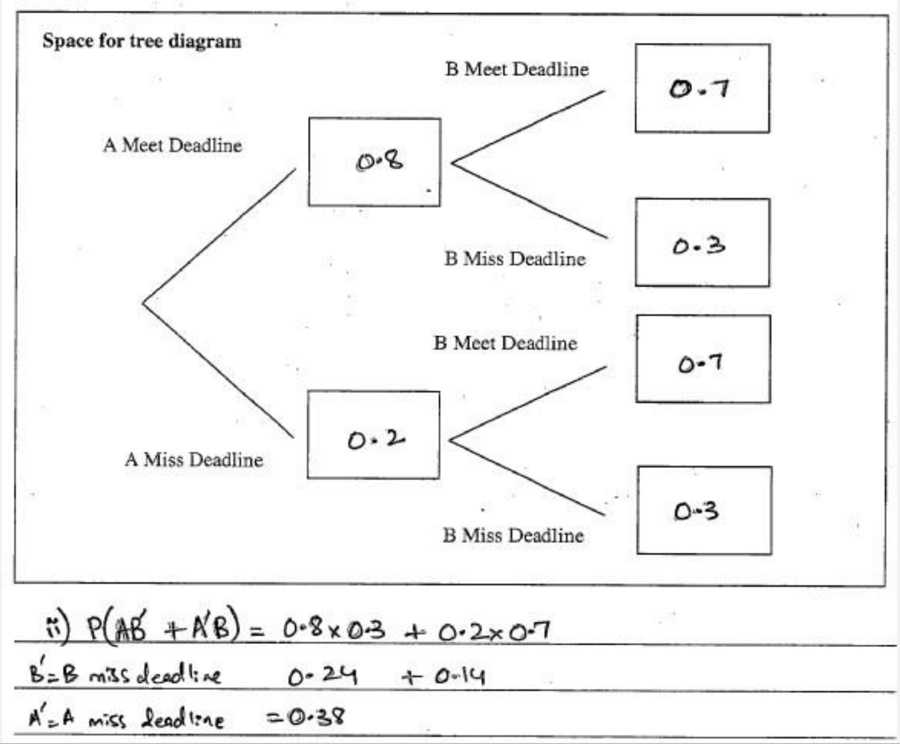
Any Additional Suggestion:

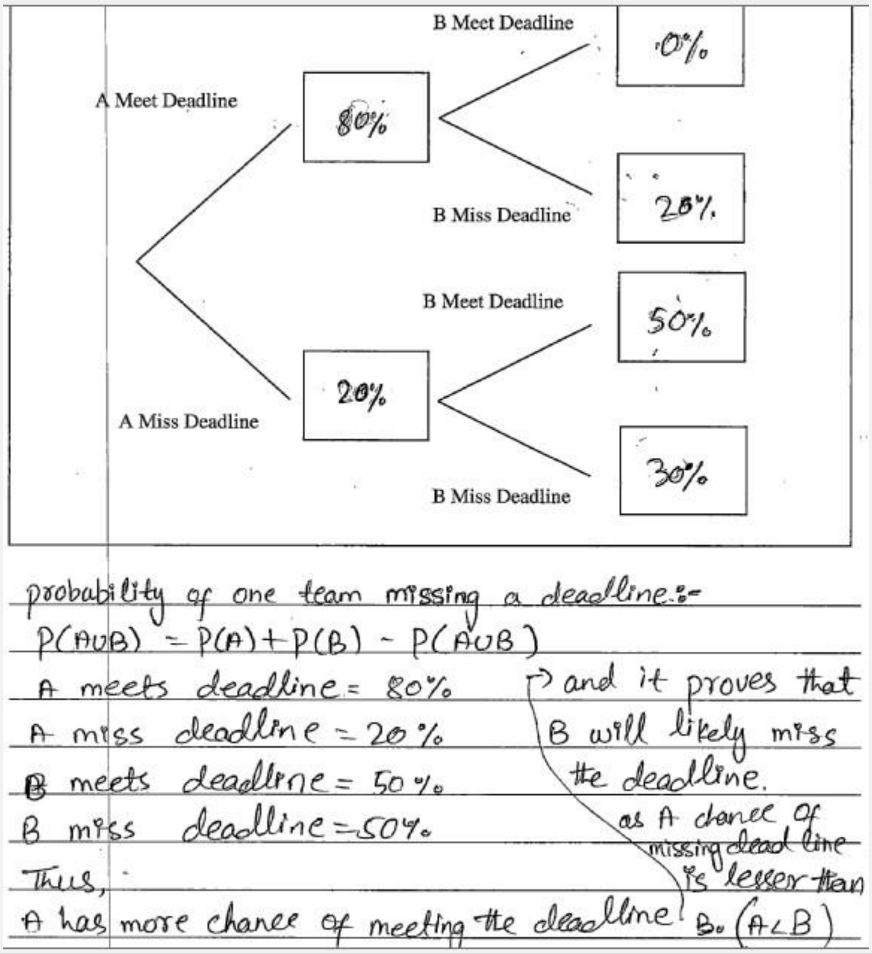
To improve students' understanding and performance, teachers may consider the following strategies;

- **Formula Clarification:** Emphasise the differences between arithmetic, geometric and harmonic means and their use. Provide clear definitions and examples for each of these means.
- **Step-by-Step Examples:** Provide detailed examples of calculating the harmonic mean and using it to find related values.
- **Practice Problems:** Offer additional practice problems focusing on harmonic and geometric means. Encourage students to solve these problems step by step to reinforce their understanding.

- **Digital Tools:** Use online tools and calculators to help students visualise the relationships between different types of means. Websites like GeoGebra can graph and illustrate these concepts interactively.
- **Online Resources:** Recommend online tutorials and resources, such as those on Coursera, which provide step-by-step explanations and practice exercises related to means and their relationships.
- **Real-Life Applications:** Discuss practical examples of where harmonic and geometric means are used, such as in averaging rates or growth factors. For instance, explain the use of harmonic mean in calculating average speeds and use of geometric mean in finance for averaging rates of return.

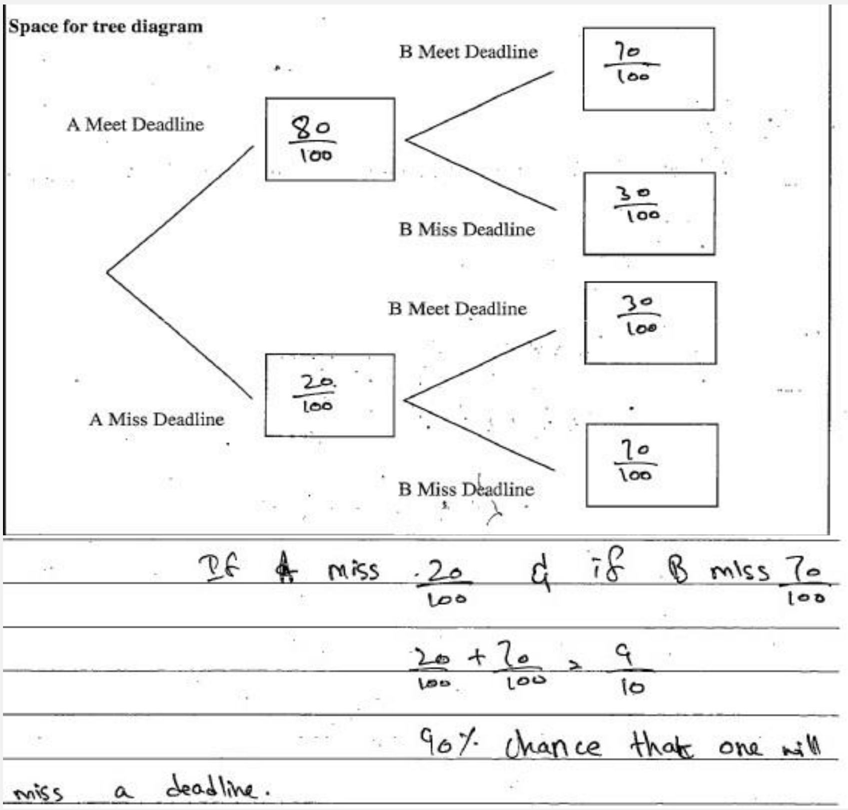
Question No. 4	
Question Text	A software company has two teams, *A* and *B*, working on different modules of a project independently. There is a 20% chance that team *A* will miss a deadline, and a 30% chance that team *B* will miss a deadline. i. Draw a tree diagram for the given situation. ii. Use tree diagram to find the probability that one team will miss a deadline.
SLO No.	5.3.4 & 5.3.9
SLO Text	Find the probability of the occurrence of an event using Venn diagram, tree diagram and probability tree diagram (with and without replacement). Describe the law of multiplication of probability $P(A \cap B) = P(A) \times P(B)$, where *A* and *B* are independent events.
Max Marks	6
Cognitive Level	A
Checking Hints	3 marks for correct tree diagram (1 mark for each correct branch) 1 mark for $P(A)$. $P(B')$ = 0.14 1 mark for $P(A')$. $P(B)$ = 0.24, 1 mark for getting 0.38
Overall Performance	This question involved using a tree diagram to represent probabilities related to two teams missing deadlines and calculating the probability that one team will miss a deadline. The question was challenging for many candidates, with confusion arising from the multiple concepts involved.
Description of Better Responses	In better responses, candidates correctly drew the tree diagram, showing the probabilities for team *A* and team *B* meeting or missing their deadlines. They correctly labeled the branches with probabilities of meeting deadlines (80% for *A* and 70% for *B*) and missing deadlines (20% for A and 30% for B). In part ii, these candidates used the tree diagram to find the correct probability that one team would miss a deadline, calculating the combined probabilities of scenarios where only one team misses the deadline.

<table>
<tr>
<td>Image of Better Response</td>
<td>


</td>
</tr>
<tr>
<td>Description of Weaker Responses</td>
<td>Weaker responses showed several common errors. Some candidates struggled to understand the word problem and made mistakes in labeling the tree diagram, confusing the probabilities for meeting and missing deadlines and mixing up the two teams. Many candidates created incorrect tree diagrams and most were unable to correctly calculate the probability that one team would miss a deadline in part (ii), while some did not attempt it at all.</td>
</tr>
<tr>
<td>Images of Weaker Responses</td>
<td>
Image (i)


</td>
</tr>
</table>

Image (ii)



Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	Preferred Pedagogy Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:

To improve students' understanding and performance, teachers may consider the following strategies;

- **Concept Clarification:** Provide clear explanations of setting up and interpreting tree diagrams. For examplein this question, emphasise the difference between meeting and missing deadlines and discuss the labelling of the branches accurately.
- **Step-by-Step Examples:** Offer detailed examples of constructing tree diagrams for similar probability problems. Show step-by-step process to label the branches and calculate the combined probabilities for different outcomes.
- **Practice Problems:** Provide additional practice problems focusing on tree diagrams and probability calculations. Encourage students to solve these problems step by step to reinforce their understanding.

- **Visual Aids:** Use visual aids such as interactive whiteboards or digital tools like GeoGebra to create and manipulate tree diagrams. These tools can help students visualise the branching and probabilities.
- **Online Resources:** Recommend online tutorials and resources, such as those on Coursera, which provide step-by-step explanations and exercises on probability and tree diagrams.
- **Real-Life Applications:** Show practical examples of probability using tree diagrams, such as predicting outcomes in games or analysing scenarios in project management. For instance, illustrate the use of tree diagrams to analyse different paths in decision-making processes.

Question No. 5	
Question Text	Consider the expression $\sqrt[3]{k+x}$. i. Prove that this expression can also be written as $k^{\frac{1}{3}}\left(1+\frac{x}{k}\right)^{\frac{1}{3}}$. ii. Expand the expression $k^{\frac{1}{3}}\left(1+\frac{x}{k}\right)^{\frac{1}{3}}$ upto 3rd term. iii. If the coefficients of x and x^2 are equal, then prove the values of k are $-\frac{1}{3}$ and 0.
SLO No.	6.3.1
SLO Text	Expand $(x+y)^n$ where n is a positive integer and extend this result for all rational values of *n*.
Max Marks	6
Cognitive Level	A
Checking Hints	1 mark for taking common $k^{\frac{1}{3}}$ 1 mark for correct substitution in formula 1 mark for expansion $\left\{1+\frac{x}{3k}-\frac{x^2}{9k^2}+....\right\}$ OR 2 marks if done directly 1 mark for selecting correct coefficients of 2nd and 3rd terms 1 mark for equating coefficients $\frac{k^{\frac{1}{3}}}{3k}=-\frac{k^{\frac{1}{3}}}{9k^2}$ 1 mark for $3k(3k+1)=0$ OR 2 marks if done directly
Overall Performance	This question involved expanding $(x+y)^n$ where *n* is a positive integer and extending the result for all rational values of *n*. Overall, most candidates successfully attempted part (i), with fewer correctly completing parts (ii) and (iii).
Description of Better Responses	In better responses, candidates showed good understanding of the binomial expansion formula and applied it to situations where the exponent is a fraction. These candidates successfully identified that when *n* is a fraction, the binomial series formula $(x+y)^n$ should be used. They correctly took the common term from the given expression and applied the binomial theorem. These candidates accurately calculated the coefficients of x and x^2, showing a clear understanding of the binomial expansion for fractional exponents.

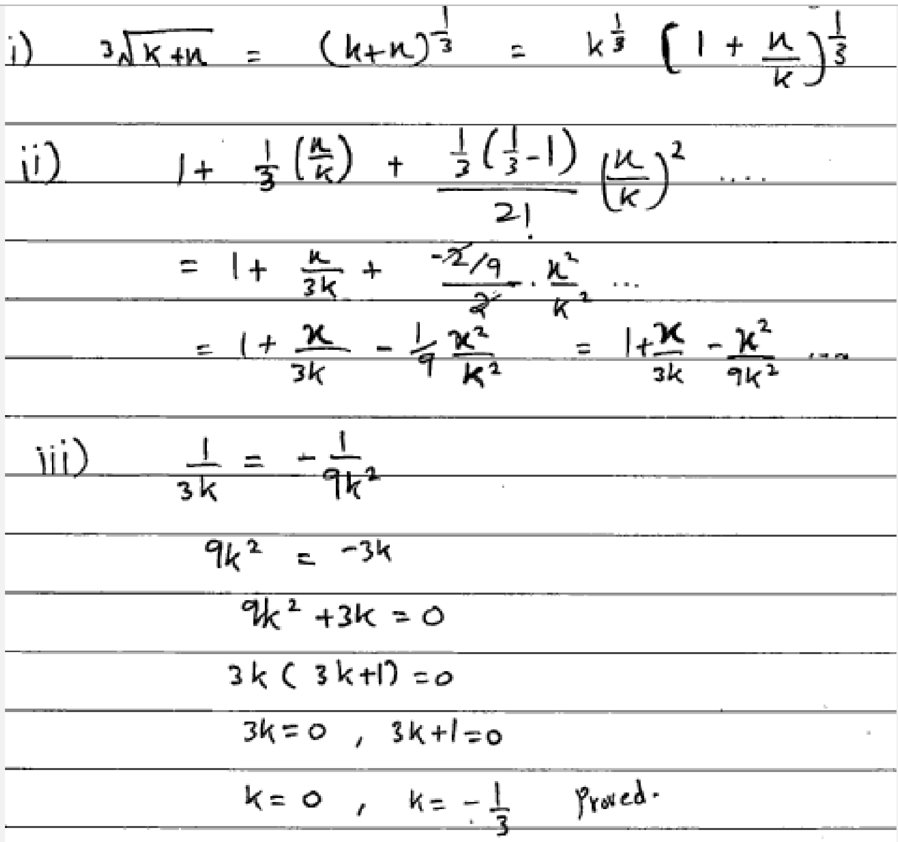
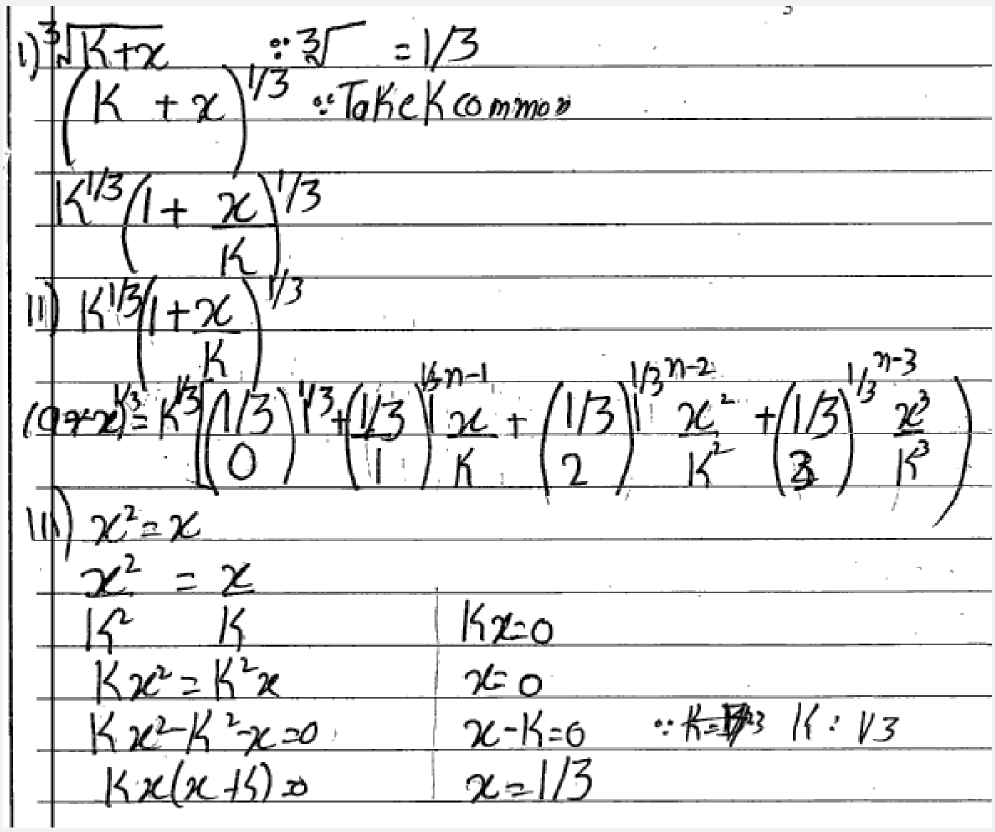
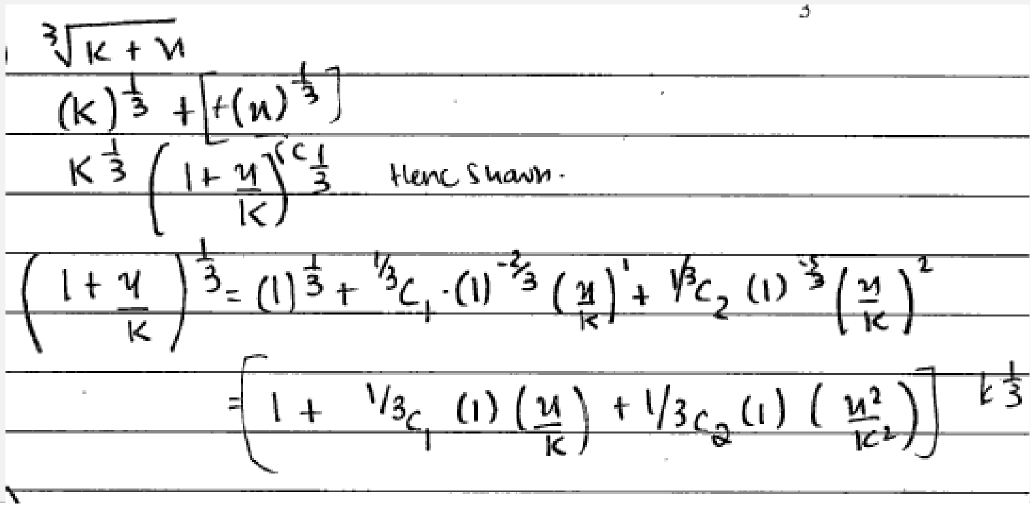
Image of Better Response	 i) $\sqrt[3]{k+x} = (k+x)^{\frac{1}{3}} = k^{\frac{1}{3}}\left[1+\frac{x}{k}\right)^{\frac{1}{3}}$ ii) $1+\frac{1}{3}\left(\frac{x}{k}\right)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{x}{k}\right)^2 \ldots$ $= 1+\frac{x}{3k}+\frac{-2/9}{2}\cdot\frac{x^2}{k^2}\ldots$ $= 1+\frac{x}{3k}-\frac{1}{9}\frac{x^2}{k^2} = 1+\frac{x}{3k}-\frac{x^2}{9k^2}\ldots$ iii) $\frac{1}{3k} = -\frac{1}{9k^2}$ $9k^2 = -3k$ $9k^2+3k=0$ $3k(3k+1)=0$ $3k=0,\ 3k+1=0$ $k=0,\ k=-\frac{1}{3}$ Proved.
Description of Weaker Responses	Weaker responses showed that many candidates misunderstood the concept of a fractional exponent in part (i), leading them to incorrectly apply the binomial theorem directly using the standard binomial coefficient ($^{n}C_{r}$). Candidates who struggled with part (ii) also failed to correctly identify and equate the coefficients of x and x^2. Additionally, many candidates did not attempt part (iii) at all.
Images of Weaker Responses	**Image (i)**  i) $\sqrt[3]{K+x}$ ∵ $\sqrt[3]{\ } = 1/3$ $(K+x)^{1/3}$ ∵ Take K common $K^{1/3}\left(1+\frac{x}{K}\right)^{1/3}$ ii) $K^{1/3}\left(1+\frac{x}{K}\right)^{1/3}$ $= K^{1/3}\left[\binom{1/3}{0}1^{1/3}+\binom{1/3}{1}1^{1/3\,n-1}\frac{x}{K}+\binom{1/3}{2}1^{1/3\,n-2}\frac{x^2}{K^2}+\binom{1/3}{3}1^{1/3\,n-3}\frac{x^3}{K^3}\right)$ iii) $x^2 = x$ $\frac{x^2}{K^2} = \frac{x}{K}$ | $Kx=0$ $Kx^2 = K^2x$ | $x=0$ $Kx^2-K^2x=0$ | $x-K=0$ ∵ $K = 1/3$ | $K: 1/3$ $Kx(x-K)=0$ | $x=1/3$

	Image (ii) 

Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	Preferred Pedagogy Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

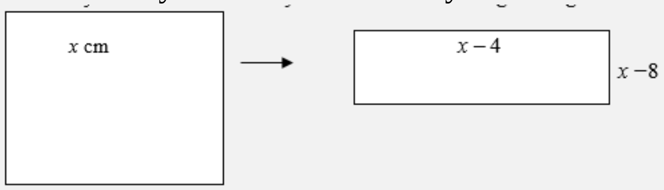
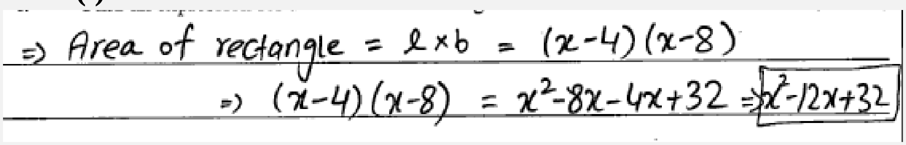
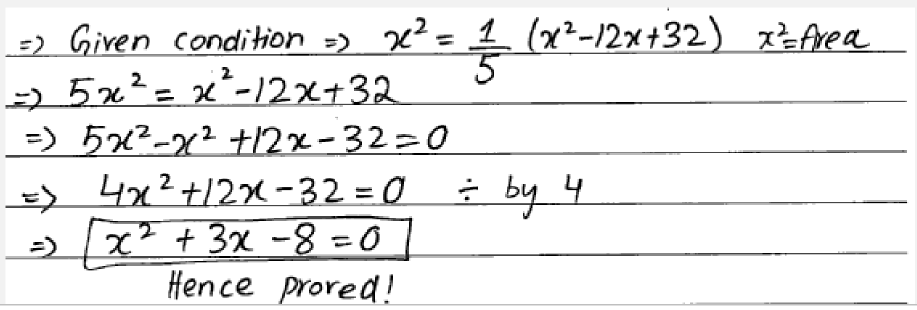
Any Additional Suggestion:

To improve students' understanding and performance, teachers may consider the following strategies;

- **Formula Clarification:** Provide clear explanations that the binomial expansion formula adapts when the exponent is a fraction. Emphasise that the standard binomial coefficients do not apply directly and introduce the generalised binomial theorem for rational exponents.
- **Practice Problems:** Provide additional practice problems focusing on binomial expansions with fractional exponents. Encourage students to solve these problems step by step to reinforce their understanding.
- **Digital Tools:** Use online tools and calculators to help students visualise and perform binomial expansions. Websites like Wolfram Alpha can automate the expansion process, allowing students to check their work.
- **Online Resources:** Recommend online tutorials and resources, such as those on Coursera, which provide step-by-step explanations and exercises on binomial expansions for rational exponents.
- **Real-Life Applications:** Show practical examples of binomial expansions, such as in probability theory or finance. For instance, illustrate the use of binomial theorem in calculating compound interest or analysing probability distributions.

Question No. 6a

Candidates were given the choice to attempt any ONE out of the two questions: 6a and 6b.

Question Text	Consider a square of side x cm. This square is changed into a rectangle by decreasing one of its sides by 8 cm and the other by 4 cm as shown in the given figure.  i. Find an expression for the area of the rectangle. ii. The area of the given square is $\frac{1}{5}$ times the area of the rectangle. Show that $x^2+3x-8=0$. iii. Without using a calculator, find the length of the side of square. [**Note:** Choose the appropriate length of the square from the values found in part iii.]
SLO No.	7.9.1
SLO Text	Solve word problems related to quadratic equations.
Max Marks	6
Cognitive Level	A
Checking Hints	1 mark for finding an expression for the area of rectangle 1 mark for finding area of square 1 mark for writing $x^2 = \frac{1}{5}(x-8)(x-4)$ 1 mark for correct simplification to prove 2 marks for applying the quadratic formula correctly to get the values −4.7 and 1.7 or $\frac{(-3\pm\sqrt{41})}{2}$
Overall Performance	This question involved transforming a square into a rectangle, forming expressions for the areas and solving a quadratic equation. Overall, most candidates performed well.
Description of Better Responses	In better responses, candidates demonstrated a strong understanding of finding the expressions for the area. They successfully calculated the areas of both, the square and the rectangle. For part (ii), candidates correctly set up the equation based on the given relationship between the areas and verified the given quadratic equation $x^2+3x-8=0$. In part (iii), they applied the quadratic formula accurately to find the appropriate value for x and chose the correct length of the square.
Images of Better Response	**Part (i)**  => Area of rectangle = l × b = (x−4)(x−8) => (x−4)(x−8) = x²−8x−4x+32 => x²−12x+32 **Part (ii)**  => Given condition => x² = 1/5 (x²−12x+32) x²=Area => 5x² = x²−12x+32 => 5x²−x²+12x−32=0 => 4x²+12x−32=0 ÷ by 4 => x² + 3x − 8 = 0 Hence proved!

	Part (iii) => Length of side of square:- => $x^2+3x-8=0$ => $\frac{-3 \pm \sqrt{41}}{2}$ => by quadratic $x = \frac{-3 \pm \sqrt{9-4(8)}}{2}$ so $x = \frac{-3 \pm \sqrt{41}}{2}$
Description of Weaker Responses	Weaker responses faced difficulty even in part (i), where candidates struggled to form the correct expression for the area of a rectangle. In part (ii), some candidates were unable to correctly derive the quadratic equation using the area of square and rectangle. In part (iii), candidates who did not correctly form the quadratic equation struggled to solve it, and those who attempted it often made errors in substituting the values in the quadratic formula.
Images of Weaker Responses	**Part (i) Image (i)** $2(x-4)\ 2(x-8)$ (Ans.). **Image (ii)** L×B = 4×8 x=4 x=8 = 32cm. **Part (ii) Image (i)** => $x^2+3x-8=0$, a = 1, b = 3, c = −8 => $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ => $\frac{3 \pm \sqrt{9+32}}{2}$ => $\frac{3 \pm \sqrt{41}}{2}$ => $\frac{3-\sqrt{41}}{2}$, $\frac{3+\sqrt{41}}{2}$ (Ans). **Part (ii) Image (ii)** a=1 b=3 c=-8 $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $x = \frac{+3 \pm \sqrt{3^2-4(1)(-8)}}{2(1)}$ $x = \frac{3 \pm \sqrt{9+16}}{2}$ $x = \frac{3 \pm 5}{2}$ $x_1 = \frac{3+5}{2}$ $x_2 = \frac{3-5}{2}$ $x_1 = 8/2$ $x_2 = -2/2$ $x_1 = 4$ $x_2 = -1$ **Part (iii) Image (i)** Length of square = 4 + length of rectangle length of square = 4+4 = 8cm **Part (iii) Image (ii)** the length of the side of the square is x and of rectangle is (x−4) (x−8).

Suggestions for improvement (Highlight all that apply)

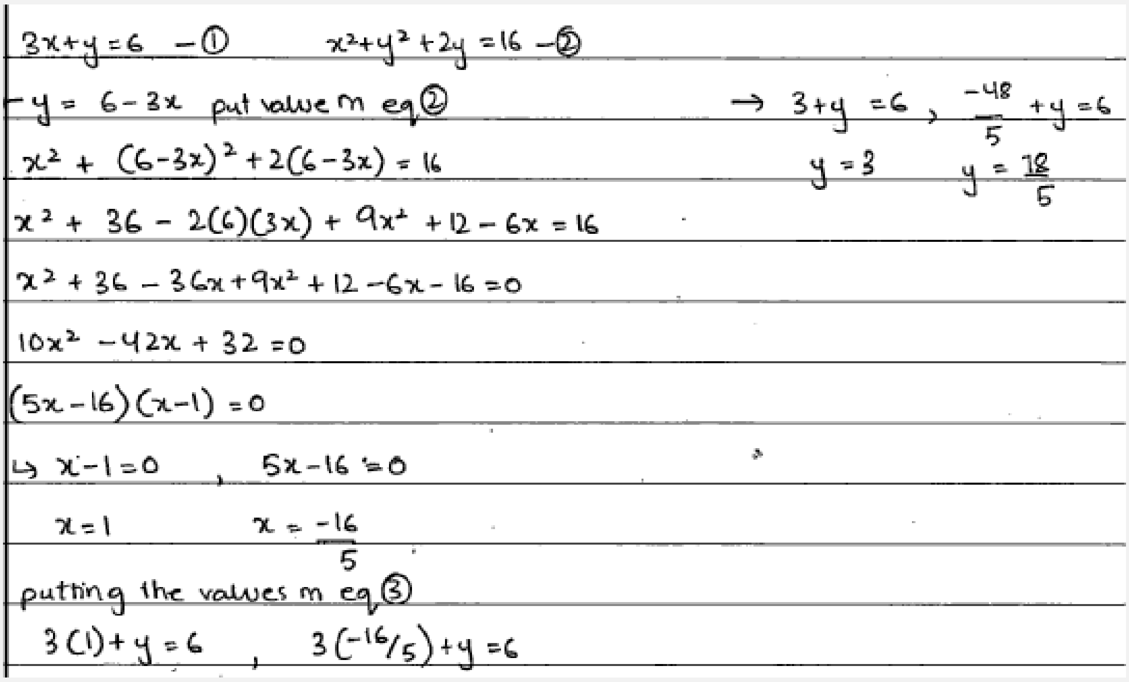
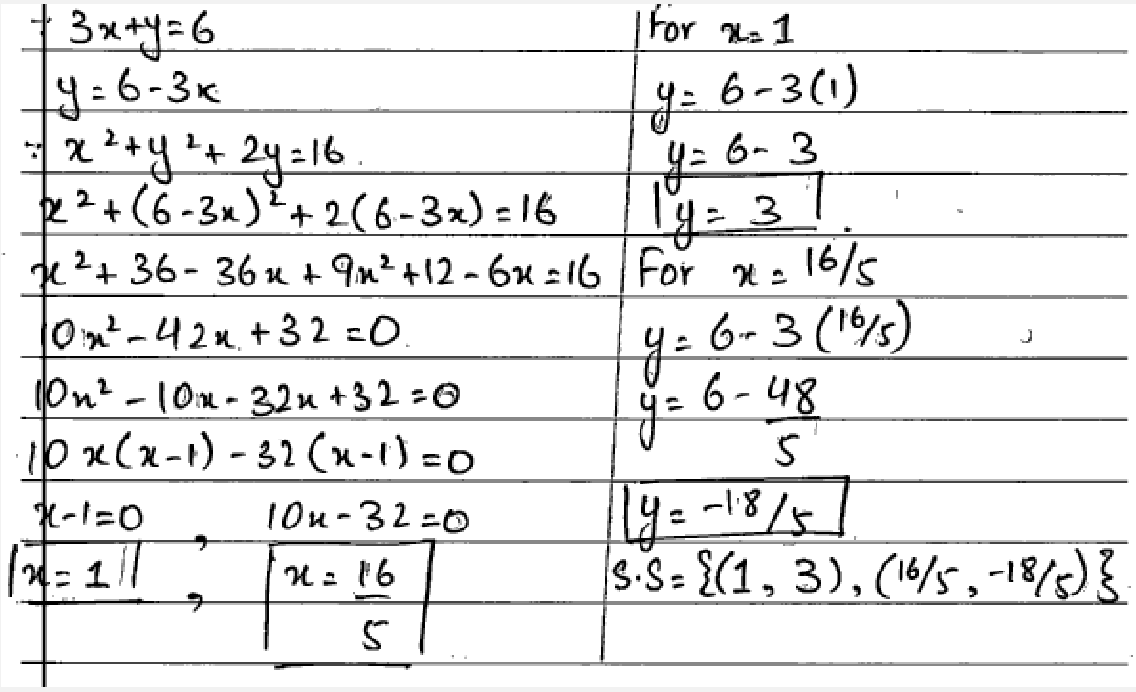
Maximising SLO Achievement	Pedagogy Used for that SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

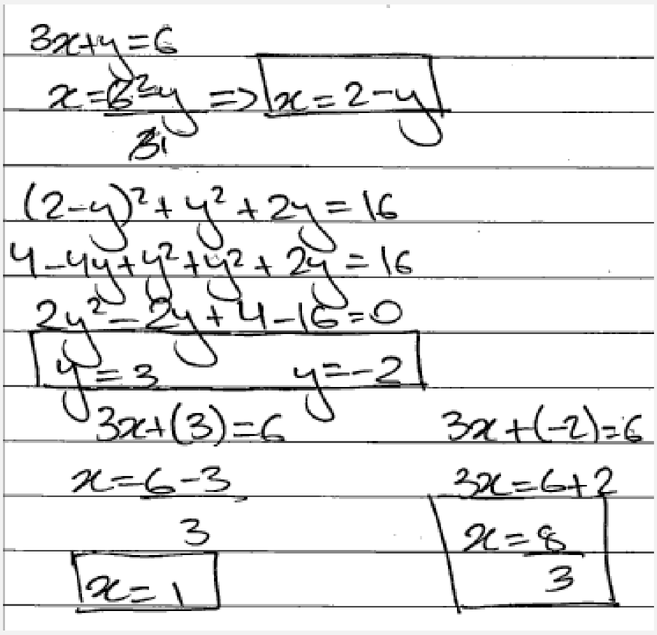
Any Additional Suggestion:

To improve students' understanding and performance, teachers may consider the following strategies;

- **Concept Clarification:** Provide clear explanations on deriving expressions for finding area of different shapes and translating word problems into mathematical equations. Emphasise the steps to follow when changing dimensions and forming equations.
- **Practice Problems:** Provide additional practice problems that involve transforming shapes and solving quadratic equations. Encourage students to solve these problems step by step to reinforce their understanding.
- **Visual Aids:** Use diagrams and visual aids to help students better understand the geometric transformations and the relationships between different areas.
- **Online Resources:** Recommend online tutorials and resources, such as those on Coursera, which provide step-by-step explanations and exercises on solving word problems and quadratic equations.
- **Real-Life Applications:** Show practical examples of word problems that involve quadratic equations, such as in physics or economics. For instance, illustrate the calculation of areas in real-life scenarios like land division or construction.

Question No. 6b **Candidates were given the choice to attempt any ONE out of the two questions: 6a and 6b.**	
Question Text	Solve the following system of equations. $x^2 + y^2 + 2y = 16$ and $3x + y = 6$
SLO No.	7.8.1
SLO Text	Solve system of two equations in two variables when: a. one equation is linear and the other is quadratic.
Max Marks	6
Cognitive Level	A
Checking Hints	1 mark for writing $y = 6 - 3x$ 1 mark for substituting $y = 6 - 3x$ in the given quadratic equation 1 mark for simplification to get $5x^2 - 21x + 16 = 0$ 1 mark for complete factorisation of $5x^2 - 21x + 16 = 0$ 1 mark for finding the value of x and y 1 mark for writing the solution set

Overall Performance	This question involved solving a system of simultaneous equations where one equation is linear and the other is quadratic. Most candidates performed well, with many choosing to attempt this part (b) of the question.
Description of Better Responses	In better responses, candidates accurately solved for the values of *x* and *y* by correctly substituting the linear equation into the quadratic equation. These candidates successfully found the correct values of *x* and *y*, demonstrating a clear understanding of handling systems involving both linear and quadratic equations. Their methodical approach ensured accurate solutions.
Images of Better Responses	**Image (i)**  $3x+y=6$ —① $x^2+y^2+2y=16$ —② $y=6-3x$ put value in eq② $x^2+(6-3x)^2+2(6-3x)=16$ $x^2+36-2(6)(3x)+9x^2+12-6x=16$ $x^2+36-36x+9x^2+12-6x-16=0$ $10x^2-42x+32=0$ $(5x-16)(x-1)=0$ $x-1=0$, $5x-16=0$ $x=1$ $x=\frac{-16}{5}$ putting the values in eq③ $3(1)+y=6$, $3(-16/5)+y=6$ → $3+y=6$, $\frac{-48}{5}+y=6$ $y=3$ $y=\frac{18}{5}$ **Image (ii)**  $3x+y=6$ $y=6-3x$ $x^2+y^2+2y=16$ $x^2+(6-3x)^2+2(6-3x)=16$ $x^2+36-36x+9x^2+12-6x=16$ $10x^2-42x+32=0$ $10x^2-10x-32x+32=0$ $10x(x-1)-32(x-1)=0$ $x-1=0$, $10x-32=0$ $x=1$, $x=\frac{16}{5}$ For $x=1$ $y=6-3(1)$ $y=6-3$ $y=3$ For $x=16/5$ $y=6-3(16/5)$ $y=6-\frac{48}{5}$ $y=-18/5$ $S.S=\{(1,3),(16/5,-18/5)\}$
Description of Weaker Responses	Weaker responses showed candidates facing difficulty in finding the correct values of *x* and *y*. Some candidates made errors in substitution, leading to incorrect solutions. Others attempted to solve both equations as quadratic equations, leading to confusion and incorrect answers. Some of the candidates struggled to complete the step-by-step solution even after

	substituting the correct values of *y* and solving the equation for *x*, leading to incorrect required values.
Image of Weaker Response	

Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	Preferred Pedagogy Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

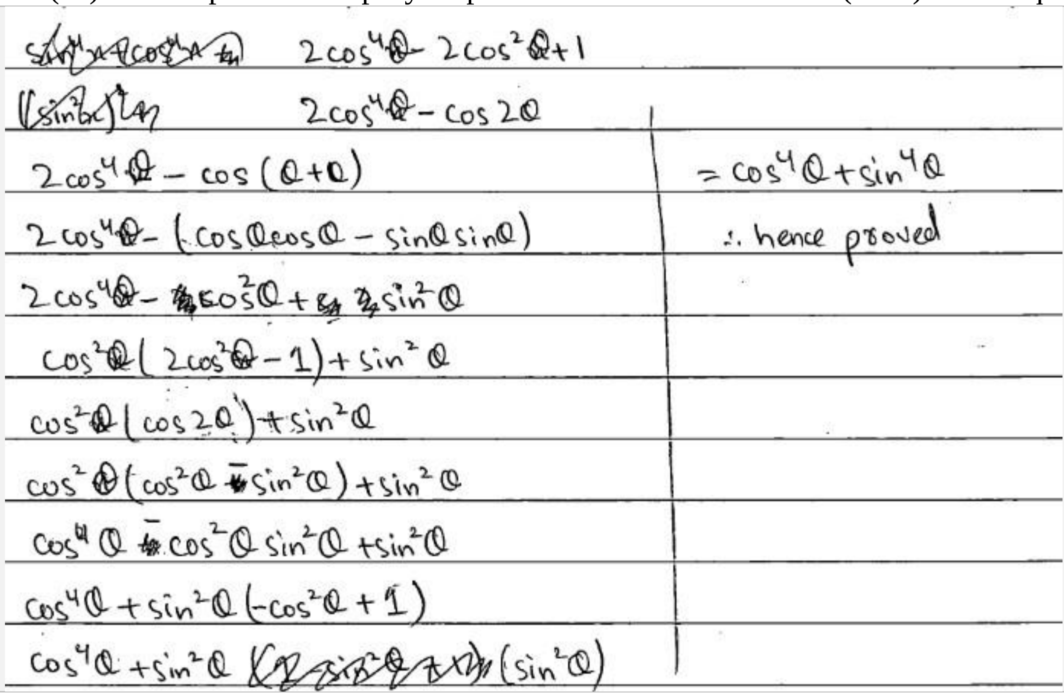
Any Additional Suggestion:

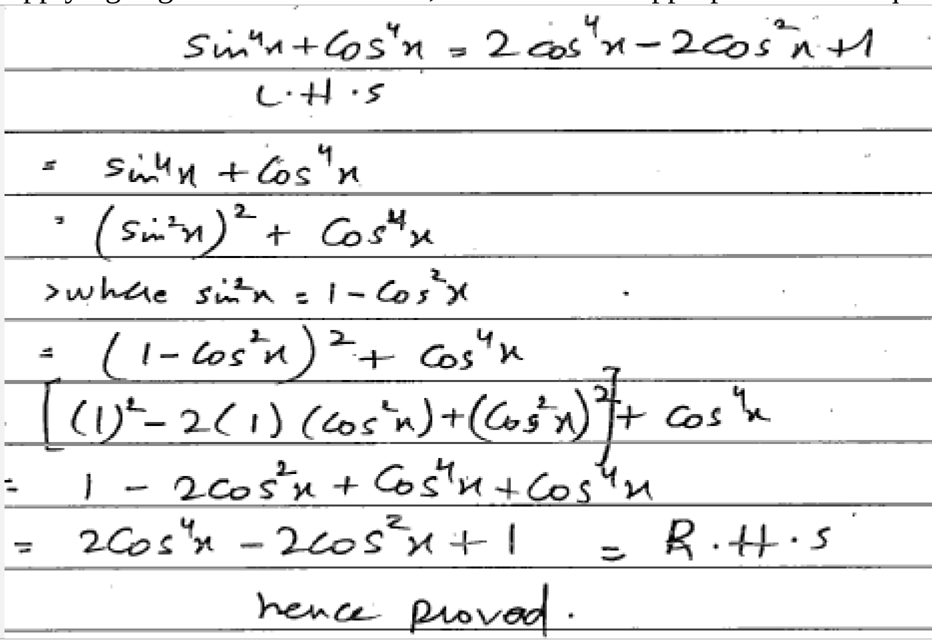
To improve students' understanding and performance, teachers may consider the following strategies;

- **Concept Clarification:** Provide clear explanations on solving systems of equations where one is linear and the other is quadratic. Emphasise the steps of substitution and solving the resulting quadratic equation.
- **Practice Problems:** Provide additional practice problems that involve solving systems of linear and quadratic equations. Encourage students to solve these problems step by step to reinforce their understanding.
- **Visual Aids:** Use graphs and visual aids to show the intersection points of the linear and quadratic equations. This can help students understand the solutions better.
- **Online Resources:** Recommend online tutorials and resources, such as those on Coursera, which provide step-by-step explanations and exercises on solving systems of equations involving linear and quadratic equations.

Question No. 7a

Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b, and 7c.

Question Text	Using half angle and double angles formulae, prove that $\sin^4 x + \cos^4 x = 2\cos^4 x - 2\cos^2 x + 1$. [**Note:** $\cos 2\theta = 2\cos^2\theta - 1$]
SLO No.	8.5.2
SLO Text	Prove different trigonometric relations using identities mentioned in SLO 8.5.1
Max Marks	5
Cognitive Level	A
Checking Hints	1 mark for $\left(\frac{1-\cos 2x}{2}\right)^2 + \left(\frac{1+\cos 2x}{2}\right)^2$ **OR** $\left(\frac{1-\cos 2x}{2}\right)^2 + \cos^4 x$ **OR** $(\sin^2 x)^2 + (\cos^2 x)^2$ 1 mark for $\frac{1-2\cos 2x+\cos^2 2x}{4} + \frac{1+2\cos 2x+\cos^2 2x}{4}$ **OR** $\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos^4 x + \cos^4 x$ **OR** $1 - \frac{1}{2}(2\sin x\cos x)^2$ 1 mark for $\frac{1}{2} + \frac{\cos^2 2x}{2}$ **OR** $\frac{1}{4} - \cos^2 x + \frac{1}{2} + \frac{4\cos^4 x - 4\cos^2 x + 1}{4} + \cos^4 x$ **OR** $\frac{2-\sin^2 2x}{2}$ 1 mark for $\frac{1}{2} + \frac{1}{2}(4\cos^4 x - 4\cos^2 x + 1)$ **OR** $\frac{1}{4} - \cos^2 x + \frac{1}{2} + \cos^4 x - \cos^2 x + \frac{1}{4} + \cos^4 x$ **OR** $\frac{1}{2}(1+\cos^2 2x)$ 1 mark for $2\cos^4 x - 2\cos^2 x + 1$
Overall Performance	Overall, many candidates struggled with this question, with only a few able to apply the correct identities and steps to reach at the solution.
Description of Better Responses	Better responses demonstrated a clear understanding of the double-angle identities of cosine. They correctly expanded the right-hand side (RHS) of the equation using the identity of cos(2*x*) and simplified it step-by-step to obtain the left-hand side (LHS) of the equation.
Image of Better Response	

Description of Weaker Responses	Weaker responses faced difficulty correctly identifying and applying double-angle and half-angle identities. Some made mistakes in expanding $\sin^4x+\cos^4x$ by using double and half angle identities or did not follow the proper steps to simplify the expression to match the RHS. Furthermore, some of the candidates also attempted to prove the question by directly applying trigonometric identities, which was not appropriate for the question.
Image of Weaker Response	 $\sin^4 x+\cos^4 x = 2\cos^4 x-2\cos^2 x+1$ L.H.S $= \sin^4 x+\cos^4 x$ $= (\sin^2 x)^2+\cos^4 x$ > where $\sin^2 x = 1-\cos^2 x$ $= (1-\cos^2 x)^2+\cos^4 x$ $[(1)^2-2(1)(\cos^2 x)+(\cos^2 x)^2]+\cos^4 x$ $= 1-2\cos^2 x+\cos^4 x+\cos^4 x$ $= 2\cos^4 x-2\cos^2 x+1 =$ R.H.S hence proved.

Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	**Preferred Pedagogy Used for this SLO**	**Assessment Strategies**
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

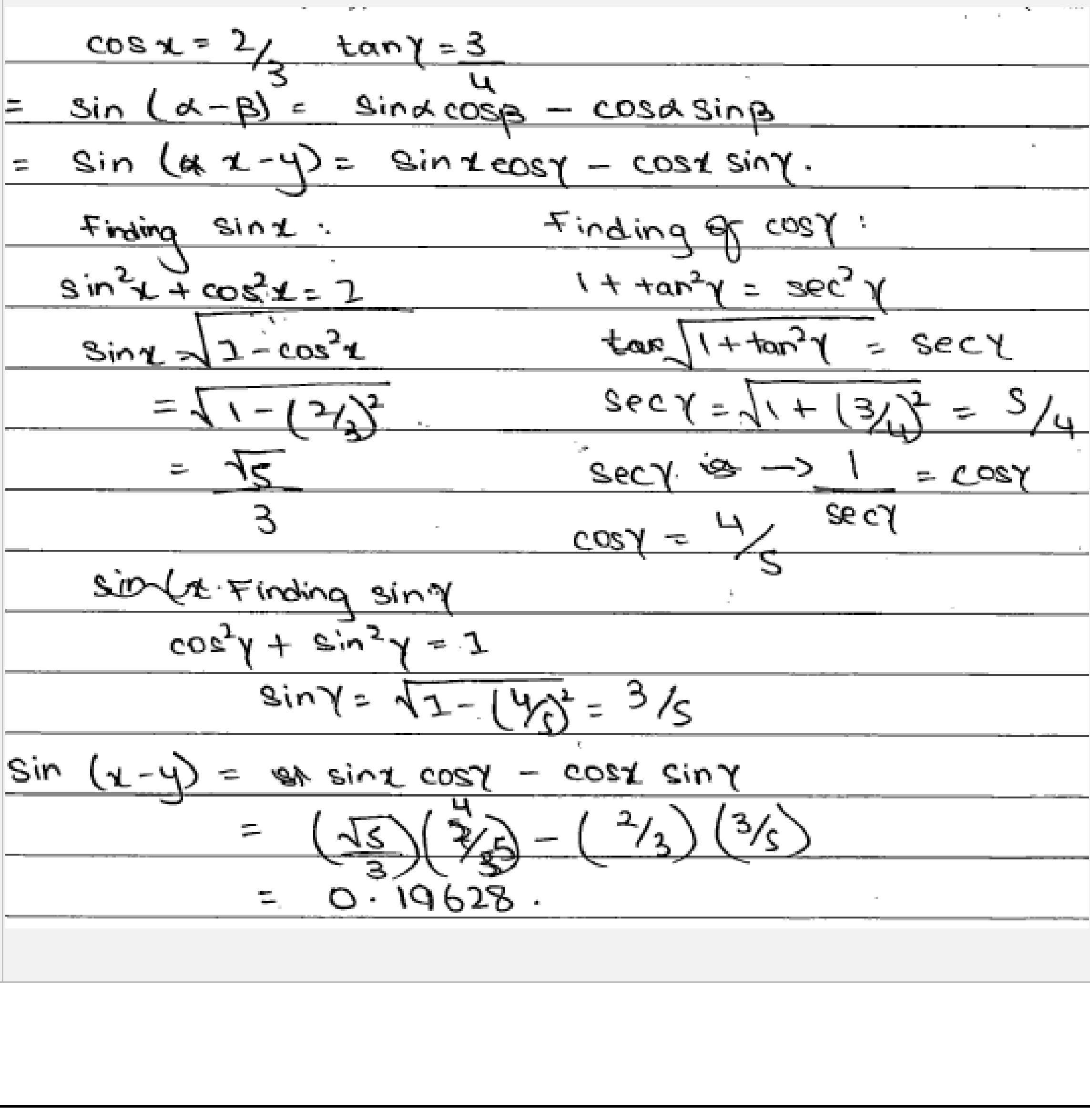
Any Additional Suggestion:

To improve students' understanding and performance, teacher may consider the following strategies;

- **Step-by-Step Examples:** Offer detailed examples of proving similar trigonometric identities, showing the step-by-step application of relevant identities and simplification processes. Demonstrate verification of each step to ensure accuracy in the solution.
- **Practice Problems:** Provide additional practice problems that involve proving trigonometric identities using double-angle and half-angle formulas. Encourage students to solve these problems step-by-step to reinforce their understanding.

Question No. 7b

Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b, and 7c.

Question Text	It is given that $\cos x = \frac{2}{3}$ and $\tan y = \frac{3}{4}$. Find the value of $\sin(x - y)$.
SLO No.	8.3.4
SLO Text	Solve problems related to fundamental law of trigonometry and its deductions.
Max Marks	5
Cognitive Level	A
Checking Hints	1 mark for $\sin x$ 1 mark for $\sin y$ 1 mark for $\cos y$ 1 mark for correct substitution in $\sin(x - y)$ 1 mark for $\frac{4\sqrt{5}-6}{15}$
Overall Performance	Most candidates performed well, correctly applying trigonometric identities and formulas to find the solution.
Description of Better Responses	In better responses, candidates accurately used the correct identities such as $\sin^2 x + \cos^2 x = 1$ and $1 + \tan^2 y = \sec^2 y$ and substituted the given values for $\cos x$ and $\tan y$ correctly to find the values of $\sin x$ and $\cos y$. Some of the candidates also applied the inverse identities to find the angles x and y. These candidates then used the formula for $\sin(x \pm y)$ to find $\sin(x - y)$. Their methodical approach and understanding of trigonometric identities led to successful and accurate results.
Images of Better Responses	**Image (i)** 

$\cos x = 2/3 \quad \tan y = \frac{3}{4}$

$= \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \sin(x - y) = \sin x\cos y - \cos x\sin y.$

Finding $\sin x$:

$\sin^2 x + \cos^2 x = 1$

$\sin x = \sqrt{1 - \cos^2 x}$

$= \sqrt{1 - (2/3)^2}$

$= \frac{\sqrt{5}}{3}$

Finding of $\cos y$:

$1 + \tan^2 y = \sec^2 y$

$\sqrt{1 + \tan^2 y} = \sec y$

$\sec y = \sqrt{1 + (3/4)^2} = 5/4$

$\sec y \rightarrow \frac{1}{\sec y} = \cos y$

$\cos y = 4/5$

Finding $\sin y$

$\cos^2 y + \sin^2 y = 1$

$\sin y = \sqrt{1 - (4/5)^2} = 3/5$

$\sin(x - y) = \sin x\cos y - \cos x\sin y$

$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{4}{5}\right) - (2/3)(3/5)$

$= 0.19628.$

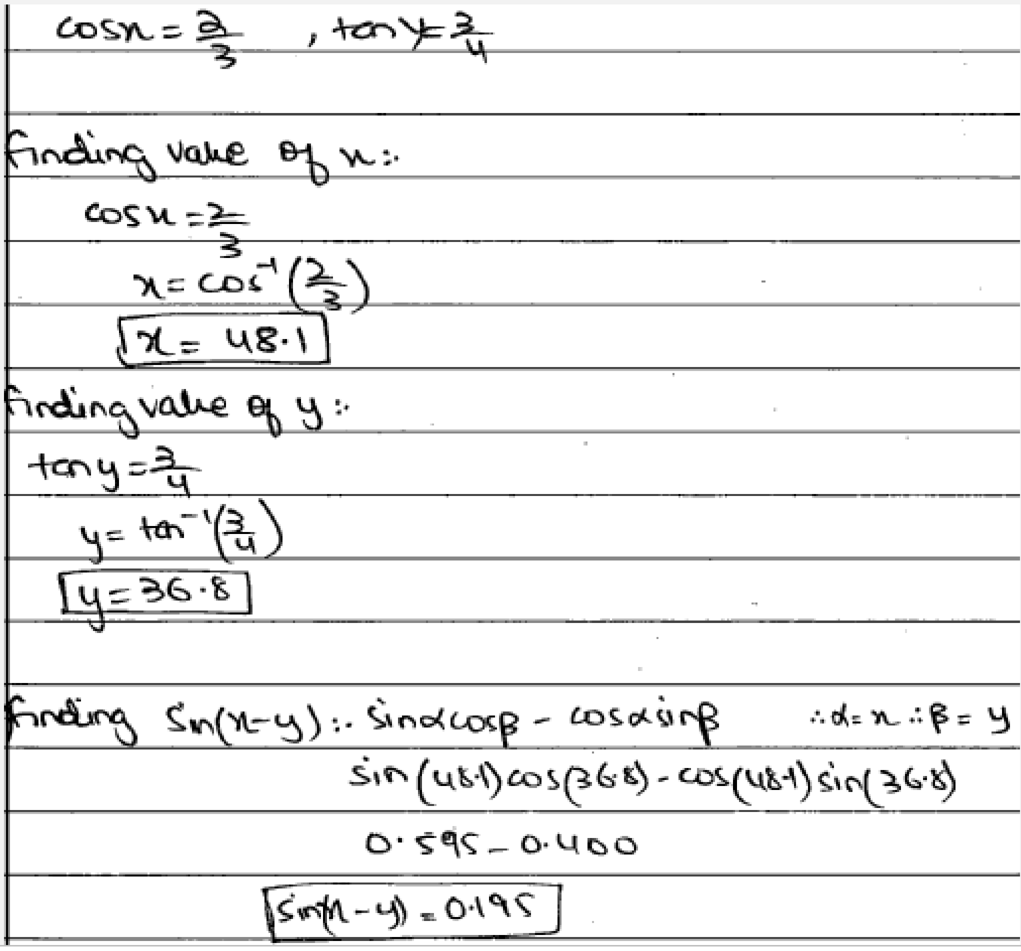
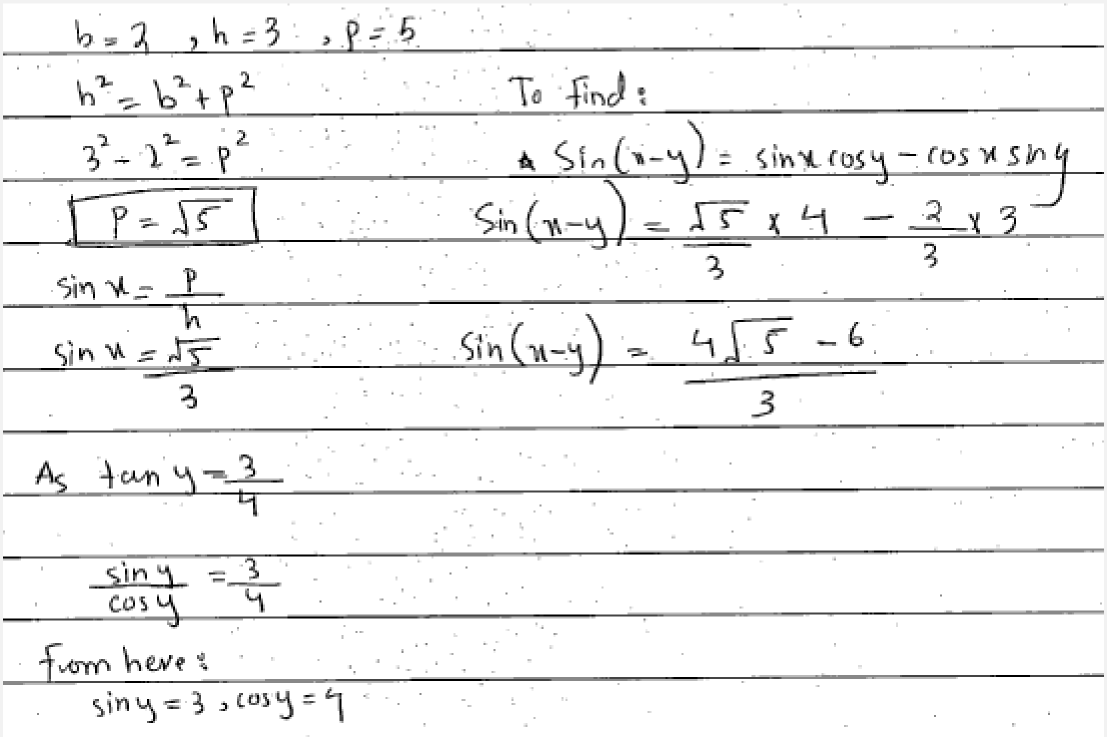
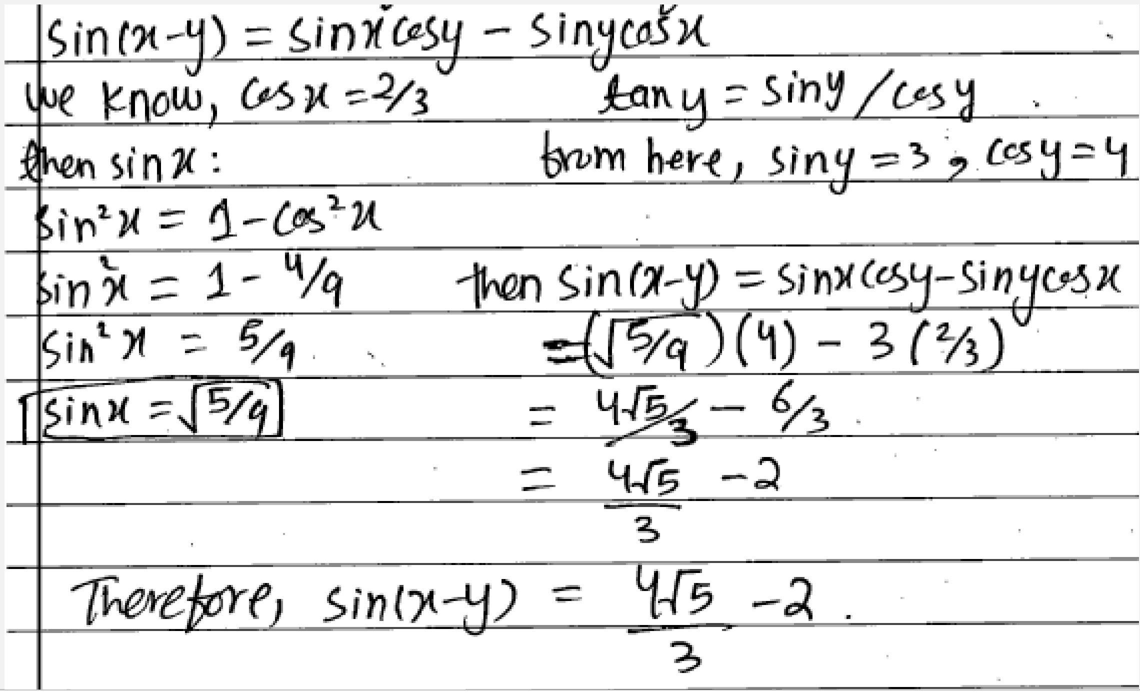
	Image (ii)  $\cos x = \frac{2}{3}$, $\tan y = \frac{3}{4}$ finding value of x: $\cos x = \frac{2}{3}$ $x = \cos^{-1}\left(\frac{2}{3}\right)$ $x = 48.1$ finding value of y: $\tan y = \frac{3}{4}$ $y = \tan^{-1}\left(\frac{3}{4}\right)$ $y = 36.8$ finding $\sin(x-y)$:- $\sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\therefore \alpha = x \therefore \beta = y$ $\sin(48.1)\cos(36.8) - \cos(48.1)\sin(36.8)$ $0.595 - 0.400$ $\sin(x-y) = 0.195$
Description of Weaker Responses	In weaker responses, some of the candidates applied the $\sin(x-y)$ correctly, however they applied incorrect identities or made substitution errors in finding the values of $\sin x$ and $\cos y$. These mistakes leading to incorrect values of $\sin(x-y)$.
Images of Weaker Responses	**Image (i)**  $b = 2$, $h = 3$, $p = 5$ $h^2 = b^2 + p^2$ $3^2 - 2^2 = p^2$ $p = \sqrt{5}$ $\sin x = \frac{p}{h}$ $\sin x = \frac{\sqrt{5}}{3}$ As $\tan y = \frac{3}{4}$ $\frac{\sin y}{\cos y} = \frac{3}{4}$ from here: $\sin y = 3$, $\cos y = 4$ To find: $\sin(x-y) = \sin x\cos y - \cos x \sin y$ $\sin(x-y) = \frac{\sqrt{5}}{3} \times 4 - \frac{2}{3} \times 3$ $\sin(x-y) = \frac{4\sqrt{5} - 6}{3}$

	Image (ii)

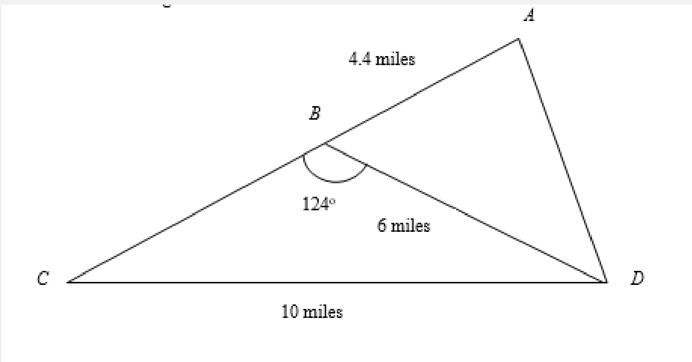
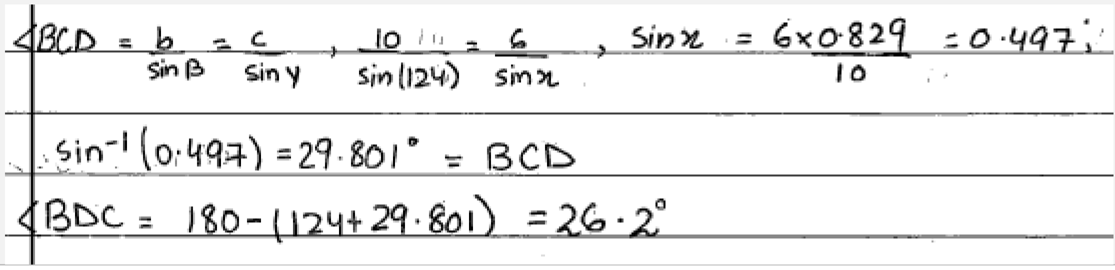


Suggestions for improvement (Highlight all that apply)

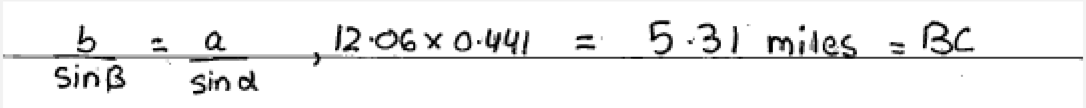
Maximising SLO Achievement	**Preferred Pedagogy Used for this SLO**	**Assessment Strategies**
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:
To improve students' understanding and performance, teachers may consider the following strategies;

- **Practice Problems:** Provide additional practice problems that involve finding values of trigonometric functions using given ratios. Encourage students to solve these problems step by step to reinforce their understanding.
- **Visual Aids:** Use unit circles and diagrams to help students better understand the relationships between different trigonometric functions and enable them to find angles from given ratios.
- **Online Resources:** Recommend online tutorials and resources such as those on Coursera, which provide step-by-step explanations and exercises related to using trigonometric identities and solving related problems.

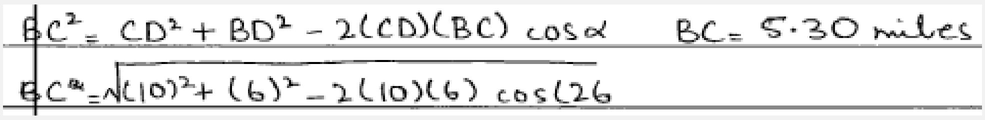
Question No. 7c **Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b, and 7c.**	
Question Text	A road map shows four cities labelled *A*, *B*, *C*, and *D*. Cities *A*, *B*, and *C* lie on a straight line with distances *AB* = 4.4 miles *BD* = 6 miles, and *CD* = 10 miles. The angle *CBD* measures 124^{o}.  i. Find the angles *BCD* and *BDC*. ii. Find the length of *BC*. iii. Find the angle *ABD*. iv. Calculate the area of triangle *ABD*.
SLO No.	9.1.5, 9.2.2
SLO Text	Solve problems related to SLOs from 9.1.1 to 9.1.4; 9.1.3(b). Apply the above formulae to find the area of a triangle, (two sides and one angle is given) 9.2.1(a).
Max Marks	5
Cognitive Level	A
Checking Hints	i. 1 mark for angle *C* 1 mark for angle *D* ii. 1 mark for length of *BC* iii. 1 mark for angle *B* iv. 1 mark for area of *ABD*
Overall Performance	This question involved solving problems related to the laws of sines and cosines and finding the area of a triangle. Overall, most candidates performed well, with many correctly applying the relevant formulas to find angles, sides and areas.
Description of Better Responses	In better responses, candidates demonstrated a strong understanding of the laws of sines and cosines and the properties of triangles. They correctly applied the law of sines to find angles *BCD* and *BDC*, then used the sum of angles in a triangle (180^{o}) to verify their calculations. For part (ii), they accurately applied the law of cosines to find the length of *BC*. In part (iii), candidates correctly used supplementary angles to find the angle *ABD*. Finally, in part (iv), they applied the appropriate formula for the area of a triangle with two sides and the included angle to find the area of triangle *ABD*. These responses showed a clear and methodical approach, with all essential steps correctly followed.
Images of Better Responses	**Part (i)** 

Part (ii) Image (i)



$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$, $12.06 \times 0.441 = 5.31$ miles $= BC$

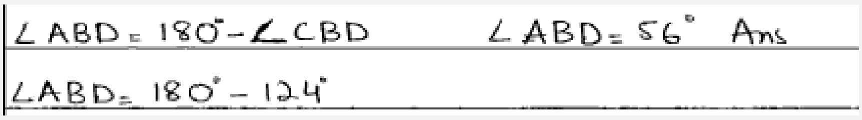
Image (ii)



$BC^2 = CD^2 + BD^2 - 2(CD)(BC)\cos\alpha$ $BC = 5.30$ miles

$BC^2 = \sqrt{(10)^2 + (6)^2 - 2(10)(6)\cos(26}$

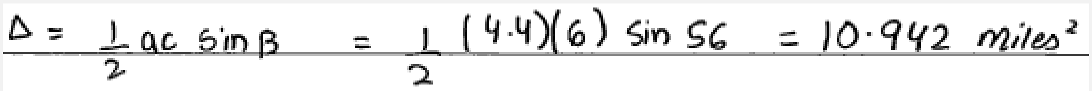
Part (iii)



$\angle ABD = 180° - \angle CBD$ $\angle ABD = 56°$ Ans

$\angle ABD = 180° - 124°$

Part (iv)



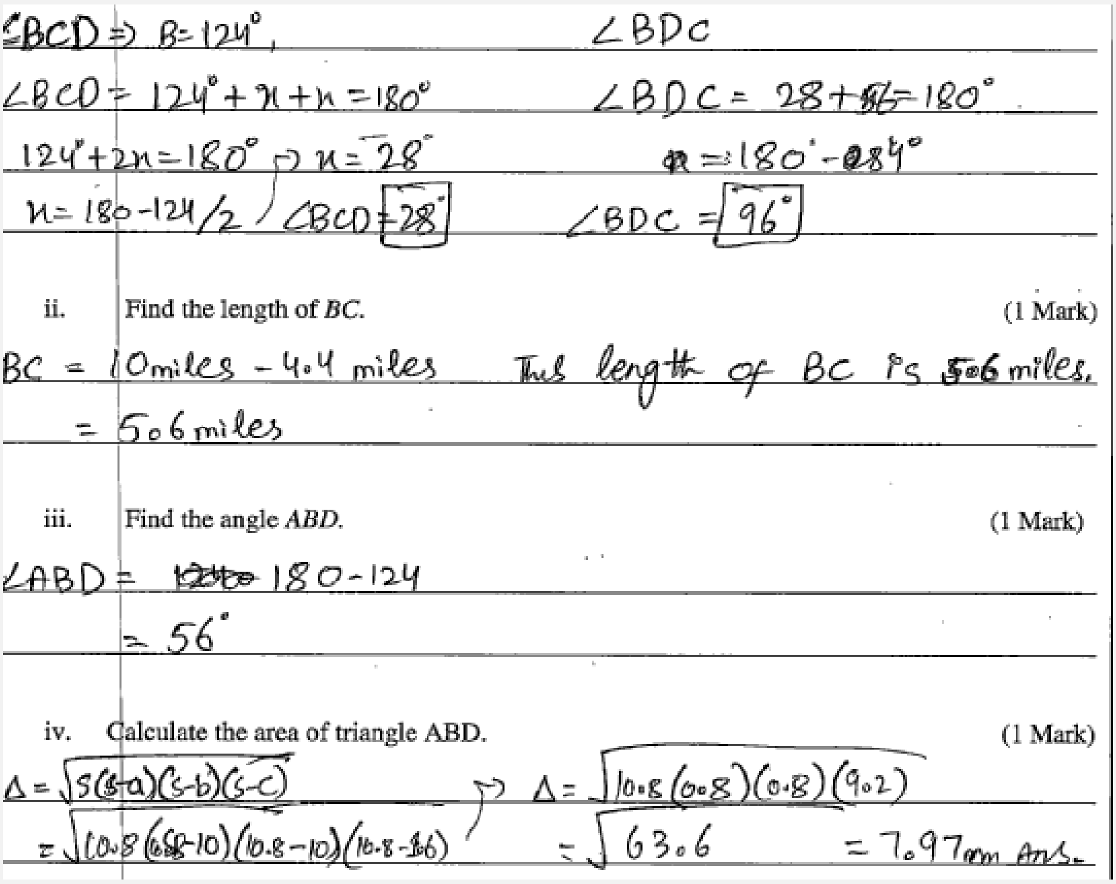
$\Delta = \frac{1}{2}ac\sin\beta = \frac{1}{2}(4.4)(6)\sin 56 = 10.942$ miles^2

Description of Weaker Responses

Weaker responses reflected that the candidates have struggled to apply the sine and cosine laws correctly. Some candidates incorrectly used the sum of angles in a triangle (180°) instead of the law of sines to find the required angles. This led to errors in calculating angles *BCD* and *BDC*. For part (ii), some candidates failed to apply the law of cosines correctly, resulting in incorrect values for *BC*. In part (iii), mistakes were made in identifying supplementary angles, leading to incorrect values for *ABD*. In part (iv), candidates who did not correctly solve the earlier parts struggled to find the correct area of triangle *ABD*.

Images of Weaker Responses

Image (i)



$\angle BCD \Rightarrow B = 124°$, $\angle BDC$

$\angle BCD = 124° + x + x = 180°$ $\angle BDC = 28 + 56 = 180°$

$124° + 2x = 180° \Rightarrow x = 28°$ $x = 180° - 84°$

$x = 180 - 124/2$ $\angle BCD = 28°$ $\angle BDC = 96°$

ii. Find the length of *BC*. (1 Mark)

BC = 10 miles − 4.4 miles The length of BC is 5.6 miles.

= 5.6 miles

iii. Find the angle *ABD*. (1 Mark)

$\angle ABD = 180 - 124$

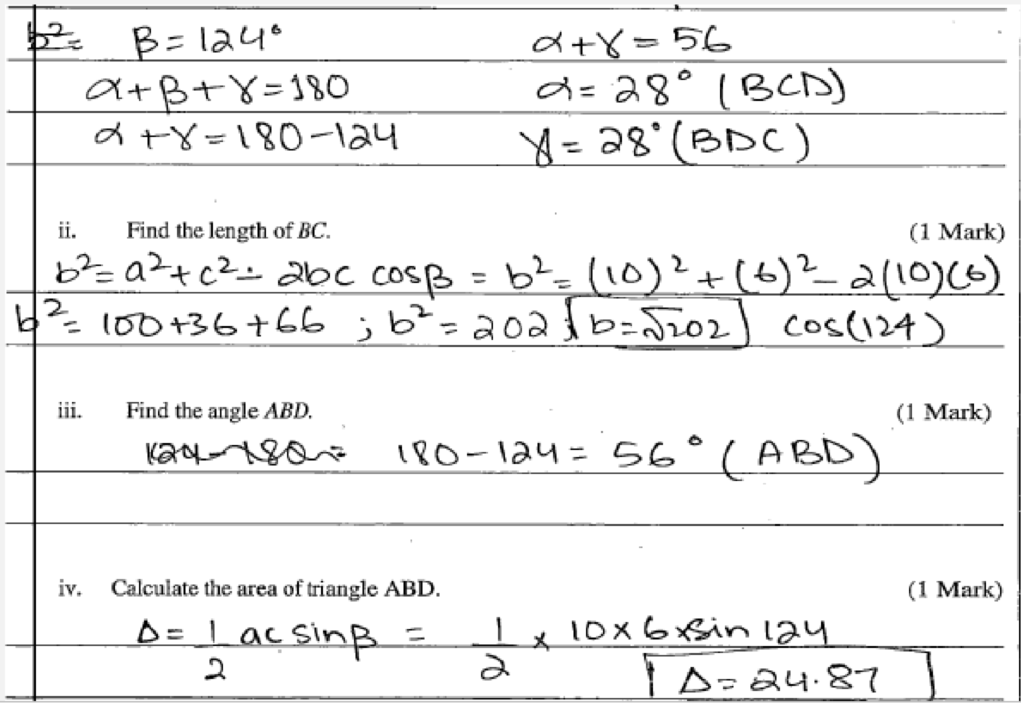
$= 56°$

iv. Calculate the area of triangle ABD. (1 Mark)

$\Delta = \sqrt{s(s-a)(s-b)(s-c)} \Rightarrow \Delta = \sqrt{10.8(0.8)(0.8)(9.2)}$

$= \sqrt{10.8(10.8-10)(10.8-10)(10.8-1.6)} = \sqrt{63.6} = 7.97$ mm Ans.

Image (ii)



$b^2=$ $\beta = 124°$ $\alpha + \gamma = 56$

$\alpha + \beta + \gamma = 180$ $\alpha = 28°$ (BCD)

$\alpha + \gamma = 180 - 124$ $\gamma = 28°$ (BDC)

ii. Find the length of *BC*. (1 Mark)

$b^2 = a^2 + c^2 - 2bc \cos\beta = b^2 = (10)^2 + (6)^2 - 2(10)(6)$

$b^2 = 100 + 36 + 66$; $b^2 = 202$ $b = \sqrt{202}$ $\cos(124)$

iii. Find the angle *ABD*. (1 Mark)

~~124 − 180 =~~ $180 - 124 = 56°$ (ABD)

iv. Calculate the area of triangle ABD. (1 Mark)

$\Delta = \frac{1}{2} ac \sin\beta = \frac{1}{2} \times 10 \times 6 \times \sin 124$

$\Delta = 24.87$

Suggestions for improvement (Highlight all that apply)

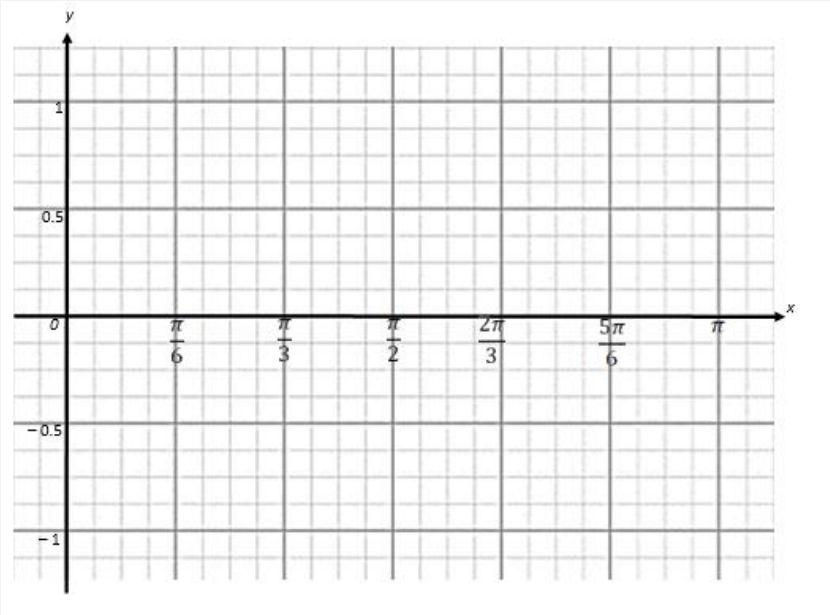
Maximising SLO Achievement	Preferred Pedagogy Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:

To improve students' understanding and performance, teachers may consider the following strategies;

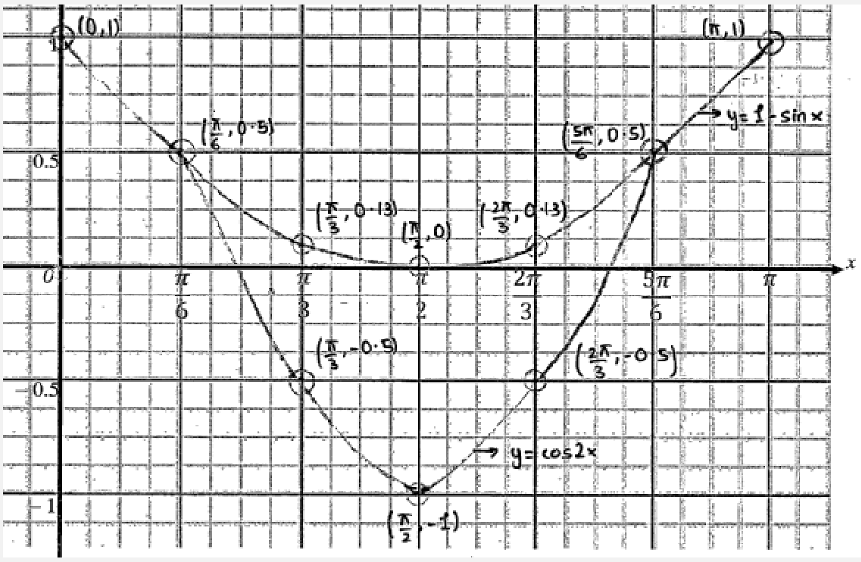
- **Concept Clarification:** Provide clear explanations related to applying the laws of sines and cosines in different situations. Emphasise the steps to follow when solving for angles, sides and areas in different triangle configurations.
- **Step-by-Step Examples:** Discuss detailed examples of solving similar problems, showing the step-by-step application of the laws of sines and cosines and the calculation of areas of the triangles. Demonstrate verification of angles using the sum of angles in a triangle and supplementary angles.
- **Practice Problems:** Provide additional practice problems that involve applying the laws of sines and cosines to find angles, sides and areas of triangles. Encourage students to solve these problems step by step to reinforce their understanding.

- **Visual Aids:** Use diagrams and visual aids to help students understand the relationships between different angles and sides in different types of triangles. Visualising the problems can help students grasp the concepts more effectively.

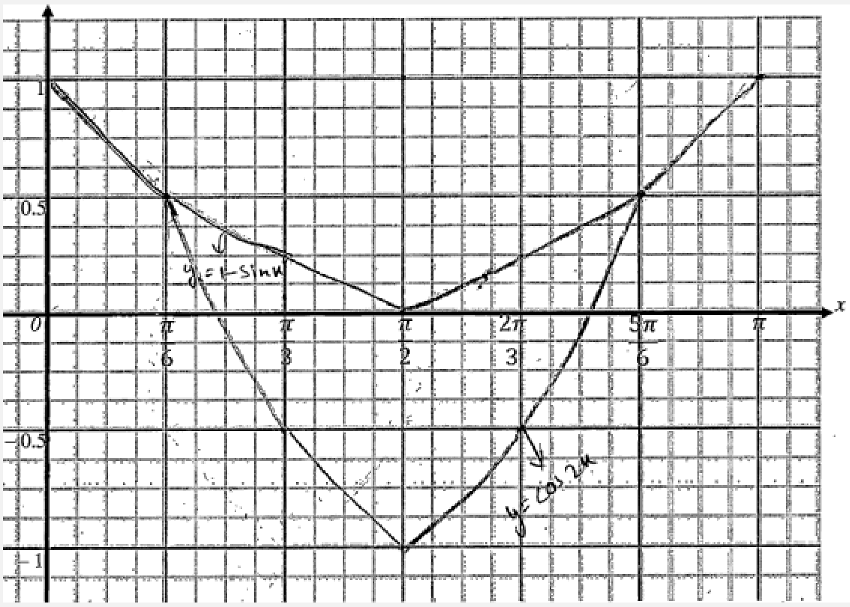
Question No. 8	
Question Text	Consider the functions $y = 1 - \sin x$ and $y = \cos 2x$. i. Sketch a single graph that shows these functions for the given interval [0, π].  ii. Determine the value of k that makes the equation $\cos 2x + k^2 \sin x = 1$ true for the x-coordinates of the points where the two graphs intersect, as mentioned in part (i).
SLO No.	10.4.3
SLO Text	Find the solution set of trigonometric equation graphically.
Max Marks	6
Cognitive Level	A
Checking Hints	i. 1 mark for correct values of $y = 1 - \sin x$ (any 5) 1 mark for correct values of $y = \cos 2x$ (any 5) 1 mark for correct graph of each function (2 required) ii. 1 mark on correction intersection points 1 mark on values of k
Overall Performance	This question involves sketching graphs of trigonometric functions and finding the value of k that satisfies the given equation at the intersection points of these graphs. Overall, most candidates successfully solved part (i), but many struggled with part (ii), specifically in calculating the correct value of k.
Description of Better Responses	In better responses, candidates correctly sketched the graphs of $y = 1 - \sin x$ and $y = \cos 2x$ over the interval [0,π]. They accurately calculated the corresponding values of x and plotted the ordered pairs correctly. These candidates successfully identified the intersection points of the two graphs. For part (ii), they correctly substituted the xxx-coordinates of the intersection points into the equation $\cos 2x + k^2 \sin x = 1$ and solved for the value of k, demonstrating a strong understanding of the graphical solution of trigonometric equations.

Images of Better Responses

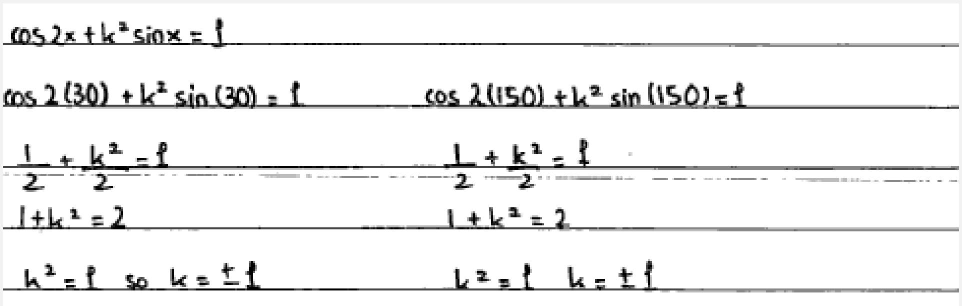
Part (i) Image (i)



Part (i) Image (ii)



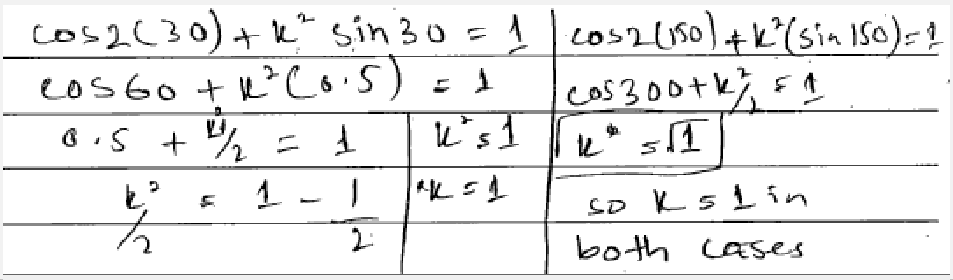
Part (ii) Image (i)



$\cos 2x + k^2 \sin x = 1$

$\cos 2(30) + k^2 \sin(30) = 1$	$\cos 2(150) + k^2 \sin(150) = 1$
$\frac{1}{2} + \frac{k^2}{2} = 1$	$\frac{1}{2} + \frac{k^2}{2} = 1$
$1 + k^2 = 2$	$1 + k^2 = 2$
$k^2 = 1$ so $k = \pm 1$	$k^2 = 1 \quad k = \pm 1$

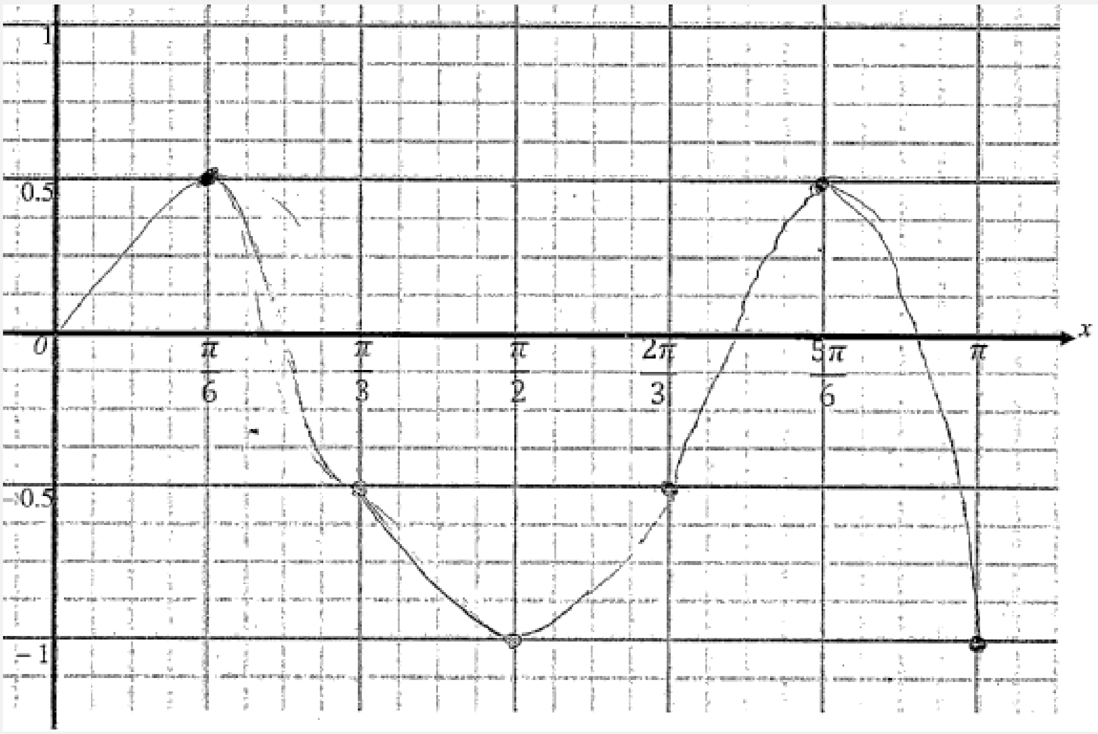
Part (ii) Image (ii)

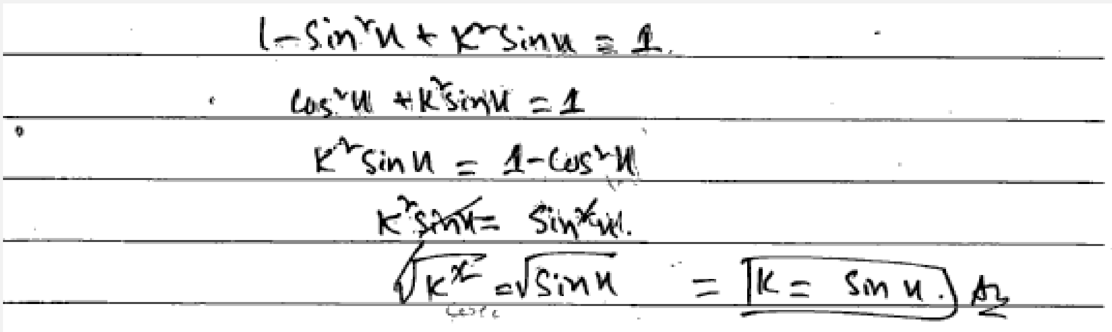


$\cos 2(30) + k^2 \sin 30 = 1$		$\cos 2(150) + k^2(\sin 150) = 1$
$\cos 60 + k^2(0.5) = 1$		$\cos 300 + k^2/2 = 1$
$0.5 + k^2/2 = 1$	$k^2 = 1$	$k^2 = \sqrt{1}$
$k^2/2 = 1 - \frac{1}{2}$	$k = 1$	so $k = 1$ in
		both cases

Description of Weaker Responses	Weaker responses substituted incorrect values of x when plotting the graphs, leading to inaccurate graphs and incorrect intersection points. Consequently, they were unable to find the correct value of k. Some candidates did not accurately plot the graphs or failed to understand the relationship between the graphs and the equation $\cos 2x + k^2 \sin x = 1$.

Image of Weaker Response





Suggestions for improvement (Highlight all that apply)

Maximising SLO Achievement	Preferred Pedagogy Used for this SLO	Assessment Strategies
• Identify the expectation of command words (use Command Word Guide) • Ensure the content is taught at the relevant cognitive level • Identify necessary content required (skills + concepts) • Review past paper questions on the concept • Utilise the resource guide for additional materials	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual Resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach)	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

	• Practical Demonstration	

Any Additional Suggestion:

To improve students' understanding and performance, consider the following strategies;

- **Concept Clarification:** Provide clear explanations related to sketching of graphs for trigonometric functions and finding their intersection points. Emphasise the importance of accurately calculating and plotting the values.
- **Step-by-Step Examples:** Discuss detailed examples of graphing similar trigonometric functions, identifying intersection points, and substituting these points into equations to solve for unknowns.
- **Practice Problems:** Provide additional practice problems that involve graphing trigonometric functions and solving related equations. Encourage students to solve these problems step by step to reinforce their understanding.
- **Graphing Tools:** Use graphing calculators or software like Desmos to help students visualise the graphs and identify intersection points accurately. These tools can also assist in verifying manual calculations.
- **Online Resources:** Recommend online tutorials and resources, such as those on Coursera, which provide step-by-step explanations and exercises on graphing trigonometric functions and solving equations graphically.

Annexure A: Pedagogies Used for Teaching the SLOs

Pedagogy: Storyboard
Description: A visual pedagogy that uses a series of illustrated panels to present a narrative, encouraging creativity and critical thinking. It helps learners organise ideas, sequence events, and comprehend complex concepts through storytelling.
Example: In a Literature class, students are tasked with creating storyboards to visually retell a novel. They draw key scenes, write captions, and present their stories to the class, enhancing their reading comprehension and fostering their imagination.

Pedagogy: Cause and Effect
Description: This pedagogy explores the relationships between actions and consequences. By analysing cause-and-effect relationships, learners develop a deeper understanding of how events are interconnected and how one action can lead to various outcomes.
Example: In a History class, students study the causes and effects of the Industrial Revolution. They research and discuss how technological advancements in manufacturing led to significant societal changes, such as urbanisation and labour reform movements.

Pedagogy: Fish and Bone
Description: A method that breaks down complex topics into main ideas (the fish) and supporting details (the bones). This visual approach enhances comprehension by highlighting essential concepts and their relevant explanations.
Example: During a Biology class on human anatomy, the teacher uses the fish and bone technique to teach about the human skeletal system. Teacher presents the main components of the human skeleton (fish) and elaborates on each bone's structure and function (bones).

Pedagogy: Concept Mapping
Description: An effective way to visually represent relationships between ideas. Learners create diagrams connecting key concepts, aiding in understanding the overall structure of a subject and fostering retention.
Example: In a Psychology assignment, students use concept mapping to explore the various theories of personality. They interlink different theories, such as Freud's psychoanalysis, Jung's analytical psychology, and Bandura's social-cognitive theory, to see how they relate to each other.

Pedagogy: Audio Visual Resources
Description: Incorporating multimedia elements like videos, images, and audio into lessons. This approach caters to different learning styles, making educational content more engaging and memorable.
Example: In a General Science class, the teacher uses a documentary-style video to teach about the solar system. The video includes stunning visual animations of the planets, interviews with astronomers, and background music, enhancing students' interest and understanding of space.

Pedagogy: Think, Pair, and Share
Description: A collaborative learning technique where students ponder a question or problem individually, then discuss their thoughts in pairs or small groups before sharing with the entire class. It fosters active participation, communication skills, and diverse perspectives.
Example: In a Literature in English class, the teacher poses a thought-provoking question about a novel's moral dilemma. Students first reflect individually, then pair up to exchange their opinions, and finally participate in a lively class discussion to explore different viewpoints.

Pedagogy: Questioning Technique (Socratic Approach)
Description: Based on Socratic dialogue, this method stimulates critical thinking by posing thought-provoking questions. It encourages learners to explore ideas, justify their reasoning, and discover knowledge through a process of inquiry.
Example: In an Ethics class, the instructor uses the Socratic approach to lead a discussion on the meaning of justice. By asking a series of probing questions, the students engage in a deeper exploration of ethical principles and societal values.

Pedagogy: Practical Demonstration
Description: A hands-on approach where learners observe real-life applications of theories or skills. Practical demonstrations enhance comprehension, skill acquisition, and problem-solving abilities by bridging theoretical concepts with real-world scenarios.
Example: In a Food and Nutrition class, the instructor demonstrates the proper technique for filleting a fish. Students observe and then practice the skill themselves, learning the practical application of knife skills and culinary precision.

(**Note:** The examples provided in this annexure serve as illustrations of various pedagogies. It is important to understand that these pedagogies are versatile and can be applied across subjects in numerous ways. Feel free to adapt and explore these techniques creatively to enhance learning outcomes in your specific context.)

Acknowledgements

The Aga Khan University Examination Board (AKU-EB) acknowledges with gratitude the invaluable contributions of all the dedicated individuals who have played a pivotal role in the development of the Mathematics HSSC-I E-Marking Notes.

We extend our sincere appreciation to Ms Shahneel Jawaid and Mr Muhammad Kasshaf Shaikh, Specialists, Mathematics at AKU-EB, for taking subject lead during the entire process of e-marking.

We particularly thank to Ms Noor ul ain Jafri, MSB, School, Karachi, and Mr Muhammad Nasir Humayun, Aga Khan Higher Secondary School, Karimabad, Karachi, for evaluating each question's performances, delineating strengths and weaknesses in candidates' responses, and highlighting instructional approaches along with recommendations for better performance.

Additionally, we express our gratitude to the esteemed team of reviewers for their constructive feedback on overall performance, better and weaker responses, and validating teaching pedagogies along with suggestions for improvement.

These contributors include:

- Mahrukh Amin Jiwa, Specialist, Curriculum Development, AKU-EB
- Zain Muluk, Manager, Examination Development, AKU-EB
- Raabia Hirani, Manager, Curriculum Development, AKU-EB
- Dr Naveed Yousuf, CEO, AKU-EB