Aga Khan University Examination Board

Notes from E-Marking Centre on HSSC-I Mathematics Examination May 2018

Introduction:

This document has been produced for the teachers and candidates of Higher Secondary School Certificate (HSSC-I) Mathematics. It contains comments on candidates' responses to the 2018 HSSC-I Examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E-Marking Notes:

This includes overall comments on students' performance on every question and *some* specific examples of students' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that require candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

Candidates need to be aware that the marks allocated to the questions are related to the answer space provided on the examination paper as a guide to the length of the required response. A longer response will not in itself lead to higher marks. Candidates need to be familiar with the command words in the SLOs which contain terms commonly used in examination questions. However, candidates should also be aware that not all questions will start with or contain one of the command words. Words such as 'how', 'why' or 'what' may also be used.

General Observations:

- In weaker responses, it was noted that the candidates failed to comprehend formulae and their applications according to given situation did not score well in the examination.
- Candidates made mistakes in the formulae of trigonometry.
- It was also noted that candidates failed to comprehend the concepts of permutation, combination and row and column operations in matrices.

Detailed Comments:

Constructed Response Questions (CRQs)

Question 1: Without using calculator, apply basic operations to separate real and imaginary parts of $\dfrac{(3+2i)^2}{1+i} \div (2+3i).$

Better responses exhibited that candidates correctly rationalised the denominator. Most of the candidates used the correct formulae of $(a+b)^2$ and a^2-b^2 that led to correct answer. In some responses, it was noted that candidates first applied the formula of $(a+b)^2$ followed by rationalisation of denominator and division process to separate the real and imaginary parts of the given complex number.

Example:

$$\frac{(3+2i)^2}{1+i} \div (2+3i$$

$$= \left(\frac{9+12i+4i^2}{1+i}\right) \times \left(\frac{1}{2+3i}\right)$$

$$= \left(\frac{5+12i}{1+i}\right) \times \left(\frac{1}{2+3i} \times \frac{2-3i}{2-3i}\right)$$

$$= \left(\frac{5+12i}{1+i} \times \frac{1-i}{1-i}\right) \times \left(\frac{2-3i}{4-9i^2}\right)$$

$$= \left(\frac{5-5i+12i-12i^2}{1-i^2}\right) \times \left(\frac{2-3i}{13}\right)$$

$$= \left(\frac{17+7i}{2}\right) \times \left(\frac{2-3i}{13}\right)$$

$$= \left(\frac{34-51i+14i-21i^2}{26}\right)$$

$$= \left(\frac{55-37i}{26}\right)$$

Real Part $= \dfrac{55}{26}$ Imaginay Part $= \dfrac{-37}{26}i$

Weaker responses showed that the candidates made mistakes in application of formulae of $(a+b)^2$ and a^2-b^2 and in basic arithmetical operations on complex numbers. Consequently, candidates were not able to separate the real and imaginary parts of the complex number.

Example 1:

Solve:

$$\frac{(3+2i)^2}{1+i} \div (2+3i)$$

$$\frac{3^2+2i^2}{1+i} \div (2+3i)$$

$$\frac{9+8-1}{1+i} \div (2+3i)$$

$$\frac{16}{1} \div (2+3i)$$

Taking square on both sides

$$\frac{16^2}{1} \div (2+3i)^2$$

$$16^2 \div (2^2 \pm 3i^2)$$

$$256 \div (8+9i)$$

$$256 \div 16$$

$$\boxed{16}$$

Example 2:

$$\frac{(3+2i)^2}{1+i} \div (2+3i).$$

$$\left(\frac{3+2i}{1+i}\right) \times \frac{1}{2+3i}.$$

$$\frac{3+2i}{(1+i)(2+3i)}$$

$$\frac{3+2i}{2+3i+2i+3i^2}$$

$$\frac{3+2i}{3i^2+5i+2}$$

$$\frac{3}{2} \quad \frac{+\;2i}{3i^2+5i}$$

Example 3:

$$\frac{(3+2i)(3+2i)}{1+i} \div (2+3i)$$

$$\frac{(3+2i)(3+2i)}{1+i} \times \frac{1}{(2+3i)}$$

$$\frac{(3+2i)}{1+i}$$

$$\frac{3+2i}{1+i} \times \frac{1-i}{1-i}$$

$$\frac{3-3i+2i-2i^2}{(1)^2-(i)^2}$$

$$\frac{3-i-2(-1)}{1-(-1)} \quad (i^2=-1)$$

$$\frac{3-i+2}{1+1}$$

$$\frac{3+2-i}{2}$$

$$\frac{5-i}{2}$$

$$\frac{5}{2}, \ -\frac{1}{2}i$$

$\frac{5}{2}$ → Real Part

$-\frac{1}{2}i$ → Imaginary part

Question 2a:

Without expansion, verify that $\begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ bc & ac & ab \end{vmatrix} = -\begin{vmatrix} a & b & b+c \\ a^2 & b^2 & b^2+c^2 \\ 1 & 1 & 2 \end{vmatrix}$.

Better responses exhibited that candidates applied the multiple methods to verify the required result. They appropriately applied the row and column operations to prove $\begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ bc & ac & ab \end{vmatrix} = -\begin{vmatrix} a & b & b+c \\ a^2 & b^2 & b^2+c^2 \\ 1 & 1 & 2 \end{vmatrix}$. The candidates multiplied and divided the column I by *a*, column II by *b* and column III by *c*. They took *abc* common from Row-III, added column-II in Column III and interchanged the Row-I and Row-II to verify the required result.

Example 1:

multipling C_1 by a, C_2 by b and C_3 by c

$\frac{1}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ abc & abc & abc \end{vmatrix}$ taking (abc) common from R_3

$\frac{abc}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ Adding, $C_3 + C_2$

$\frac{abc}{abc}\begin{vmatrix} a^2 & b^2 & c^2+b^2 \\ a & b & c+b \\ 1 & 1 & 2 \end{vmatrix}$

interchanghing R_1 and R_2

$\frac{abc}{abc}\begin{vmatrix} a & b & b+c \\ a^2 & b^2 & b^2+c^2 \\ 1 & 1 & 2 \end{vmatrix}$

$-\begin{vmatrix} a & b & b+c \\ a^2 & b^2 & b^2+c^2 \\ 1 & 1 & 2 \end{vmatrix}$ hence proved

Example 2

$L.H.S = \begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ bc & ac & ab \end{vmatrix}$

$= \frac{1}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ abc & abc & abc \end{vmatrix}$ (by multipliying and dividing abc)

$= \frac{abc}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ (common in Row3)

$= -\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix}$ (interchanging R1 and R2)

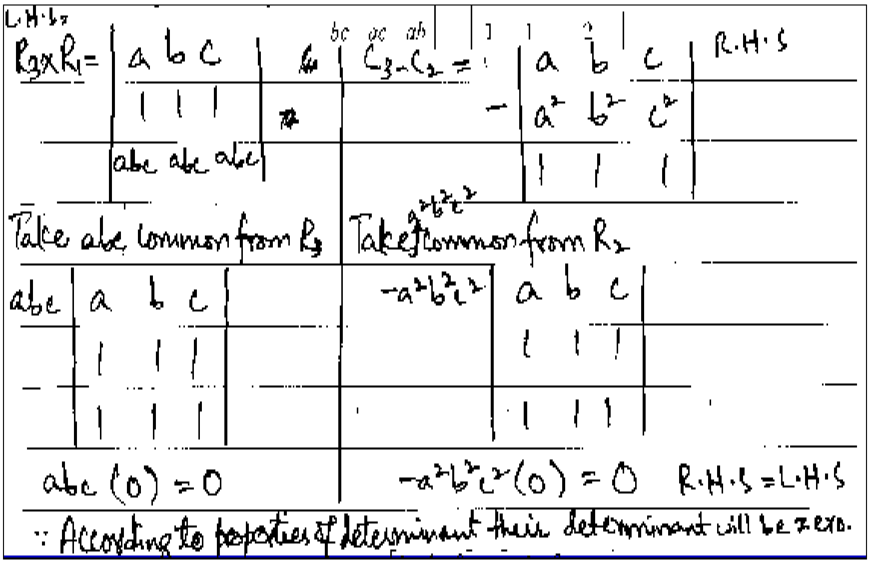
$= -\begin{vmatrix} a & b & c+b \\ a^2 & b^2 & c^2+b^2 \\ 1 & 1 & 1+1 \end{vmatrix}$ (by C_3+C_2)

$= -\begin{vmatrix} a & b & b+c \\ a^2 & b^2 & b^2+c^2 \\ 1 & 1 & 2 \end{vmatrix} = R.H.S$

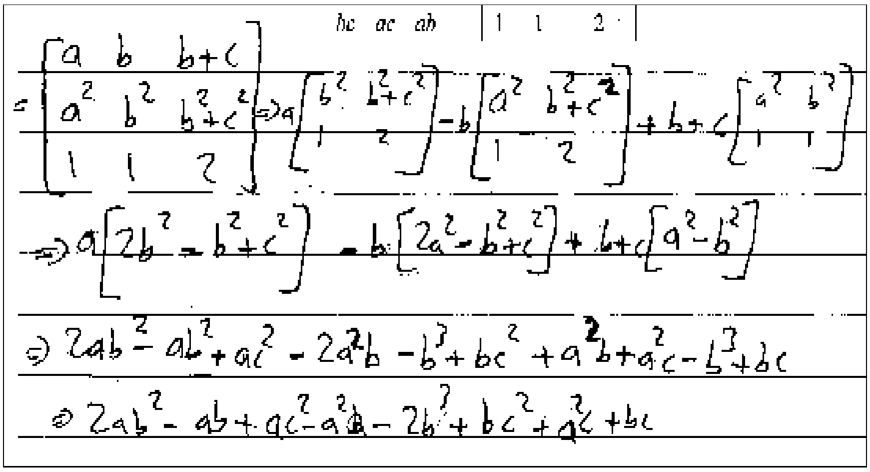
Weaker responses showed that candidates mainly failed to perform row and column operation properly. They exhibited the knowledge of row and column operations but were unable to apply them correctly to verify the result.

Although it was clearly mentioned in the question that without expansion, verify the result but few weaker responses reflected that candidates expanded the determinant.

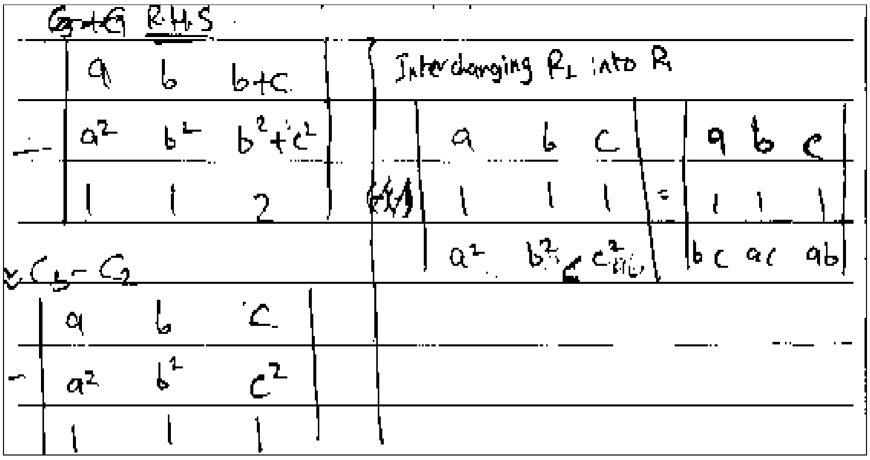
Example 1:



Example 2:



Example 3:



C_3+C_1 RHS

$\begin{vmatrix} a & b & b+c \\ a^2 & b^2 & b^2+c^2 \\ 1 & 1 & 2 \end{vmatrix}$

C_3-C_2

$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix}$

Interchanging R_2 into R_3

$(-1)\begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ a^2 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ bc & ac & ab \end{vmatrix}$

Question 2b:

The multiplicative inverse of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ is $\begin{bmatrix} -1 & 0 & a \\ 2 & 0 & -1 \\ -4 & 1 & 2 \end{bmatrix}$. Find the value of *a*.

Better responses exhibited that candidates applied the multiple methods of solution to find the value of *a*. Mostly candidates applied the fact that $A \times A^{-1} = I = A^{-1} \times A$. They multiplied the given two matrices and compared the result with identity matrix of order 3×3 to find the value of *a*. In other responses, it was noted that instead of multiplying whole matrices, the smarter candidates only multiplied the first row to the third column to get the required value.

Few other responses reported that candidates calculated the inverse of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ and compared it with $\begin{bmatrix} -1 & 0 & a \\ 2 & 0 & -1 \\ -4 & 1 & 2 \end{bmatrix}$ to get the value of *a*.

Example 1:

As $A \cdot A^{-1} = I_3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & a \\ 2 & 0 & -1 \\ -4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1+2+0 & 0+0+0 & a-1+0 \\ 0+4-4 & 0+0+1 & 0-2+2 \\ -2+2+0 & 0+0+0 & 2a-1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & a-1 \\ 0 & 1 & 0 \\ 0 & 0 & 2a-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From above we get

$a-1=0$ OR $2a-1=1$

$a=1$ $\quad$ $2a=2$

$a=1$

Example 2:

(4 Marks)

$|A| = 1\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = 1(0-1) - 1(0-2) = 1$

$$\begin{bmatrix} -1 & 2 & -4 \\ 0 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}^t$$

$A_{11} = 1\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 0-1 = -1$

$A_{12} = -1\begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = -1(0-2) = +2$

$A_{13} = +1\begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = 0-4 = -4$

$A_{21} = -1\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = -1(0-0) = 0$

$A_{22} = +1\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} = +1(0) = 0$

$A_{23} = -1\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1(1-2) = +1$

$A_{31} = +1\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = +1(1-0) = 1$

$A_{32} = -1\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$

$A_{33} = +1\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 2$

$$\begin{bmatrix} -1 & 0 & 1 \\ 2 & 0 & -1 \\ -4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & a \\ 2 & 0 & -1 \\ -4 & 1 & 2 \end{bmatrix}$$

by comparing $a=1$

Example 3:

Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} -1 & 0 & a \\ 2 & 0 & -1 \\ -4 & 1 & 2 \end{bmatrix}$

Using $AA^{-1} = I$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & a \\ 2 & 0 & -1 \\ -4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow 1(a) + 1(-1) + 0(2) = 0$

$a - 1 = 0$

$\therefore \boxed{a = 1}$

Weaker responses reflected lack of understanding of property $A \times A^{-1} = I = A^{-1} \times A$. Few candidates used correct property but made mistakes in multiplication and addition of corresponding elements of a row or a column and failed to find the value of *a*. There were few responses showed that candidates inappropriately applied the concept of determinant to find the required value.

Example 1:

$A^{-1} = \frac{Adj}{|A|}$

$|A| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{vmatrix}$

$1(0-1) - 1(0-2) + 0(4-1)$

$= -1 + 2$

$= 1$

$Adj = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 2 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & a \\ 2 & 0 & -1 \\ -4 & 1 & 2 \end{bmatrix}$

$\boxed{a = 2}$

Example 2:

$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & a \\ 2 & 0 & -1 \\ -4 & 1 & 2 \end{bmatrix}$$ (4 M

$$A^{-1} = \frac{1}{|A|} \text{Adj of } A$$

$$|A| = 1$$

$$a_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$a_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = 0$$

$$a_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = -4$$

here a is a_{13} so value of a comes

$$a = -4$$

Question 3:

Find the first term (a_1), common difference (d) and the n^{th} term of an arithmetic sequence which satisfies conditions $4 \times a_6 = a_{26}$ and $a_{15} = 47$.

This was generally a well-attempted question.

Better responses exhibited that candidates applied correct formula of general term of an arithmetic sequence and successfully translated the given conditions to find the values of a and d by solving two equations, i.e. $4 \times (a + 5d) = a + 25d$ and $a + 14d = 47$. Finally, candidates were able to find the n^{th} term of the arithmetic sequence.

Example 1:

$a_1 = ??$

$d = ??$

$n = ??$

$4 \times a_6 = a_{26}$ $\quad a_{15} = 47$

$4 \times (a + 5d) = a + 25d$

$4(a + 5d) = a + 25d$

$4a + 20d = a + 25d$

$4a - a = 25d - 20d$

$3a = 5d$

$a = \frac{5}{3}d \rightarrow (2)$

$a_{15} = a + 14d \rightarrow (1)$

$47 = \frac{5}{3}d + 14d$

$47 = \frac{5d + 42d}{3}$

$141 = 47d$

$d = \frac{141}{47}$

$\boxed{d = 3}$

Putting d in eq 2

$a = \frac{5}{3}(3)$

$\boxed{a = 5}$

$a_n = a_1 + (n-1)d$

$= 5 + (n-1)3$

$= 5 + 3n - 3$

$\boxed{a_n = 2 + 3n}$

Example 2:

$\Rightarrow a_{15} = a_1 + (15-1)d$

$\boxed{47 = a_1 + 14d}$ — ①

$\Rightarrow a_6 = a_1 + (6-1)d$

$\boxed{a_6 = a_1 + 5d}$

$\Rightarrow a_{26} = a_1 + (26-1)d$

$\boxed{a_{26} = a_1 + 25d}$

Putting values of 'a_6' and a_{26} in

$4 \times a_6 = a_{26}$

$4(a_1+5d) = a_1 + 25d$

$4a_1 + 20d = a_1 + 25d$

$4a_1 - a_1 + 20d - 25d = 0$

$\boxed{3a_1 - 5d = 0}$ — ②

By eq ① and ②

$a_1 + 14d = 47$

$3a_1 - 5d = 0$

$\boxed{a_1 = 5}$, $\boxed{d = 3}$

$\Rightarrow a_n = a_1 + (n-1)d$

$= 5 + (n-1)3$

$a_n = 5 + 3n - 3 = \boxed{2+3n}$

verify:

$4 \times a_6 = a_{26}$

$4(a_1+5d) = a_1 + 25d$

$4(5+5(3)) = 5 + 25(3)$

$4(5+15) = 5 + 75$

$4(20) = 80$

$80 = 80$ proved.

Weaker responses reflected that candidates either failed to write the correct formula or made mistakes in translation of the given conditions and therefore, failed to find the first term and common difference of the required arithmetic sequence.

Example 1:

$4 \times a_6 = a_{26}$ and $a_{15} = 47$

$a_{15} = 47$

$47 = a + 14d \longrightarrow$ (i)

$4 \times a_6 = a_{26}$ then

$\boxed{d = 2} \Longrightarrow$ common difference

Now finding first term

if $a_5 = 47$ and $4 \times a_6 = a_{26}$ then

we get

$47 - 46 = a$

$\boxed{a = 1} \Longrightarrow$ first term

n^{th} term will be $a_n = a + (n-1)d$

Example 2:

$a_1 = ?$

$d = ?$

Answer

$4 \times a_6 = a_{26}$ & $a_{15} = 47$.

$4 \times a_1 + 5d = a_1 + 25d$ — ①

$4 \times a_1 + 5d - a_1 - 25d$.

$4x - 20d \Rightarrow -80 = d$.

$\Rightarrow 4 \times a_1 + 5(-80) = a_1 + 25(-80)$.

$\Rightarrow 4 \times a_1 + (-400) = a_1 + 25(-2000)$.

$\Rightarrow 4 \times a_1 - 400 = a_1 - 2000$

$\Rightarrow a_1 - 400 = \frac{a_1 - 2000}{4} \Rightarrow a_1 - 500$.

$\Rightarrow 2a_1 = 100 \Rightarrow a_1 = \frac{100}{2} \Rightarrow 50$.

So, $a_1 = 50$, $d = -80$.

Example 3:

$a_1 = ?$ $d_1 = ?$ n^{th} term of sequence

$4 \times a_6 = a_{26}$

$a_{26} = 1 + a_{25}$

$4 \times a_{26} = 1 + a_{25}$

we know $a + (n-1)d$

$a + (15-1)d = 47$

$a + 14d = 47$

$a + (n-1)d = 47$

$a + (n-1)d = 4 \times a_6$

$\frac{47}{4} = a_6$

$11.75 = a_6$

$a + (n-1)d = a_6$

$a + (6-1)d = 11.75$

$\cancel{a + (n-1)d}$ $a + 5d = 11.75$

$\cancel{a + (\ldots)d}$ $a_{15} = 47$, $a_6 = 11.75$

$a +$ ⟸ 11.75, 9 terms, 47

$d = 4$

Question 4:

This question offered a choice between part **a** and **b**. Most candidates performed well in this question. Majority of the students attempted part **b** and avoided the word problem given in part **a.**

Question 4a:
Over a period of five years in a certain city, the number of road accidents increased by 20% per year. If there were 10,240 accidents in 2010 and the road accidents follow geometric sequence, then how many accidents occurred in 2015?
(**Note:** The answer should be the nearest whole number.)

Better responses indicated that the candidates understood the question well and correctly found the common ratio i.e. $r = 1 + \frac{20}{100} = 1.2$ and applied the formula of $a_n = ar^{n-1}$ to find the number of accidents occurred in 2015. In few other responses, it is noted that the candidates applied an alternate way as cited in example **2.**

Example 1:

2010, 2011, 2012, 2013, 2014, 2015

$n = 6$

$a_1 = 10240$

$a_6 = ?$

$r = \left(1 + \frac{20}{100}\right) = (1 + 0.2) = 1.2$

$a_n = a_1 r^{n-1}$

$a_6 = (10240)(1.2)^{6-1}$

$= 10240(1.2)^5$

$= 10240 \times 2.48832$

$= 25480.397$

$a_6 \approx 25480$ accidents occured in 2015.

Example 2:

a) Each year accidents increased by 20%

so, accidents × 20%.

In 2010 = $\boxed{10240}$

2011 = 10240 × 20% => 2048 + 10240 = $\boxed{12288}$

2012 = 12288 × 20% => 2457.6 + 12288 = $\boxed{14745.6}$

2013 = 14745.6 × 20% => 2949.12 + 14745.6 = $\boxed{17694.72}$

2014 = 17694.72 × 20% => 3538.944 + 17694.72 = $\boxed{21233.664}$

2015 = 21233.664 × 20% => 4246.7328 + 21233.664 = $\boxed{25480.3968}$

Geometric Sequence:

10240, 12288, 14745.6, 17694.72, 21233.664, 25480.3968, ...

Accidents occured in 2015 = 25480

Weaker responses reflected various types of mistakes to convert the given word problem into the mathematical model. The candidates were unable to identify the correct common ratio and in some cases, they were unable to identify the correct formula. Other weaker responses revealed that candidates wrote the correct formula but failed to identify correct value of *a* and *r*, hence they failed to fulfill the requirement of the question.

Example 1:

10,240 accidents in 2010

20% increase per year

totall ~~add~~ accidents in 2015?

$T_n = a, r^{n-1}$

$T_n = 10{,}240\ (20)^{5-1}$

$T_n = 10{,}240 \times 160000$

$T_n = 1638400000.$

Example 2:

a: five years = 20% , if 10,240 accidents in 2010

then in 2015 = ? accidents

∴ 20% = 2048 accidents

∴ 10,240 increased till 2015

∴ total accidents occurred in

2015 is = $\boxed{20,480}$ Ans.

Question 4b:

Find TWO harmonic means between $\frac{1}{17}$ and $\frac{1}{32}$.

Better responses correctly converted the given terms into their associated terms in arithmetic progression (AP) and then applied the formula $a_n = a + (n-1)d$ aptly to find the value of first term and common difference. After that they found the two terms in AP and converted them into the harmonic progression by taking reciprocal of the terms.

Example:

If $\frac{1}{17}$ and $\frac{1}{32}$ are in harmonic progression	Then $A_1 = a_1 + d$
then 17 and 32 are in arithmetic progression.	$= 17 + 5$
$17, A_1, A_2, 32.$	$A_1 = 22.$
$a_1 = 17,\ a_4 = 32.$	$A_2 = a_1 + 2d$
$a_4 = a_1 + 3d.$	$= 17 + 10$
$32 = 17 + 3d$	$A_2 = 27$
$32 - 17 = 3d$	write the arithmetic means in harmonic ~~forme~~ we get =
$\frac{15}{3} = d$	$\frac{1}{22}, \frac{1}{27}$
$d = 5.$	∴ The two harmonic means are found to be $\frac{1}{22}$ and $\frac{1}{27}$.

Weaker responses suggested that candidates were failed to select or apply the correct formula and consequently, were failed to find the required harmonic means. In other weaker responses, it was noted that they took the reciprocal of the formula of arithmetic progression, i.e. $\dfrac{1}{a_n = a + (n-1)d}$ instead of calculating associated terms in arithmetic progression and then took their reciprocals.

Example 1:

$\frac{1}{17}, \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{32}$

$\cancel{a_1 = \frac{1}{17} + \left(\frac{1}{n-1}\right)\left(\frac{1}{d}\right)}$

$a_1 = 17 + (n-1)(d)$

$a_1 = 17 + (1-1)d$

$a_1 = 17 + (0)d$

$a_1 = 17.$

$a_2 = 17 + (2-1)d$

$a_2 = 17 + d$

$a_3 = 17 + (3-1)d$

$a_3 = 17 + 2d$

$\frac{1}{17} + \frac{1}{17+d} + \frac{1}{17+2d} + \frac{1}{32}$

Example 2:

$H.M. = \frac{4ab}{a+b}$

$1/17, H_1, H_2, 1/32$

$a = 17$

$d = \cancel{H_1 - 1/17}\ H_1 - 1/17$

$32 = 17 + 3(H_1 - 1/17)$

$15 = 3H_1 - 3/17$

$H_1 = 86/17$

$\boxed{H_1 = 17/86}$

$\cancel{H_n}\ d = 5$

$H_2 = 17 + 2(5)$

$H_2 = 27$

$\boxed{H_2 = 1/27}$

$\dfrac{4\left(\frac{17}{86}\right)\left(\frac{1}{27}\right)}{\frac{17}{86} + \frac{1}{27}} = \dfrac{\frac{34}{1161}}{\frac{545}{2322}} = \frac{68}{545}$ Ans.

Example 3:

$$a_1 = \frac{1}{17} \;,\; a_4 = \frac{1}{32}$$

$$a_n = \frac{1}{a_1 + (n-1)d}$$

$$\frac{1}{32} = \frac{1}{17 + (4-1)d}$$

$$\frac{1}{32} = \frac{1}{17+3d}$$

$$17 + 3d = 32$$

$$3d = 32 - 17$$

$$d = \frac{15}{3}$$

Question 5a: i. How many different words can be formed with the letters of word **BREAD** if I. all letters are used? II. all letters are used and B and R always come together? III. only three letters are used?

This question offered a choice between part **a** and part **b**.

Better responses exhibited that candidates correctly applied techniques or formula to find the number of different words that can be formed using the given conditions. They showed they were conceptually well versed with the concept of counting techniques.

Example 1:

I. If all letters are used: $^nP_r = {}^nP_n$

$\Rightarrow \frac{5!}{(5-5)!} \Rightarrow 120$

II. BR together $\Rightarrow$

BR _ _ _ 3!

_ BR _ _ 3!

_ _ BR _ 3! $\rightarrow 3! \times 4 = 4! = 24$

_ _ _ BR 3!

Similarly RB _ _ _ $4! = 24$

Total BR together $= 24 + 24 = 48$

III $^5P_3 = \frac{5!}{(5-3)!} = 5 \times 4 \times 3 = 60$

Example 2:

III. only three letters are used? (1 Mark)

I) all letters are used then

$n = 5$

$r = 5$

$^nP_r = {}^5P_5 = \frac{5!}{(5-5)!} = 5! = 5.4.3.2.1 = 120$

II) If B and R always come together then consider B and R as them. B and R does come together in following ways

BR _ _ _, then permutation $= {}^4P_4 = 4! = 24$ ways

RB _ _ _, then Permutation $= {}^4P_4 = 24$ ways

Total permutation $= 24 + 24$ ways $= 48$.

III) Only three letter are used then

$n = 5$

$r = 3$

$^nP_r = {}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5.4.3 = 60$

Weaker responses reflected lack of concept of counting techniques and therefore, failed to find the number of words under given conditions. They used wrong formula or techniques and hence were unable to fulfill the requirement of the question.

Example 1:

$5! = 120$

$4! + 2! + 2! = 166$

$3! = 6$

Example 2:

BREAD = 5!

BREAD = 5! , RBEAD = 5!

= 5! × 5!

5 | 3 | 2 | = 5 × 3 × 2 = 30

Example 3:

I: all letters are used

then BREAD, 5! = 5·4·3·2·1

120 different words can be formed.

II: if all letters are used and B and R always

come together then 96 words can be

formed in BR always come together.

III: only 3 letters are used then 3! = 6 words can be formed.

Question 5a:

ii. A basket contains 6 white balls and 4 black balls. If all the balls are identical, then how many selections of 4 balls can be made such that at least 3 of them are white balls?

Better responses exhibited good understanding of concept of combination and were able to clearly differentiate between combination and permutation. Candidates understood the question well and were able to find the correct number of selections of balls as per given condition in the question. The better responses also indicated that candidates were clear about the usage of the term “at least”.

Example 1:

$${}^6C_4 \times {}^4C_0 + {}^6C_3 \times {}^4C_1$$

$$\frac{6!}{4!(6-4)!} \times \frac{4!}{0!(4-0)!} + \frac{6!}{3!(6-3)!} \times \frac{4!}{1!(4-1)!}$$

$$\frac{6!}{(4!)(2)!} \times \frac{4!}{1 \times 4!} + \frac{6!}{3!(3)!} \times \frac{4!}{1 \times 3!}$$

$$15 \times 1 + 20 \times 4$$

$$15+80 = \boxed{95} \text{ Ans}$$

Example 2:

Total balls: 10 , white balls: 6 , black balls: 4 , $r = 1$

Condition: atleast 3 are white.

Possible Combinations $= 6C_4 \times 4C_0 + 6C_3 \times 4C_1$

$= 15 + 80$

$= 95$

Hence 95 selections of 4 balls can be made if atleast 3 are white.

Weaker responses reflected that students were unable to find the possible selections of balls as requirement of the question. The candidates used wrong formula or were unable to comprehend the term '**at least**' and failed to solve the question correctly. It was generally not a well attempted question, which is indicative of the fact that counting techniques need more attention of teachers and students.

Example 1:

$${}^{10}C_4 = 210$$

$${}^6C_3 + {}^4C_1 = 20 + 4 = 24$$

$${}^6C_4 + {}^4C_0 = 15 + 1 = 16$$

$24 + 6 = 30$ selections of 4 balls.

Example 2:

3 white balls out 6 and 1 black ball out of ? is

$= 6C_3 \times 4C_1$

$\boxed{= 80 \text{ selections}}$

Example 3:

For other black ball $= 4C_1 = 4$

For white ball $= 6C_3 = 20$

$nC_r = 4C_1 \times 4C_3 = 20 \times 4 = 80$

There are 80 selection of 4 ball in which ~~[illegible]~~ least 3 ball come ~~[illegible]~~.

Question 5b:

Two fair dice are rolled simultaneously and score on both dice is added together.

i. Complete the given table to show all possible outcomes.

Die-1 / Die-2	1	2	3	4	5	6
1	2					7
2		4			7	
3			6	7		
4			7	8		
5		7			10	
6	7					12

ii.	Find the probability of obtaining the score of
	I. exactly 10.
	II. at least 10.
	III. at most 10.
	IV. other than 10.

Better responses exhibited good understanding of probability theory. Candidates understood the question well and were able to complete the given table. They correctly used the given table to find the probability of the given situations in the next part of the question. The better responses also indicated that candidates were clear about the terms **'exactly'**, '**at least'**, '**at most'** and **other than** in the context of probability to find the probabilities of the given events.

Example 1:

Die-2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

ii. Find the probability of obtaining the score of

I. exactly 10.

II. at least 10.

III. at most 10.

IV. other than 10.

I) $\frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12} = 0.083$

II) $\frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6} = 0.166$

III) $\frac{n(A)}{n(S)} = \frac{\cancel{33}}{\cancel{36}} = \frac{11}{12} = 0.91$

IV) $\frac{n(A)}{n(S)} = \frac{\cancel{33}}{\cancel{36}} = \frac{11}{12} = 0.91$

Weaker responses reflected that candidates were able to fill the table but failed to find the probabilities asked in the question. Weaker responses also exhibited that candidates were not clear about the difference of **'at least'**, **'at most'** and **'exactly'** and therefore, failed to find the probabilities of the given events. .

Example 1:

Die 2 \ Die 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

ii. Find the probability of obtaining the score of

I. exactly 10.

II. at least 10.

III. at most 10.

IV. other than 10.

Example 2:

Die-2 \ Die-1st	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

ii. Find the probability of obtaining the score of

I. exactly 10. (1)

II. at least 10. (1)

III. at most 10. (1)

IV. other than 10. (1)

(i) exactly 10 = $\frac{3}{48}$ = $\boxed{\frac{1}{18}}$ = $\boxed{0.125}$

(ii) at least 10 = $\frac{3}{48}$ = $\boxed{\frac{1}{16}}$ = $\boxed{0.0625}$

(iii) at most 10 = $\frac{3}{48} \times 10$ = $\boxed{\frac{5}{6}}$ = $\boxed{0.625}$

(iv) other than 10 = $\frac{45}{48}$ = $\frac{15}{16}$ = 0.93

Question 6:

Prove by mathematical induction that for all positive integral values of n, $7^n - 1$ is divisible by 6.

Better responses indicated that candidates systematically followed the steps of mathematical induction. They proved that the given statement is true for n = 1 by substituting n = 1 and to prove that the statement is true for n = k+1, they considered $7^{k+1} - 1$ and tactfully converted it into $7^{k+1} - 7 + 7 - 1 = 7(7^k - 1) + 6$. Finally, they proved the required result for all positive integral values of n.

Example 1:

① for n=1

$7^1 - 1 = 6$ proved

② for n=K

$= 7^K - 1$ — ①

③ let asume n=K+1

$= 7^{K+1} - 1$

$= 7^K \cdot 7 - 1$

$= 7^K \cdot 7 - 7 - 1 + 7$ (adding and subtracting 7 on both sides)

$= 7(7^K - 1) + 6$

which is divisible by 6

Hence proved.

Example 2:

A/c to fundamental principle of inductions

(i) Put n=1 in $7^n - 1$; $7^1 - 1 = 6$

Since 6 is divisible by 6 (Q=1), hence 1st condition is satisfied

(ii) Now put n=k in $7^n - 1$; $7^k - 1$

We assume that $7^k - 1$ is ~~true~~ divisible by 6

Now to prove for n=k+1;

Put n=k+1 in $7^n - 1$; $7^{k+1} - 1$

$7^k \cdot 7 - 1$

$7^k \cdot 7 - 7 + 6$ {∵ Breaking -1 into -7+6}

$7(7^k - 1) + 6$

According to our assumption $7^k - 1$ is ~~true~~ divisible by 6, ∴ $7(7^k - 1)$ is divisible by 6

Also, 6 is also divisible by 6 (Q=1)

∴ Second condition is also satisfied.

Hence, $7^n - 1$ is divisible by 6 for all positive integral values of n if n=k is true.

Weaker responses displayed that candidates were able to prove the result for $n = 1$, but failed to considered the correct term to prove the truth of the statement for $n=K+1$ and consequently unable to prove the given statement by using principle of mathematical induction.

Example 1:

$n = 1$

$7^1 - 1 = 6$

$6 = 6$

$n = K + 1$

$7^K - 1 = 6^{\#}$

$7^{K+1 \cdot K} - 1 = 6$

Take common K^2

$= K^2(7^1 - 1) = 6$

$= \quad 6 = 6$ proved.

Example 2:

$n = 1$

$7^1 - 1$ is divisby by 6

$6 \div 6$ Yes ✓

$n = k$

$7^k - 1$ is divisible by 6 (assume)

~~4~~ $n = k+1$

$7^{k+k+1} - 1 \quad \div 6$

$7^k . 7^k . 7 - 1$

~~If~~ As $7^k - 1$ is divisible by 6

so any number multiplied by $(7^k - 1)$ it will also be divisible by 6.

$n = 2$; ~~48~~ $49 - 1 = 48$ divisibly by 6 ✓

$n = 3$; $343 - 1 =$

Question 7

This question offered a choice between part **a** and **b**. Candidates chose to attempt part **b** more than part **a**.

Question 7a:
Find the solution set of the equation $x^4 - 7x^3 + 12x^2 - 7x + 1 = 0$.

Better responses showed that candidates re-arranged the given equation $x^4 - 7x^3 + 12x^2 - 7x + 1 = 0$ as $x^4 + 1 - 7x^3 - 7x + 12x^2 = 0$ and then, divided both sides by x^2 to get $x^2 + \frac{1}{x^2} - 7\left(x + \frac{1}{x}\right) + 12 = 0$. They made right supposition to reduce the given quartic equation into quadratic equation. To solve the newly obtained quadratic equation, mostly candidates applied the method of breaking of middle term to get the two values of *y*. Consequently, they solved the resulting two quadratic equations by using quadratic formula to get the values of variable *x* and wrote the required solution set correctly.

Example 1:

$$\frac{x^4}{x^2} - \frac{7x^3}{x^2} + \frac{12x^2}{x^2} - \frac{7x}{x^2} + \frac{1}{x^2} = 0$$

$$x^2 - 7x + 12 - \frac{7}{x} + \frac{1}{x^2} = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 7\left(x + \frac{1}{x}\right) + 12 = 0$$

let $y = x + \frac{1}{x}$

$$y^2 = x^2 + 2 + \frac{1}{x^2}$$

$$y^2 - 2 = x^2 + 1/x^2$$

$$y^2 - 2 - 7y + 12 = 0$$

$$y^2 - 7y + 10 = 0$$

$$y^2 - 5y - 2y + 10 = 0$$

$$y(y-5) - 2(y-5) = 0$$

$$(y-5)(y-2) = 0$$

$y = 2,\ y = 5$ S.S. $\left\{1, \frac{5 \pm \sqrt{21}}{2}\right\}$

$x + \frac{1}{x} = 2$, $x + \frac{1}{x} = 5$

$x^2 + 1 = 2x$ $\quad$ $x^2 + 1 = 5x$

$x^2 - 2x + 1 = 0$ $\quad$ $x^2 - 5x + 1 = 0$

$(x-1)^2 = 0$ $\quad$ $x = \frac{5 \pm \sqrt{25-4}}{2}$

$x = 1$ $\quad$ $x = \frac{5 \pm \sqrt{21}}{2}$

Example 2:

$x^4 - 7x^3 + 12x^2 - 7x + 1 = 0$

Divide by x^2

$\frac{x^4}{x^2} - \frac{7x^3}{x^2} + \frac{12x^2}{x^2} - \frac{7x}{x^2} + \frac{1}{x^2} = 0$

$x^2 - 7x + 12 - \frac{7}{x} + \frac{1}{x^2} = 0$

$x^2 + \frac{1}{x^2} - 7x - \frac{7}{x} + 12 = 0 \rightarrow$ eq(A)

$x^2 + \frac{1}{x^2} - 7\left(x + \frac{1}{x}\right) + 12 = 0$

let $x + \frac{1}{x} = y \rightarrow$ eq(B)

S.B.S

$\left(x + \frac{1}{x}\right)^2 = y^2$

$x^2 + 2\cancel{x}\frac{1}{\cancel{x}} + \frac{1}{x^2} = y^2$

$x^2 + \frac{1}{x^2} + 2 = y^2$

$x^2 + \frac{1}{x^2} = y^2 - 2$

Put these values in eq(A)

$(y^2 - 2) - 7(y) + 12 = 0$

$y^2 - 2 - 7y + 12 = 0$

$y^2 - 7y + 10 = 0$

$y^2 - 5y - 2y + 10 = 0$

$y(y-5) - 2(y-5) = 0$

$(y-2)(y-5) = 0$

$\boxed{y=2}$ $\boxed{y=5}$

Put the value of y in eq B

For $y = 2$

$x + \frac{1}{x} = 2 \Rightarrow x^2 + 1 = 2x$

$x^2 - 2x + 1 = 0 \Rightarrow x^2 - 1x - 1x + 1 = 0$

$x(x-1) - 1(x-1) = 0 \Rightarrow (x-1)(x-1) =$

$\boxed{x=1}$ $\boxed{x=1}$

For $y = 5 \Rightarrow x + \frac{1}{x} = 5$

$x^2 + 1 = 5x \Rightarrow x^2 - 5x + 1 = 0$

$a = 1$ $b = -5$ $c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-(-5) \pm \sqrt{5^2 - 4(1)}}{2(1)}$

$\frac{5 \pm \sqrt{25 - 4}}{2} \Rightarrow \frac{5 \pm \sqrt{21}}{2}$

$\boxed{x = \frac{5 + \sqrt{21}}{2}}, \boxed{\frac{5 - \sqrt{21}}{2}}$

PLEASE TURN OVER THE PAGE S.S $\left\{1, 1, \frac{5 \pm \sqrt{21}}{2}\right\}$

Weaker responses reflected that candidates were clueless to convert given quartic equation into quadratic form to solve it further. This was the most common mistake noted in the weaker responses. Other common mistakes were of re-arrangement of terms of the given equation, mistakes in taking common and simple arithmetic errors which resulted in the loss of marks.

Example 1:

$x^4 - 7x^3 + 12x^2 - 7x + 1 = 0$

put $x = 1$

$(1)^4 - 7(1)^3 + 12(1)^2 - 7(1) + 1 = 0$

$1 - 7 + 12 - 7 + 1 = 0$

$1 + 5 - 7 + 1 = 0$

$0 = 0$

Solution set $x = 1$

$$
\begin{array}{c|ccccc}
1 & 1 & -7 & 12 & -7 & +1 \\
 & & 1 & 6 & 18 & 11 \\
\hline
 & 1 & 6 & 18 & 11 & \boxed{12}
\end{array}
$$

$x^3 + 6x^2 + 18x + 11$

Example 2:

dividing by x^2

$$x^2 - 7x + 12 - \frac{7}{x} + \frac{1}{x^2} = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 7\left(x - \frac{1}{x}\right) + 12 = 0$$

Suppose $x - 1/x = y$, $(x - 1/x)^2 = x^2 - 2 + 1/x^2 \Rightarrow$ $x^2 - 1/x^2 = y^2 + 2$ | substituting in eq

$$y^2 + 2 - 7y + 12 = 0$$

$y^2 - 7y + 14 = 0$ $\boxed{a = 1 \quad b = -7 \quad c = 14}$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{7 \pm \sqrt{49 - 56}}{2}$$

$$\Rightarrow y = \frac{7 + \sqrt{7}\,i}{2} , \frac{7 - \sqrt{7}\,i}{2}$$

$x - 1/x = \frac{7 + \sqrt{7}\,i}{2}$ | $x - 1/x = \frac{7 - \sqrt{7}\,i}{2}$

$x^2 - \left(\frac{7 + \sqrt{7}\,i}{2}\right)x - 1 = 0$ | $x^2 - \left(\frac{7 - \sqrt{7}\,i}{2}\right) - 1 = 0$

Example 3:

b) $x^4 - 7x^3 + 12x^2 - 7x + 1 = 0$

divide whole eq by x^2

$x^2 - 7x + 12 - \frac{7}{x} + \frac{1}{x^2} = 0$

$x^2 + \frac{1}{x^2} - 7x + \frac{7}{x} + 12 = 0$

$x^2 + \frac{1}{x^2} - 7\left(x + \frac{1}{x}\right) + 12 = 0$

$x + \frac{1}{x} = y$

$y^2 - 7y + 12 = 0$

$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-7) \pm \sqrt{-7^2 - 4(1)(12)}}{2(1)}$

$y = \cancel{4, 3}\ \frac{7 \pm \sqrt{-13}}{2}$

$y = \frac{7 + \sqrt{+3}}{2}$, $x = \frac{7 - \sqrt{+3}}{2}$

$= \frac{7}{2} + \frac{\sqrt{3}}{2}i$, $\frac{7}{2} - \frac{\sqrt{3}}{2}i$

$= \cancel{4}$, $\cancel{3}$

$y = \frac{7}{2} + \frac{\sqrt{3}}{2}i$, $y = 3$

$x + \frac{1}{x} = \frac{7}{2} + \frac{\sqrt{3}}{2}i$, $x + \frac{1}{x} = \frac{7}{2} - \frac{\sqrt{3}}{2}i$

$x^2 + 1 = 3.5x + 0.86ix$, $x^2 + 1 = 3.5x - \sqrt{3}i$

$x^2 - (3.5 + 0.86i)x + 1 = 0$, $x^2 - (3.5 - 0.86i)x + 1 = 0$

$= \frac{-b(-4) \pm \sqrt{-4^2 - 4(1)(1)}}{2(1)}$, $= \frac{-(-3) \pm \sqrt{-3^2 - 4(1)(1)}}{2a}$

$= \frac{4 \pm \sqrt{12}}{2}$, $= \frac{3 \pm \sqrt{5}}{2}$

$x = \frac{-7 \pm \sqrt{33} + 0.86i}{4}$, $2 - \sqrt{3}$, $\frac{3 + \sqrt{5}}{2}$, $\frac{3 - \sqrt{5}}{2}$

$\left\{ \cancel{2 + \sqrt{3}}, \cancel{2 - \sqrt{3}} \right\} \left\{ \frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2} \right\}$

$\left\{ \frac{-7 + \sqrt{33} + 0.86i}{4}, \frac{7 - \sqrt{33} - 0.86i}{4} \right\}$

Question 7b:
i. Solve the following system of equations.
$x^2 + y^2 + 2y = 16$ $3x + y = 6$
ii. Prove that $(\omega^7 + 1)^2 = \omega$.

Better responses of part **i** showed that candidates found the value of $y = 6 - 3x$ and substituted the value in the given quadratic equation and followed all necessary steps to get the required solution set, i.e. $\left\{(1,\ 3), \left(\frac{16}{5}, -\frac{18}{5}\right)\right\}$. The candidates showed their understanding and skills of solving algebraic equations.

In part **ii**, better responses reflected that candidates were well aware with the properties of cube roots of unity. First, they wrote $\omega^7 = \omega^6 \times \omega = \left(\omega^3\right)^2 \times \omega$ and then, they applied the fact that $\omega^3 = 1$ to reach the result $(\omega + 1)^2$. They further applied the formula of $(a+b)^2$ and the property $\omega^2 + \omega + 1 = 0$ to prove the required result.

Example 1:

b) (i) $x^2+y^2+2y=16$ — ①

$3x+y=6$ — ②

$y=6-3x$ — ③

put ③ into ①

$x^2+(6-3x)^2+2(6-3x)=16$

$x^2+36-36x+9x^2+12-6x-16=0$

$10x^2-42x+32=0$

$(5x-16)(x-1)=0$

$\boxed{x=\frac{16}{5}}$, $\boxed{x=1}$

S.S (x,y): $\left\{\left(\frac{16}{5}, \frac{-18}{5}\right)(1,3)\right\}$ Ans

$y=6-3\left(\frac{16}{5}\right)$, $y=6-3(1)$

$\boxed{y=\frac{-18}{5}}$ $\boxed{y=3}$

ii) $(w^7+1)^2$

$[(w^3)^2 . w+1]^2$

$[w+1]^2$ $\therefore w^3=1$

w^2+2w+1 $\therefore w^2+w+1=0$

$w^2=-1-w$

$\cancel{1}-w+2w\cancel{+1}$

$\boxed{w=w}$ Proved!

Example 2:

i- $x^2+y^2+2y=16 \rightarrow (1)$

$3x+y=6$

$y=6-3x \rightarrow (2)$

put $y=6-3x$ in eq (1)

$x^2+36-36x+9x^2+12-6x=16$

$10x^2-42x+32=0$

$2(5x-21x+16)=0$

$5x^2-5x-16x+16=0$

$5x(x-1)-16(x-1)=0$

$(5x-16)(x-1)=0$

$5x-16=0 \qquad x-1=0$

$x=16/5 \qquad x=1$

put $x=16/5$ in (2) put $x=1$ in (2)

$y=6-3\left(\frac{16}{5}\right)$

$y=6-\frac{48}{5} \qquad y=6-3(+1)$

$y=\frac{30-48}{5} \qquad y=6-3$

$y=3$

$\boxed{y=\frac{-18}{5}}$

$S.S=(x, y)\{(16/5, 1), (-18/5, 3)\}$

ii. $(\omega^7+1)^2=\omega$

$((\omega^3)^2\cdot\omega+1)^2=\omega$

$((1)^2\cdot\omega+1)^2=\omega$

$(\omega+1)^2=\omega$

$\omega^2+\omega+1+\omega=\omega$

$\therefore \omega^2+\omega+1=0$

$0+\omega=\omega$

$\omega=\omega$ proved

Weaker responses showed that in part **i**, the candidates failed to solve the question. They were able to find the value of *y* from linear equation but failed to substitute it into the given quadratic equation correctly. Consequently, they were unable to find the solution set of the given system of equations. This question was parallel in difficulty level to the questions given in the recommended textbook of the syllabus and hence, better performance was expected.

In part **ii**, weaker responses showed that candidates failed to comprehend or apply the properties of cube roots of unity. They also showed mistakes in the process of simplification to get the required results. The given examples are reflection of such mistakes.

Example 1:

$((w)^7+1)^2$	$3x+y=6$
	$y=6-3x$
$=(w^6 \cdot w+1)^2$	$x^2+y^2+2y=16$ (Putting y value in eq 2).
	$x^2+(6-3x)^2-2(6-3x)=16$
$=((w^3)^2 \cdot w+1)$	$x^2+36+9x^2-12+6x=16$
$((1)^2 \cdot w+1)$	$10x^2+24+6x=16$
$(w+1)^2$	$10x^2+8+6x=16$
w^2+2w	
$w(w+2)$	
w	

Example 2:

	$x^2+y^2+2y=16$
$(w^7+1)^2=w$	$3x+y=6$
	$3x-6=y \Rightarrow \frac{x-6}{3}=y$
$((w^3)^2 \cdot w+1)^2$	
$(1)^2 \cdot w^2+1$	$x^2+\left(\frac{6-6}{3}\right)^2+2\left(\frac{x-6}{3}\right)=16$
$1+w^2=w$	$x^2+\left(\frac{x-6}{3}\right)^2+2\left(\frac{x-6}{3}\right)=16$
$x=-3$ / $x=4$	$\frac{x^2+x^2-36}{9}+\frac{2x-12}{3}$
$3(-3)+y=6$	$3(x^2+x^2-36)+9(2x-12)$
$-9+y=6$	$\frac{(3x^2+3x^2-108)+18x-108}{27}=16$
$y=-3$	$6x^2-108+18x-108=16\times27$
$3(4)+y=6$	$6x^2-108+18x-108=432$
$12+y=6$	$6x^2-18x+216=0$
$18=y$	$x^2-3x+36=0$
	$x-x+12=0$
	$x^2+4x+3x+12$
	$x(x+3)-4(x+3)$
	$(x+3)\ (x-4)$

Question 8ai:

Find the remaining trigonometric ratios, if $\sin\theta = \dfrac{5}{13}$ and the terminal ray of θ is not in the first quadrant.

Generally it was a well attempted question. *Better responses* showed that candidates used the given trigonometric ratio to find the required trigonometric ratios. Most responses reflected that the candidates applied the correct trigonometric identities aptly such as $\cos\theta = \pm\sqrt{1-\sin^2\theta}$ or $\operatorname{cos}ec\theta = \dfrac{1}{\sin\theta}$, $\operatorname{cos}ec\theta = \pm\sqrt{1+\cot^2\theta}$ and correctly found the other trigonometric ratios with correct sign. Some candidates applied the concept of Pythagorean Theorem to find the required trigonometric ratio.

Example:

1. $\sin\theta = \frac{5}{13}$ sin +ve → Quadrant I, II

Quadrant II

$\cos\theta = \pm\sqrt{1-\sin^2\theta}$ cos = −ve, tan = −ve

sec = −ve, cot = −ve

$\cos\theta = \pm\sqrt{1-\left(\frac{5}{13}\right)^2}$ cosec = +ve, sin = +ve

$\cos\theta = \pm\frac{12}{13}$

2. $\cos\theta = -\frac{12}{13}$ (Quadrant II, cos −ve)

$\sec\theta = \frac{1}{\cos\theta}$

3. $\sec\theta = -13/12$

4. $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\operatorname{cosec}\theta = \frac{13}{5}$

5. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{5/13}{-12/13}$

$\tan\theta = -5/12$

6. $\cot\theta = 1/\tan\theta$

$\cot\theta = -12/5$

Weaker responses reflected that the candidates applied wrong trigonometric identities or formulae and failed to find the trigonometric ratio. In few other responses, it was noted that the candidates failed to perform simple arithmetic operations and have no clue about the sign of trigonometric ratio in the given quadrant.

Example 1:

Since θ [illegible] is +ve and not in quad I it is in quad II

$x + y = 1$

$x + \frac{5}{13} = 1$ $\qquad \sin\theta = \frac{5}{13}, \quad \csc\theta = \frac{13}{5}$

$x = 1 - \frac{5}{13}$ $\qquad \cos\theta = \frac{8}{13}, \quad \sec\theta = \frac{13}{8}$

$x = \cos\theta = \frac{8}{13}$ $\qquad \tan\theta = \frac{5}{8}, \quad \cot\theta = \frac{8}{5}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\frac{5}{13}}{\frac{8}{13}} = \frac{5}{8}$

Example 2:

$\boxed{\operatorname{cosec}\theta = \frac{13}{5}}$

$1 + \operatorname{cosec}^2\theta = \cot^2\theta$

$1 + \left(\frac{13}{5}\right)^2 = \cot^2\theta$

$1 + \frac{169}{25} = \cot^2\theta$

$\frac{25 + 169}{25} = \cot^2\theta$

$\sqrt{\frac{194}{25}} = \sqrt{\cot^2\theta}$

$\boxed{\cot\theta = \frac{\sqrt{194}}{5}}$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta + \left(\frac{5}{13}\right)^2 = 1$

$\cos^2\theta = 1 - \frac{25}{169}$

$\cos^2\theta = \frac{169 - 25}{169}$

$\boxed{\cos\theta = \frac{12}{13}}$

$\boxed{\sec\theta = \frac{13}{12}}$

$\boxed{\tan\theta = \frac{5}{\sqrt{194}}}$

Question 8a ii:

Prove that $\dfrac{1-\cot^2\theta}{1+\cot^2\theta} = \sin^2\theta - \cos^2\theta.$

Better responses showed that the candidates used the left hand side and applied the correct trigonometric identities aptly to prove the given identity correctly. The candidates converted $\tan^2\theta$ and $\cot^2\theta$ terms of $\sin\theta$ and $\cos\theta$ to prove the given trigonometric equation.

Example 1:

$$1-\frac{\cos^2\theta}{\sin^2\theta} \div 1+\frac{\cos^2\theta}{\sin^2\theta}$$

$$\frac{\sin^2\theta-\cos^2\theta}{\sin^2\theta} \div \frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}$$

$$\sin^2\theta+\cos^2\theta = 1$$

$$= \frac{\sin^2\theta-\cos^2\theta}{\cancel{\sin^2\theta}} \times \frac{\cancel{\sin^2\theta}}{1}$$

$$\sin^2\theta-\cos^2\theta = \sin^2\theta-\cos^2\theta$$

Example 2:

Taking LHS

$$\frac{1-\cot^2\theta}{1+\cot^2\theta}$$

$$\frac{1-\frac{\cos^2\theta}{\sin^2\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}}$$

$$\frac{\frac{\sin^2\theta-\cos^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}$$

$$\frac{\sin^2\theta-\cos^2\theta}{\sin^2\theta+\cos^2\theta} \quad \therefore \sin^2\theta+\cos^2\theta=1$$

$$\frac{\sin^2\theta-\cos^2\theta}{1}$$

$$\sin^2\theta-\cos^2\theta$$

LHS = RHS proved!

Weaker responses applied wrong trigonometric formula or incorrectly performed basic arithmetic operations like cancellation, multiplication or division and failed to prove the required result.

Example 1:

$$\frac{1-\left(\frac{\cos}{\sin}\right)^2}{1+\left(\frac{\cos}{\sin}\right)^2} \qquad \frac{1-\cos\cdot\sin\theta}{1+\cos\cdot\sin\theta}$$

$$\frac{1-\frac{\cos}{\sin}}{1+\frac{\cos}{\sin}} \qquad \frac{1+\cos\cdot\sin\theta\cdot 1-\cos\cdot\sin}{\sin^2\theta-\cos^2\theta}$$

$$\frac{1-\cos\cdot\sin}{1+\cos\cdot\sin}$$

Example 2:

$$\frac{1-\frac{1}{\tan^2\theta}}{\csc^2\theta} = \frac{\tan^2\theta-1}{\tan^2\theta\sin^2\theta}$$

$$= \frac{\sec^2\theta-1-1}{\frac{\sin^4\theta}{\cos^2\theta}}$$

$$= \frac{\sec^2\theta-2}{\frac{\sin^4\theta}{\cos^2\theta}} = \sec^2\theta-2$$

Question 8b:

i. Prove that $\cos(\alpha+\beta)\times\cos(\alpha-\beta)=1-(\sin^2\alpha+\sin^2\beta)$.

Better responses showed that candidates used the formula of $\cos(\alpha+\beta)$ and $\cos(\alpha-\beta)$. They skilfully multiplied $\cos(\alpha+\beta)\times\cos(\alpha-\beta)$ using formula of a^2-b^2 and made correct conversion of $\cos^2\alpha$ and $\cos^2\beta$ into $1-\sin^2\alpha$ and $1-\sin^2\beta$ respectively and succeeded to prove the required result, i.e. $\cos(\alpha+\beta)\times\cos(\alpha-\beta)=1-(\sin^2\alpha+\sin^2\beta)$.

Example:

$(\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$(\cos\alpha\cos\beta)^2 - (\sin\alpha\sin\beta)^2$

$\cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta \qquad 1-\sin^2\alpha = \cos^2\alpha$

$(1-\sin^2\alpha)(1-\sin^2\beta) - \sin^2\alpha\sin^2\beta$

$(1-\sin^2\beta-\sin^2\alpha+\sin^2\alpha\sin^2\beta) - \sin^2\alpha\sin^2\beta$

$1-\sin^2\beta-\sin^2\alpha+\cancel{\sin^2\alpha\sin^2\beta}-\cancel{\sin^2\alpha\sin^2\beta}$

$1-\sin^2\beta-\sin^2\alpha$

$1-(\sin^2\beta+\sin^2\alpha)$

$1-(\sin^2\alpha+\sin^2\beta) = 1-(\sin^2\alpha+\sin^2\beta)$

Weaker responses showed the candidates lost marks mainly because of the incorrect use of formulae, i.e. $\cos(\alpha+\beta)$ and $\cos(\alpha-\beta)$. They made mistakes in in writing correct signs, multiplication process, conversion of $\cos^2\alpha$ and $\cos^2\beta$ into $1-\sin^2\alpha$ and $1-\sin^2\beta$ and therefore failed to prove the required result. Some other mistakes are noted in the process of simplification.

Example 1:

L.H.S $(\cos\alpha\cos\beta - \sin\alpha\sin\beta)\times(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$\cos^2\alpha\cos^2\beta - \cos\alpha\cos\beta\sin\alpha\sin\beta$

$- \cos\alpha\cos\beta\sin\alpha\sin\beta - \sin^2\alpha\sin^2\beta$

$= \cos^2\alpha\cos^2\beta - 2\cos\alpha\cos\beta -$

$\sin^2\alpha\sin^2\beta$

$= -\sin^2\alpha + 1 - \sin^2\beta$

$= +1 - \sin^2\alpha - \sin^2\beta$

$= 1 - (\sin^2\alpha + \sin^2\beta)$

Hence, it is proved that L.H.S = R.H.S

Example 2:

$(\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

Question 8b ii:

With the help of $\cos\alpha$, prove that $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$.

Better responses exhibited correct use of trigonometric formulae which led to prove the correct result as required in the given question.

Example:

Solution

As we know

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = 1 - \sin^2\alpha - \sin^2\alpha$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

putting $\alpha = \alpha/2$

$\cos 2(\alpha/2) = 1 - 2\sin^2\alpha/2$

$\cos\alpha = 1 - 2\sin^2\alpha/2$

$2\sin^2\alpha/2 = 1 - \cos\alpha$

$\sin^2\alpha/2 = \frac{1-\cos\alpha}{2}$

$\sin\alpha/2 = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

Proved.

Weaker responses exhibited that candidates used incorrect trigonometric formulae and failed to verify the required result.

Example 1:

$\cancel{\cos\alpha}$

$\sin^2\alpha/2 = 1 - \cos\alpha$

$\sin\alpha/2 = \sqrt{1-\cos\alpha}$

$\sin\alpha/2 = \sqrt{\frac{1-\cos\alpha}{2}}$

Proved

Example 2:

As $\cos\alpha/2 = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

$\sin\alpha/2 - 1 = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

$\sin\alpha/2 = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

Question 9:

This question offered a choice between part **a** and **b**. Candidates preferred to chose to attempt part **a.**

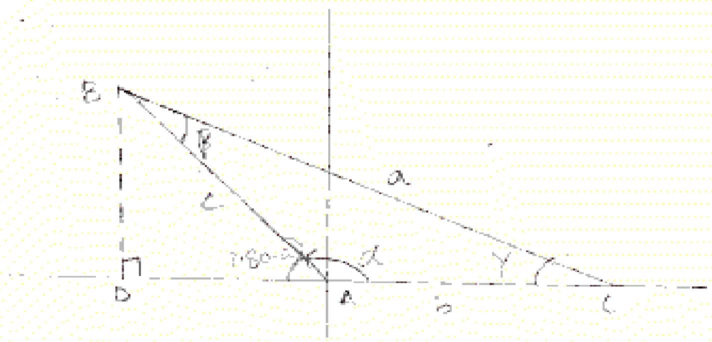
Question 9ai.

With the help of suitable diagram of an oblique triangle *ABC*, prove that $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$.

Better responses showed that in part i, the candidates drew the labelled diagram. The diagram provided them basis to correctly find the coordinate of the points labelled in the diagram. They skilfully changed the origin and compared the ordinate correctly to prove the given part of the law of sines.

Example:

Space for diagram



for an oblique triangle ~~pot~~ ~~mark~~ we mark a point in the coordinate plane such ~~so~~ that ~~α~~ ~~we~~ A (at origin) makes an angle α with the positive axis. A ~~comes~~ be comes ~~a~~ standard. By Dropping a perpendicular on point D, we can prove $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

from trigonometric ratios

~~since~~ $\sin(180-\alpha) = \frac{BD}{c}$

~~BD~~ $c\sin\alpha = \overline{BD}$ —(1) $\quad \because \sin(180-\alpha) = \sin\alpha$

for $\angle\gamma$

$\sin\gamma = \frac{BD}{a}$

$a\sin\gamma = \overline{BD}$ —(2)

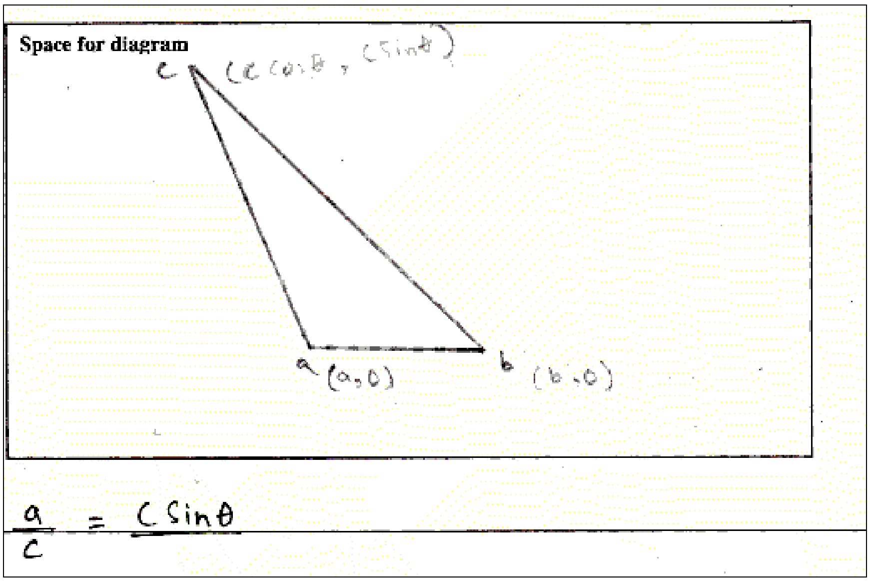
comparing both the equations

$c\sin\alpha = a\sin\gamma$

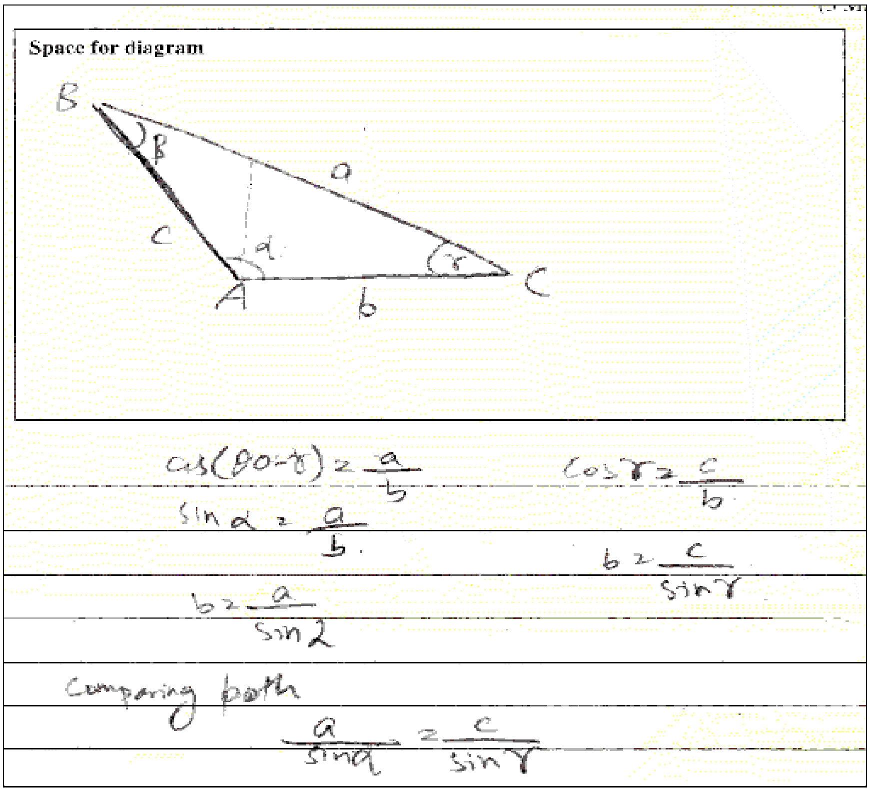
$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$ hence proved. for an oblique triangle ABC.

Weaker responses showed that most of the candidates were unable to draw the correct diagram and hence, failed to proceed further. In some cases, it was noted that the coordinates of the points were not in accordance with the diagram drawn for this purpose. Hence, it resulted in a variety of mistakes in their proof. Few common mistakes have been presented in the following examples.

Example 1:

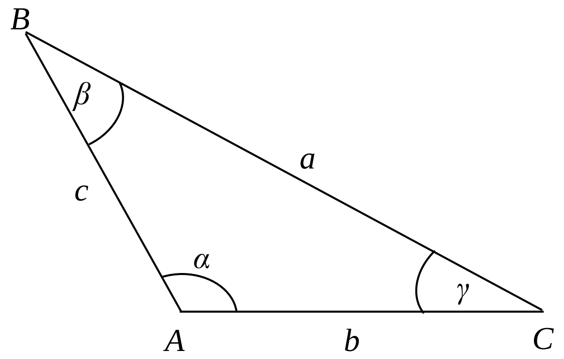


Example 2:



Question 9a:

ii. In the given diagram, find the value of a when b = 25 cm, $\beta = 35^{o}$ and $\gamma = 30^{o}$



NOT TO SCALE

Better responses exhibited that the candidates found the missing angle α followed by sine law to find the value of a. Candidates applied the formulae and simplified correctly to find the required value in the question.

Example 1:

$a = ?$

$b = 25$ $\quad\quad \alpha + \beta + \gamma = 180$

$\beta = 35^{o}$ $\quad\quad \alpha = 180 - (35 + 30)$

$\gamma = 30^{o}$ $\quad\quad \alpha = 115^{o}$

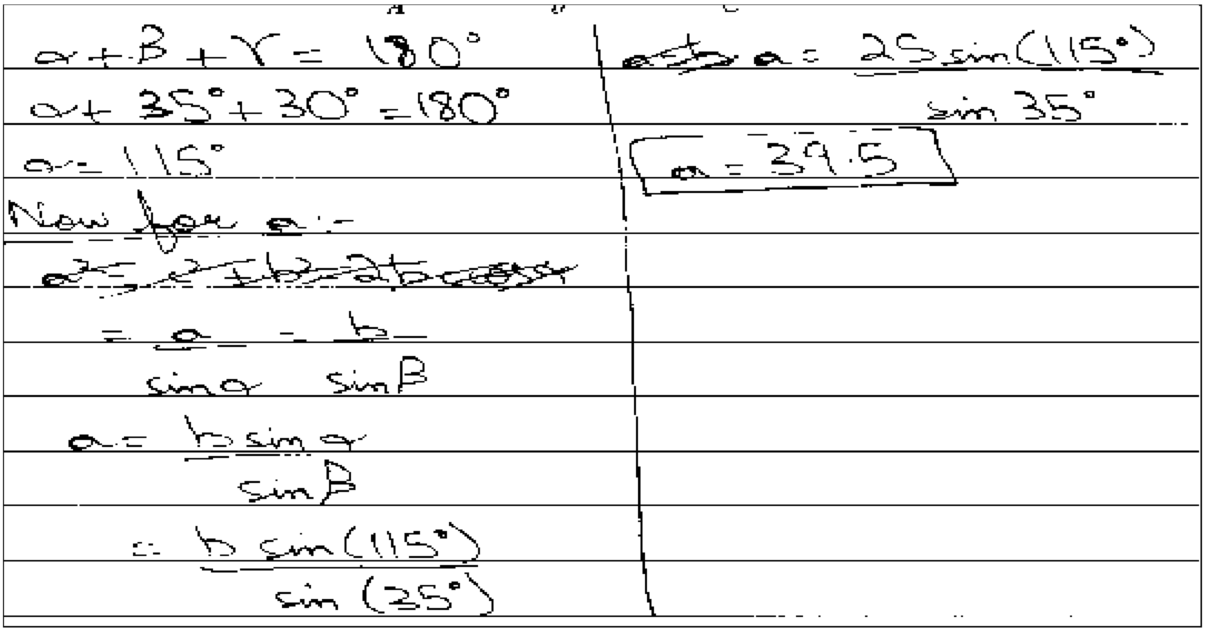
Apply law of sine

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\frac{a}{\sin(115)} = \frac{25}{\sin(35)}$$

$a = 39.50$ cm

Example 2:


$\alpha+\beta+\gamma = 180°$

$\alpha + 35° + 30° = 180°$

$\alpha = 115°$

Now for a:-

$\cancel{a^2 = c^2 + b^2 - 2bc\cos\alpha}$

$= \frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$a = \frac{b\sin\alpha}{\sin\beta}$

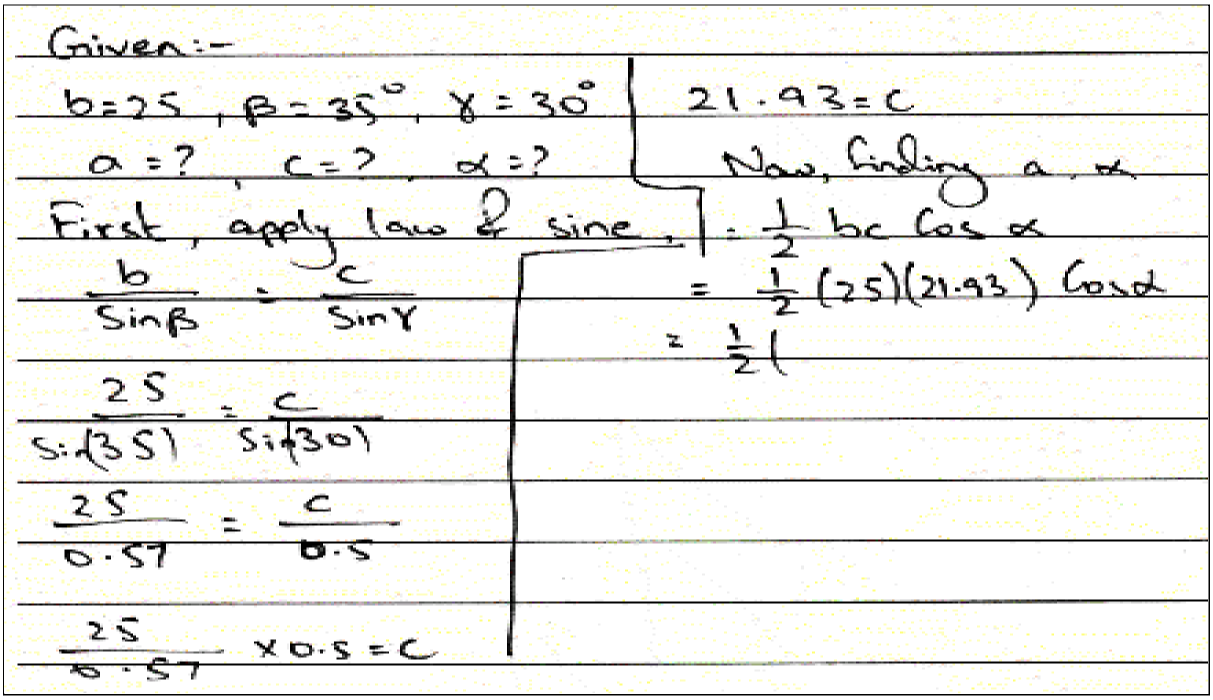
$= \frac{b\sin(115°)}{\sin(35°)}$

$a = \frac{25\sin(115°)}{\sin 35°}$

$\boxed{a = 39.5}$

Weaker responses reflected that the candidates failed to comprehend the situation given in the question and they made wrong choice of the formulae and failed to obtain the required value. In most of the weaker responses, it was noted that candidates chose the formula of area of triangle which is clear indication of lack of understanding and practice.

Example 1:


Given:-

$b=25$, $\beta = 35°$, $\gamma = 30°$

$a = ?$, $c = ?$, $\alpha = ?$

First, apply law of sine,

$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

$\frac{25}{\sin(35)} = \frac{c}{\sin(30)}$

$\frac{25}{0.57} = \frac{c}{0.5}$

$\frac{25}{0.57} \times 0.5 = c$

$21.93 = c$

Now, finding a, α

$= \frac{1}{2} bc \cos\alpha$

$= \frac{1}{2}(25)(21.93)\cos\alpha$

$= \frac{1}{2}($

Example 2:

$$a = \frac{1}{2}\, \frac{b^2 \sin\alpha \sin\gamma}{\sin\beta} \quad —(1)$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\alpha + 35^\circ + 30^\circ = 180^\circ$$

$$\alpha + 65^\circ = 180^\circ$$

$$\alpha = 180^\circ - 65^\circ$$

$$\alpha = 115^\circ$$

Subs. in (1)

$$a = \frac{1}{2}\, \frac{b^2 \sin\alpha \sin\gamma}{\sin\beta}$$

$$a = \frac{1}{2}\, \frac{(25)^2 \sin(115^\circ)\sin(30^\circ)}{\sin(35^\circ)}$$

$$= \frac{1}{2}\, \frac{625(0.906)(0.5)}{0.573}$$

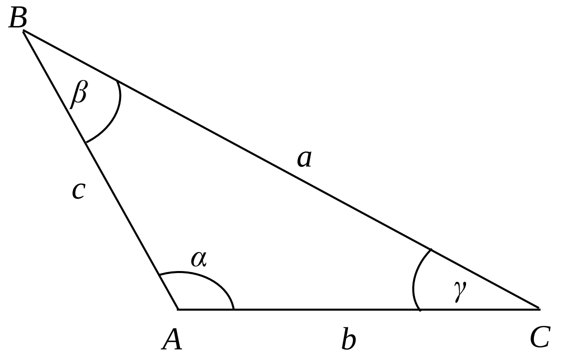
$$= \frac{283.125}{1.146}$$

$$\boxed{a = 247.05\text{cm}} \text{ Ans.}$$

Question 9b:

Prove that $\sin\dfrac{\beta}{2} = \sqrt{\dfrac{(s-c)(s-a)}{ac}}$.

(**Hint:** $\cos\beta = \cos\left(\dfrac{\beta}{2} + \dfrac{\beta}{2}\right)$ and $2s = a + b + c$)



It proved to be a difficult question for the candidates, although the required prove is given in the textbook, the only change in the question was instead of asking proof for $\sin\frac{\alpha}{2}$ the question required to prove it for $\sin\frac{\beta}{2}$.

Better responses exhibited that candidates selected the correct formulae and systematically followed the essential steps to establish the required proof. The question required grip on many formulae along with understanding to change in formulae as per given situation. It also required the use of arithmetic operation skillfully to arrive at the required result.

Example 1:

L.H.S =

using half angle formula

$\cos 2\beta = 1 - 2\sin^2\beta$

$\cos\beta = 1 - 2\sin^2\beta/2$

$2\sin^2\frac{\beta}{2} = 1 - \cos\beta$

$\sin^2\frac{\beta}{2} = \frac{1-\cos\beta}{2}$ $\qquad \because \cos\beta = \frac{a^2+c^2-b^2}{2ac}$

$\frac{\sin^2\beta}{2} = \frac{1 - \frac{a^2+c^2-b^2}{2ac}}{2}$

$\frac{\sin^2\beta}{2} = \frac{2ac - a^2 - c^2 + b^2}{4ac}$

$\frac{\sin^2\beta}{2} = \frac{-(a^2 - 2ac + c^2) + b^2}{4ac}$

$\frac{\sin^2\beta}{2} = \frac{-[(a-c)^2] + b^2}{4ac}$ $\qquad 2s = a+b+c$

$a = 2s - b - c$

$\frac{\sin^2\beta}{2} = \frac{-[(2s-b-2c)^2] + b^2}{4ac}$

$\sin^2\beta/2 = \frac{-(4s^2 + b^2 + 4c^2 - 4bs - 8cs + 4bc) + b^2}{4ac}$

$\sin^2\beta/2 = \frac{-(s^2 + c^2 - bs - 2cs + 2bc)}{ac}$

$\sin^2\beta/2 = \frac{-\{s^2 + c^2 - b(s-c)\} - 2cs)}{ac}$

$\sin^2\beta/2 = \frac{-(s^2 + c^2 + (2s - a + c - 2s)(s-c))}{ac}$

$\sin^2\beta/2 = \frac{-(s^2 + c^2 + as - ac + sc - c^2 - 2s^2)}{ac}$

$\sin^2\beta/2 = \frac{-(s^2 - s^2 + sc + as - ac)}{ac}$

$\sin^2\beta/2 = \frac{s^2 - sc - as + ac}{ac}$

$\frac{\sin^2\beta}{2} = \frac{s(s-c) - a(s-c)}{ac}$

$\sin^2\beta/2 = \frac{(s-a)(s-c)}{ac}$

$\sin\beta/2 = \sqrt{\frac{(s-a)(s-c)}{ac}}$ = R.H.S ---- proved.

Example 2:

$$2\sin^2\frac{\beta}{2} = 1-\cos\beta$$

$$2\sin^2\frac{\beta}{2} = 1-\frac{a^2+c^2-b^2}{2ca} \qquad \because b^2 = a^2+c^2-2ac\cos\beta$$

$$2\sin^2\frac{\beta}{2} = \frac{2ca-a^2-c^2+b^2}{2ca} \qquad 2ca\cos\beta = a^2+c^2-b^2$$

$$2\sin^2\frac{\beta}{2} = \frac{b^2-(a^2-2ac+c^2)}{2ca} \qquad \cos\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$2\sin^2\frac{\beta}{2} = \frac{b^2-(a-c)^2}{2ac} \qquad \because a^2-b^2=(a-b)(a+b)$$

$$\sin\frac{\beta}{2} = \frac{(b-a+c)(b+a-c)}{4ac} \qquad 2s = a+b+c$$

$$\sin^2\frac{\beta}{2} = \frac{(2s-a-a)(2s-c-c)}{4ac} \qquad \because 2s-a = b+c$$

$$\sin^2\frac{\beta}{2} = \frac{2(s-a)\cdot 2(s-c)}{4ac} \qquad \because 2s-c = a+b$$

$$\sin^2\frac{\beta}{2} = \frac{4(s-c)(s-a)}{4ac}$$

$(\sqrt{\ })$ on b/s

$$\sin\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{ac}}$$ Hence proved.

Weaker responses exhibited that candidates failed to write the formula correctly or made mistakes in substitution of values and hence, failed to complete the required proof. Two such examples are cited below.

Example 1:

$$\sin\frac{\beta}{2} = \sqrt{\frac{1-\cos\beta}{2}}$$

$$\sin\frac{\beta}{2} = \sqrt{\frac{1-\cos\left(\frac{a}{2}+\frac{b}{2}\right)}{2}}$$

$$\sin\frac{\beta}{2} = \sqrt{\frac{1-\cos\beta}{2}}$$

$$\sin\frac{\beta}{2} = \sqrt{\frac{2 = \cos\beta}{2}}$$

$$\sin\frac{\beta}{2} = \sqrt{\frac{a+b+c-2s}{2ac}}$$

$$\sin\frac{\beta}{2} = \sqrt{\frac{5(s-c)(s-a)}{5ac}}$$

$$\sin\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{ac}}$$

Example 2:

$$\left(\sin\frac{B}{2}\right) = \sqrt{\frac{1-\cos\alpha}{2}}$$

$$\left(\sin\frac{B}{2}\right)^2 = \left(\sqrt{\frac{1-\cos\alpha}{2}}\right)^2$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$2\sin^2\frac{\beta}{2} \quad \frac{1-\cos\alpha}{2}$$

$$2ac\cos\beta = a^2 + c^2 - b^2$$

$$\cos\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$2\sin^2\frac{\beta}{2} = 1 - \frac{a^2+c^2-b^2}{2ac}$$

$$S = \frac{a+b+c}{2}$$

$$2\sin^2\frac{\beta}{2} = \frac{2ac-a^2+c^2-b^2}{2ac}$$

$$2S = a+b+c$$

$$2\sin^2\frac{\beta}{2} = \frac{a^2-(c^2+b^2-2ac)}{2ac}$$

$$= a^2 - (2S-c)^2 - 2ac$$

$$= \sin\frac{B}{2} = \sqrt{\frac{(S-c)(S-a)}{ac}}$$

$$a^2 - (c+b)^2 - 2ac$$

Hence proved

$$a^2 - (c+b-b)$$

$$\sin\frac{B}{2}$$

$$a^2 - ($$

Question 10:

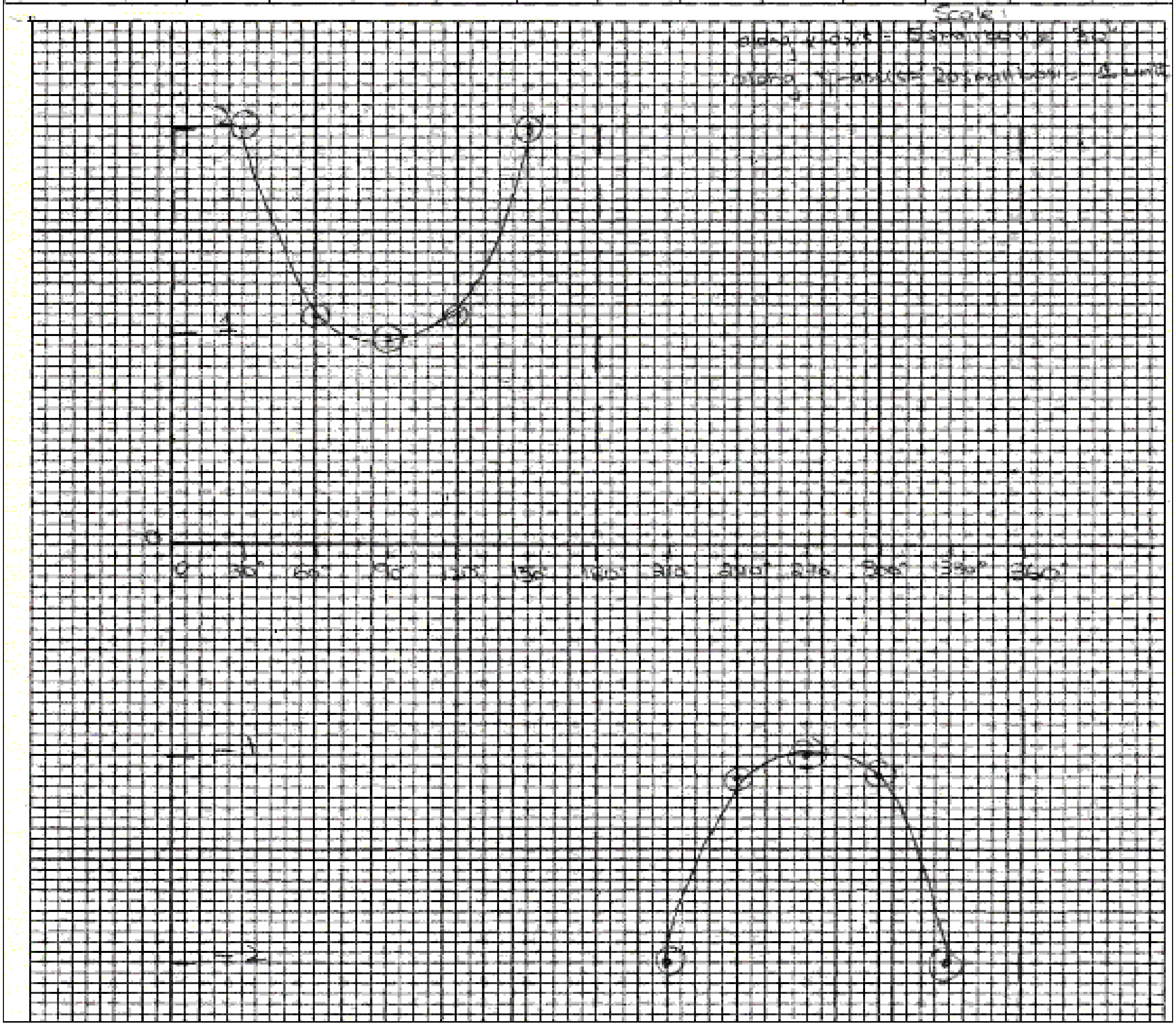
Complete the following table to draw the graph of $\operatorname{cosec}\theta$ on the given graph. Also write the range of the $\operatorname{cosec}\theta$.

θ	0	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\operatorname{cosec}\theta$													

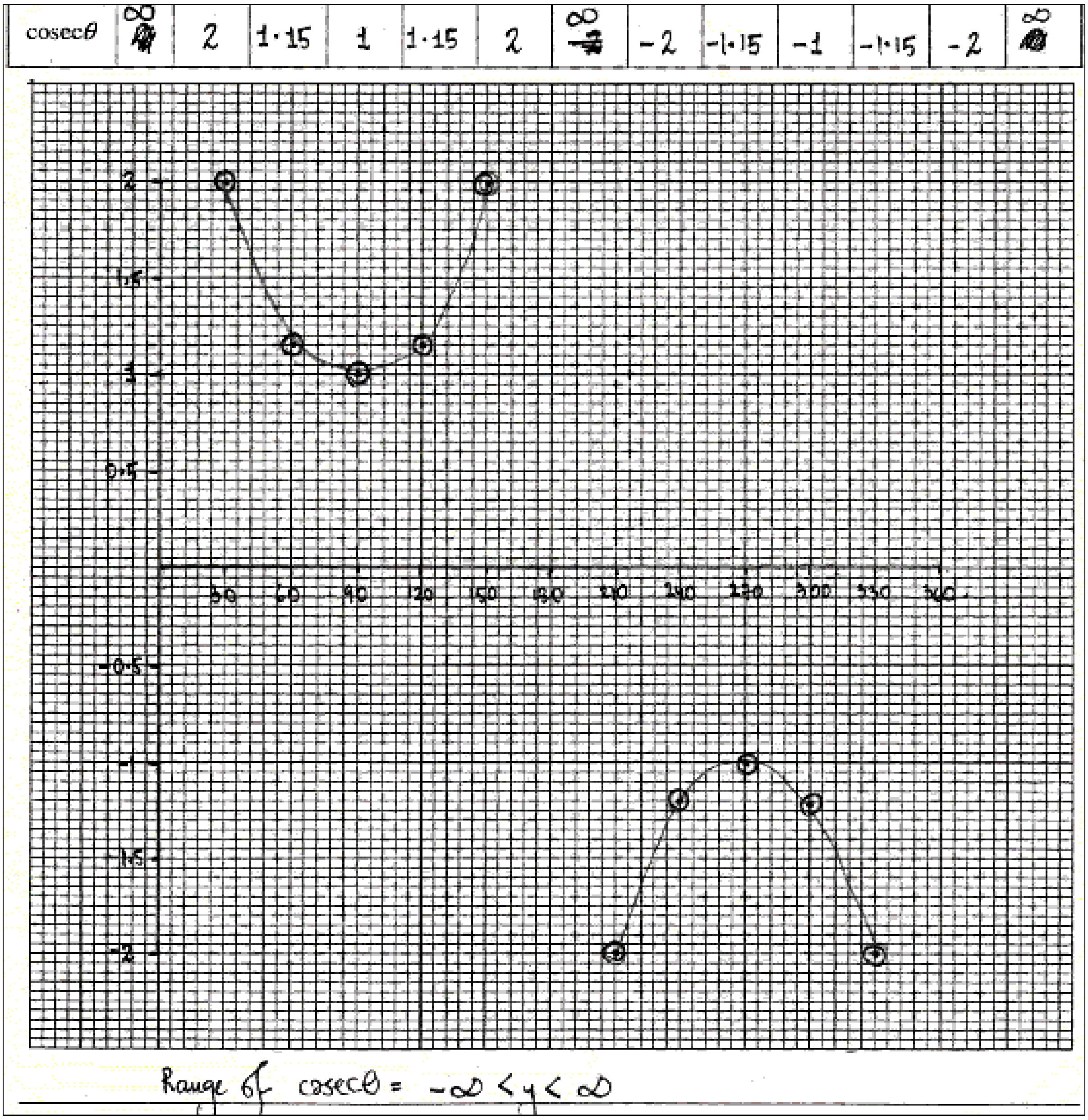
Better responses showed that candidates correctly calculated all values of $\cos ec\theta$ with the help of calculator and filled the given table. The candidates appropriately chose the scale on *x-axis* and *y-axis* and located the point on the given graph. Moreover, they skillfully marked the asymptotes of the graph. Also they were able to find the range of the $\cos ec\theta$.

Example 1:

θ	0	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
cosecθ	∞	2	1.154	1	1.154	2	∞	−2	−1.154	−1	−1.154	−2	∞



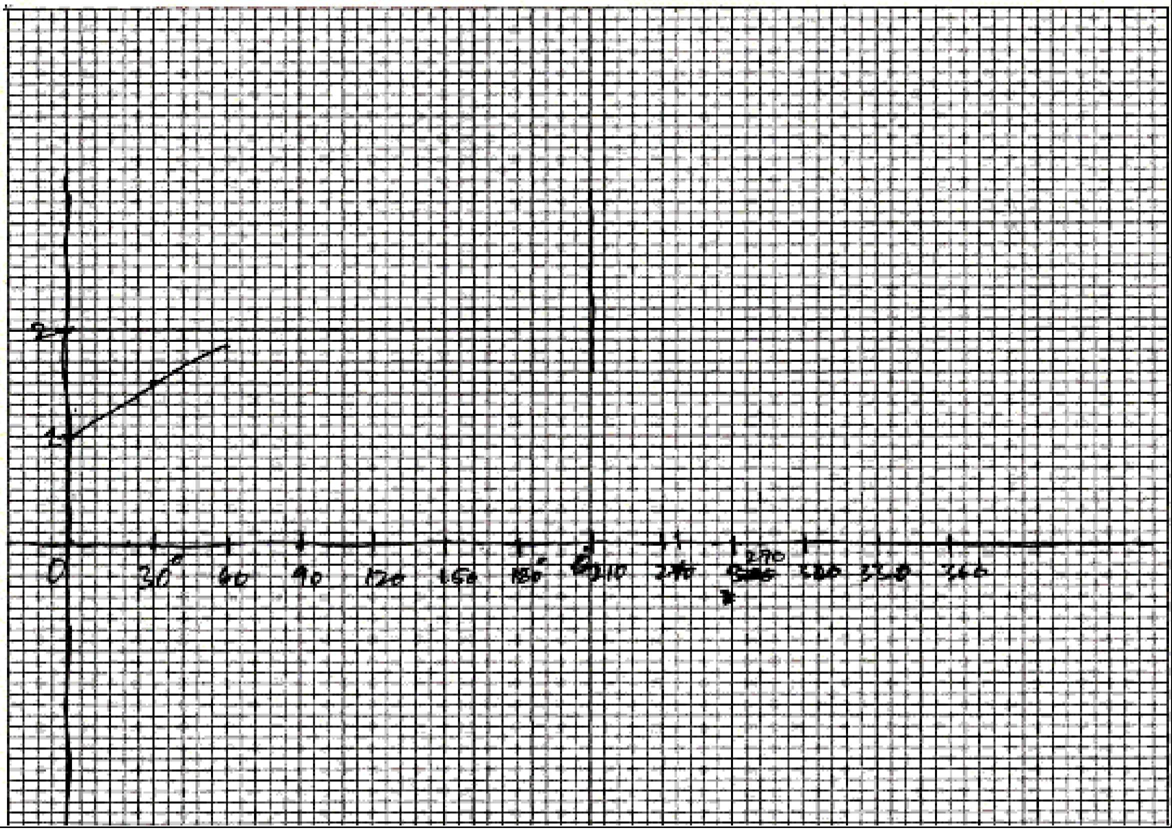
Example 2:



Weaker responses exhibited that candidates found the incorrect value of $\cos ec\theta$ for the given value of θ. Specifically, they failed to find the correct value of $\cos ec\theta$ at 0 ,180 and 360. Similarly, they failed to locate values of $\cos ec\theta$ and θ on the given graph paper. In a few responses, it was noted that candidates failed to select appropriate scale on *x-axis* and *y-axis*. It was also evident from the weaker responses that candidates were clueless about the asymptotes of the given graph.

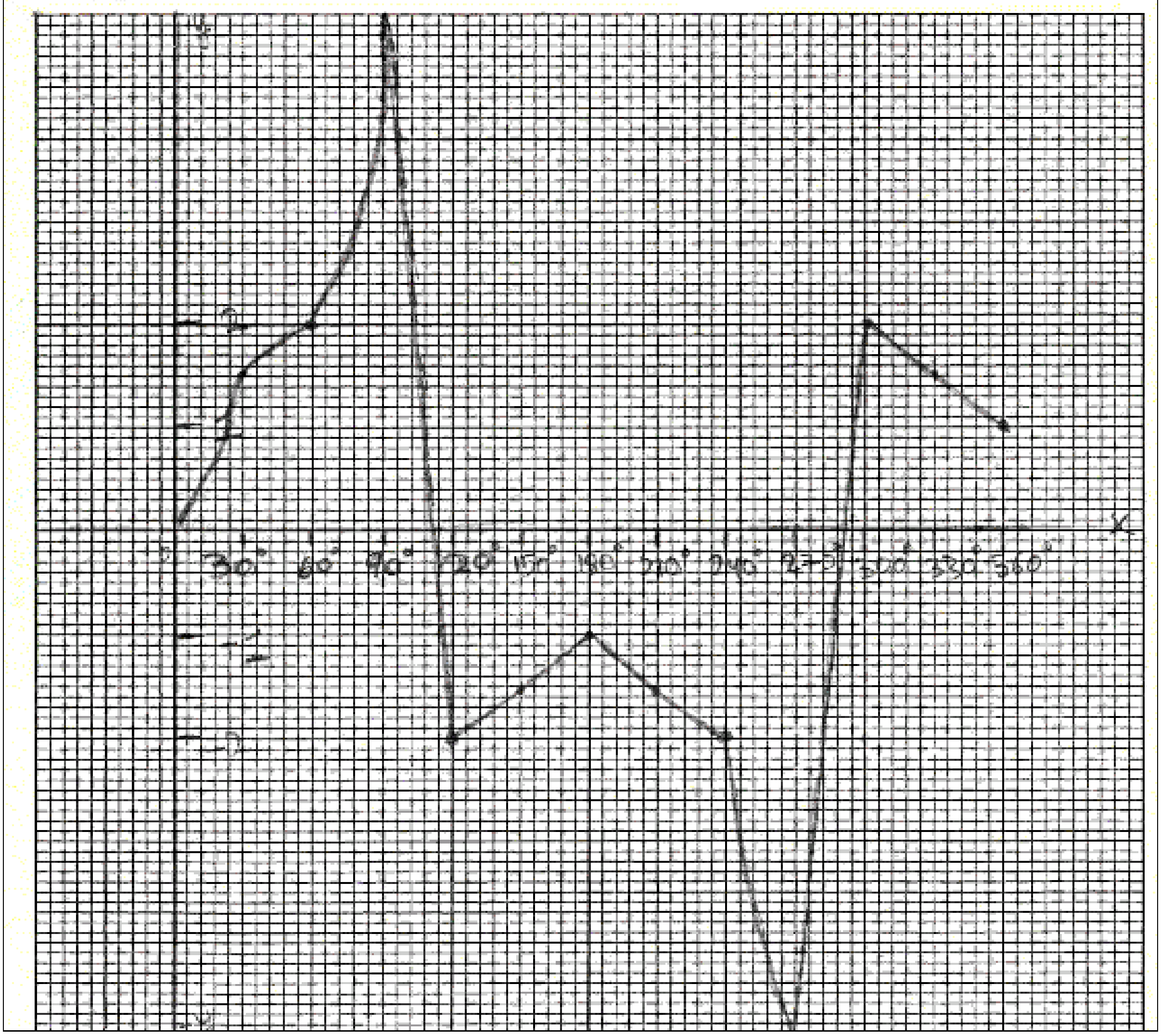
Example 1:

θ	0	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
cosecθ	1	1.15	1.86	2	1.15	−2	∝	−2	−1.1	−1	−1.15	−2	∝



Example 2:

θ	0	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
cosecθ	1	1.154	2	∞	-2	-1.154	-1	-1.154	-2	∞	2	1.154	1



Question 10b:

i. Find the solution set of the trigonometric equation $\sin 2x = \cos x$ when $0 \le x \le 2\pi$.

Better responses of part **i**, exhibited that candidates have good understanding of concept of solution of trigonometric equations. They converted the given trigonometric equation as $2\sin x\cos x - \cos x = 0$ and factorised it to get $\cos x = 0$ *or* $2\sin x - 1 = 0$. Then, they found the values satisfying the equation $\cos x = 0$ and $2\sin x - 1 = 0$. Finally, they wrote the required solution set.

Example 1:

$$2\sin x\cos x = \cos x \qquad \pi - \alpha = \pi - \frac{\pi}{6} = \frac{6\pi - \pi}{6} = \frac{5\pi}{6}$$

$$2\sin x\cos x - \cos x = 0$$

$$\cos x(2\sin x - 1) = 0$$

$$\cos x = 0 \qquad 2\sin x = 1$$

$$x = \frac{\pi}{2} \qquad \sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$2\pi - \alpha = 2\pi - \frac{\pi}{2} = \frac{4\pi - \pi}{2} = \frac{3\pi}{2}$$

$$\left[\frac{\pi}{2} + 2n\pi\right] \cup \left[\frac{3\pi}{2} + 2\pi n\right) \cup \left[\frac{\pi}{6} + 2n\pi\right] \cup \left[\frac{5\pi}{6} + 2n\pi\right]$$

Example 2:

$$2\sin n\cos n - \cos n = 0 \qquad \because \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos n(2\sin n - 1) = 0 \qquad \cos n = 0, \quad \sin n = \frac{1}{2}$$

$$\cos^{-1}(0) = n \qquad \sin^{-1}(1/2) = n$$

$$n = \frac{\pi}{2}, \frac{3\pi}{2} \qquad n = \frac{\pi}{6}, \frac{5\pi}{6}$$

Solution set: $\left\{\left(\frac{\pi}{6} + 2n\pi\right) \cup \left(\frac{\pi}{2} + 2n\pi\right) \cup \left(\frac{5\pi}{6} + 2\pi n\right) \cup \left(\frac{3}{2} + 2n\pi\right)\right\}$

Weaker responses showed that candidates failed to apply the correct technique to solve the given equation. In a few cases, candidates were able to find the principal angle; however, they failed to find the other angle satisfying the given trigonometric equation and consequently, failed to write the solution set of the equation. In some of the responses, it is noted that candidates had cancelled $\cos x$ in the equation $2\sin x\cos x = \cos x$, which is a pure misconception, as term containing variable cannot be cancelled under any condition while solving equations.

Example 1:

$\sin 2x = 2\cos x \sin x$

$2\cos x \sin x = \cos x$

÷ both side of equation by $\cos x$

$2\tan x = 1$

$\tan x = \frac{1}{2}$ as period of is π so

$x = \tan^{-1}\frac{1}{2}$ s.s $\{26.56, \pi + 26.56\}$ – solution set

$x = 26.56°$

Example 2:

$\sin 2x = \cos x$

$2\sin x \cos x = \cos x$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right)$ (+ve in QI & II)

$x = \frac{\pi}{6}$ (Q_1)

$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ (Q_2)

$\therefore S.S = \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\}$

Question 10b:

ii. Find the solution set of the trigonometric equation $\theta\tan^2\theta + 3\theta = 0$, when $0 \le \theta \le 2\pi$.

Better responses exhibited that candidates took θ common from the equation $\theta\tan^2\theta + 3\theta = 0$ and converted the given equation to $\theta(\tan^2\theta + 3) = 0$ and were able to find the values of θ satisfying the given equation. They were also cognizant to the fact that no value of θ may satisfy the equation $(\tan^2\theta) = -3$ and then, wrote the solution set of the given equation accordingly.

Example:

$$\theta(\tan^2\theta + 3) = 0$$

$\theta = 0$ or $\tan^2\theta + 3 = 0$

$$\tan^2\theta = -3$$

$$\tan\theta = \sqrt{-3}$$

$$\theta = \tan^{-1}\sqrt{-3}$$

not possible

$$S.S = \{0\}$$

Weaker responses exhibited that candidates were unable to take θ common and if they took it common, they again cancelled the variable which is not permissible. It is also noted that they found the value θ for the equation $(\tan^2\theta) = -3,$ although equation did not have any real solution.

Example 1:

$$\theta\tan^2\theta + 3\theta = 0$$

$$\theta\tan^2\theta = -3\theta$$

$$\tan^2\theta = -3$$

Hence Solution does not exist.

Example 2:

$$\theta(\tan^2\theta + 3) = 0$$

$$\tan^2\theta + 3 = 0$$

$$\tan^2\theta = -3$$

$$\tan\theta = \pm\sqrt{3}$$

$\theta = \tan^{-1}(\sqrt{3})$ $\therefore$ $\tan\theta$ is +ive in Ist and III quad

$\theta = 60°$ or $\pi/3$

Ist quad :- $\theta = R.A$ IIIrd quad = $\theta = \pi + R.A$

$\theta = \pi/3$ $\theta = \pi + \pi/3$

$\theta = 4\pi/3$

General solution set = $\{(\pi/3 + 2n\pi) \cup (4\pi/3 + 2n\pi)\}$